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Mathematical Model for Nonlinear Budget Constraint: Economic Activities on Increased Budget

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Abstract

In this study economic predictions of the various inputs are analyzed when the budget of the organization increases. Method of Lagrange multiplier is applied here to work with nonlinear budget constraint for the achievement of the profit maximization atmosphere. In the study 6×6 bordered Hessian matrix and 6×6 Jacobian matrix are also operated for the prediction of economic analysis. In mathematical economics, efficient and wise decisions can provide profit maximization setting, which is essential for the sustainability of the industrial organizations.

Keywords: Lagrange multiplier, nonlinear budget constraint, increased budget

1. Introduction

In modern economics, mathematical modeling becomes popular to the applied mathematicians (Samuelson, 1947). At present it becomes an essential part of many branches of social sciences, such as in economics, sociology, psychology, political science, etc. (Carter, 2001). Profit maximization practice is essential for the sustainability of an industrial firm (Eaton & Lipsey, 1975; Islam et al. 2010). Mathematics is extensively used in economics to solve optimization problems and many other problems of welfare economics (Zheng & Liu, 2022). To create profit maximization environment, an organization must be sincere in every step of its total operation, such as in production, financial balance, inventory, transportation, assignment, supply chain management, total management system, etc. (Ferdous & Mohajan, 2022; Mohajan & Mohajan, 2022a).

Lagrange multipliers method is a very useful and powerful practice in multivariable calculus that is applied as a device for transforming a constrained problem to a higher dimensional unconstrained problem (Baxley & Moorhouse, 1984). In this study we have used Cobb-Douglas production function as our profit function to discuss economic effects of future production procedures (Cobb & Douglas, 1928; Husain, 2012). In the study we have used the determinant of 6×6 bordered Hessian matrix, 6×6 Jacobian matrix, and four input variables to provide economic predictions precisely.

2. Literature Review

The literature review section is an introductory unit of any research that exhibits the works of previous researchers in the same field (Polit & Hungler, 2013). In 1928, two US scholars; mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976), have taken a bold attempt to derive a formula on production functions which is known as "*Cobb-Douglas production function*" (Cobb & Douglas, 1928). Later in 1984, another two US Professors; mathematician John V. Baxley and economist John C. Moorhouse have worked on the Cobb-Douglas production function for optimization (Baxley & Moorhouse, 1984). Professor Jamal Nazrul Islam (1939-2013) is an eminent mathematician of Bangladesh. He and his coauthors have discussed profit maximization for the welfare of the mathematical economics (Islam et al., 2009a,b, 2010, 2011).

Recently Jannatul Ferdous and Haradhan Kumar Mohajan have worked taking three inputs variables, such as capital, labor, and raw materials and other inputs on profit maximization of an industry (Ferdous & Mohajan, 2022). Professor Pahlaj Moolio and his coworkers have worked on the Cobb-Douglas production functions to analyze the mathematical structure of profit maximization and utility maximization (Moolio et al., 2009; Islam et al., 2011). Lia Roy and her coauthors have established a series of theorems with proofs in a cost minimization analysis paper (Roy et al., 2021). Devajit Mohajan and Haradhan Kumar Mohajan have worked on various types of optimization problems, such as sensitivity analyses of profit maximization, cost minimization, and utility maximization (Mohajan & Mohajan, 2022a-d, 2023a-d).

3. Research Methodology of the Study

Research is a hard-working search, scholarly inquiry, and investigation that aims to discover new facts and findings (Adams et al., 2007). In any kind of research, a researcher collects data and information, and then analyzes and interprets them efficiently to present a research paper or so on (Groh, 2018). Research always searches for truth and tries to develop the storehouse of human knowledge (Pandey & Pandey, 2015). It uses scientific methods to explain, predict, and control the observed phenomenon of a researcher (Babbie, 2017). Methodology is a systematic guideline for the accomplishment of a good research (Kothari, 2008). It tries to make relationship with the nature and power to science, truth, and epistemology (Ramazanoglu & Holland, 2002). It shows the research design and analysis procedures (Hallberg, 2006). Hence, we have realized that research methodology is the specific procedures that are used to identify, select, process, and analyze materials related to the research matters (Somekh & Lewin, 2005; Schwandt, 2014).

A well-developed outline of the study and an efficient understanding are essential to reach the goal of a research (Tie et al., 2019). To prepare this study we have used the mathematical logics and depended on the secondary data sources that are related to the profit maximization. We have also unsparingly consulted valuable articles and books of famous authors (Mohajan, 2017b, 2018a). To enrich this paper, we have managed some research materials from the internet and websites (Mohajan, 2017a, 2018b, 2020).

4. Objective of the Study

The principal objective of this article is to discuss the economic strategies of various inputs when the budget of the industry increases. Other minor related objectives of the study are as follows:

- to provide the mathematical calculations in some details,
- to give the economic predictions properly, and
- to show the physical significances efficiently.

5. Lagrange Function

We consider that an organization tries to make a maximum profit from its products and it wants to establish a sustainable environment in the economic world. Let the organization uses ℓ_1 amount of capital, ℓ_2 quantity of labor, ℓ_3 quantity of principal raw materials, and ℓ_4 quantity of irregular raw material for its usual production process. Let us consider the Cobb-Douglas production function $f(\ell_1, \ell_2, \ell_3, \ell_4)$ as a profit function for our economic model (Cobb & Douglas, 1928; Islam et al., 2011; Mohajan & Mohajan, 2022c),

$$P(\ell_1, \ell_2, \ell_3, \ell_4) = f(\ell_1, \ell_2, \ell_3, \ell_4) = A\ell_1^a \ell_2^b \ell_3^c \ell_4^d, \tag{1}$$

where A is the efficiency parameter that reflects the level of technology, i.e., technical process, economic system, etc., which represents total factor productivity. Moreover, A reflects the skill and efficient level of the workforce. Here a, b, c, and d are parameters; a indicates the output of elasticity of capital, and measures the percentage change in $P(\ell_1, \ell_2, \ell_3, \ell_4)$ for 1% change in ℓ_1, while ℓ_2, ℓ_3, and ℓ_4 are held constants. Similarly, b indicates the output of elasticity of labor, c indicates the output of elasticity of principal raw material, and d indicates the output of elasticity of irregular raw material. These four parameters a, b, c, and d must satisfy the following four inequalities (Islam et al., 2010; Moolio et al., 2009; Mohajan, 2022; Mohajan & Mohajan, 2023a):

$$0 < a < 1,\ 0 < b < 1,\ 0 < c < 1, \text{ and } 0 < d < 1. \tag{2}$$

A strict Cobb-Douglas production function, in which $\Delta = a + b + c + d > 1$ indicates increasing returns to scale, $\Delta = 1$ indicates constant returns to scale, and $\Delta < 1$ indicates decreasing returns to scale. Now we consider that the profit function is subject to a nonlinear budget constraint as (Roy et al., 2021; Mohajan & Mohajan, 2022c, 2023d),

$$B(\ell_1, \ell_2, \ell_3, \ell_4) = k\ell_1 + l\ell_2 + m\ell_3 + n(\ell_4)\ell_4, \tag{3}$$

where k is rate of interest or services of per unit of capital ℓ_1; l is the wage rate per unit of labor ℓ_2; m is the cost per unit of principal raw material ℓ_3; and n is the cost per unit of irregular raw material ℓ_4. In nonlinear budget equation (3) we consider (Moolio et al., 2009; Mohajan & Mohajan, 2023c),

$$n(\ell_4) = n_0\ell_4 - n_0, \tag{4}$$

where n_0 being the discounted price of the irregular input ℓ_4. Therefore, the nonlinear budget constraint (3) takes the form (Mohajan, 2021a; Mohajan & Mohajan, 2023b);

$$B(\ell_1, \ell_2, \ell_3, \ell_4) = k\ell_1 + l\ell_2 + m\ell_3 + n_0\ell_4^2 - n_0\ell_4. \tag{5}$$

We now formulate the maximization problem for the profit function (1) in terms of single Lagrange multiplier λ by defining the Lagrangian function $L(\ell_1, \ell_2, \ell_3, \ell_4, \lambda)$ as (Ferdous & Mohajan, 2022; Mohajan & Mohajan, 2023a),

$$L(\ell_1, \ell_2, \ell_3, \ell_4, \lambda) = A\ell_1^a\ell_2^b\ell_3^c\ell_4^d + \lambda\{B(\ell_1, \ell_2, \ell_3, \ell_4) - k\ell_1 - l\ell_2 - m\ell_3 - n_0\ell_4^2 + n_0\ell_4\}. \tag{6}$$

Relation (6) is a 5-dimensional unconstrained problem that is formed combining (1) and 4-dimensional constrained problem (3), where Lagrange multiplier λ, is considered as a device in our profit maximization model.

6. Four Variable Inputs

For maximization, first order differentiation equals to zero; then from (6) we can write (Islam et al., 2011; Mohajan, 2021c; Mohajan & Mohajan, 2022d),

$$L_\lambda = B - k\ell_1 - l\ell_2 - m\ell_3 - n_0\ell_4^2 + n_0\ell_4 = 0, \tag{7a}$$

$$L_1 = aA\ell_1^{a-1}\ell_2^b\ell_3^c\ell_4^d - \lambda k = 0, \tag{7b}$$

$$L_2 = bA\ell_1^a\ell_2^{b-1}\ell_3^c\ell_4^d - \lambda l = 0, \tag{7c}$$

$$L_3 = cA\ell_1^a\ell_2^b\ell_3^{c-1}\ell_4^d - \lambda m = 0, \tag{7d}$$

$$L_4 = dA\ell_1^a\ell_2^b\ell_3^c v_4^{d-1} - \lambda n_0(2\ell_4 - 1) = 0, \tag{7e}$$

where, $\frac{\partial L}{\partial \lambda} = L_\lambda$, $\frac{\partial L}{\partial \ell_1} = L_1$, $\frac{\partial L}{\partial \ell_2} = L_2$, etc. indicate first-order partial differentiations of multivariate Lagrangian function.

Using equations (2) to (7) we can determine the values of ℓ_1, ℓ_2, ℓ_3, and ℓ_4 as follows (Ferdous & Mohajan, 2022; Mohajan, 2021b; Mohajan & Mohajan, 2022c):

$$\ell_1 = \frac{aB}{k\Delta}, \tag{8a}$$

$$\ell_2 = \frac{bB}{l\Delta}, \tag{8b}$$

$$\ell_3 = \frac{cB}{m\Delta}, \tag{8c}$$

$$\ell_4 = \frac{dB}{n\Delta}. \tag{8d}$$

7. Bordered Hessian

Let us consider the determinant of the 5×5 bordered Hessian matrix as (Islam et al. 2010; Mohajan & Mohajan, 2023b),

$$|H| = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & L_{11} & L_{12} & L_{13} & L_{14} \\ -B_2 & L_{21} & L_{22} & L_{23} & L_{24} \\ -B_3 & L_{31} & L_{32} & L_{33} & L_{34} \\ -B_4 & L_{41} & L_{42} & L_{43} & L_{44} \end{vmatrix}. \tag{9}$$

Taking first-order partial differentiations of (5) we get,

$$B_1 = k, \quad B_2 = l, \quad B_3 = m, \text{ and } \quad B_4 = 2n_0\ell_4 - n_0. \tag{10}$$

Taking second-order and cross-partial derivatives of (6) we get (Roy et al., 2021; Mohajan & Mohajan, 2023b),

$$L_{11} = a(a-1)A\ell_1^{a-2}\ell_2^b\ell_3^c\ell_4^d,$$

$$L_{22} = b(b-1)A\ell_1^a\ell_2^{b-2}\ell_3^c\ell_4^d,$$

$$L_{33} = c(c-1)A\ell_1^a\ell_2^b\ell_3^{c-2}\ell_4^d,$$

$$L_{44} = d(d-1)A\ell_1^a\ell_2^b\ell_3^c\ell_4^{d-2},$$

$$L_{12} = L_{21} = abA\ell_1^{a-1}\ell_2^{b-1}\ell_3^c\ell_4^d,$$

$$L_{13} = L_{31} = acA\ell_1^{a-1}\ell_2^b\ell_3^{c-1}\ell_4^d,$$

$$L_{14} = L_{41} = adA\ell_1^{a-1}\ell_2^b\ell_3^c\ell_4^{d-1}, \tag{11}$$

$$L_{23} = L_{32} = bc\,A\ell_1^a\ell_2^{b-1}\ell_3^{c-1}\ell_4^d,$$

$$L_{24} = L_{42} = bd\ A\ell_1^a \ell_2^{b-1} \ell_3^c \ell_4^{d-1},$$

$$L_{34} = L_{43} = cd\ A\ell_1^a \ell_2^b \ell_3^{c-1} \ell_4^{d-1}.$$

where $\frac{\partial^2 L}{\partial \ell_1 \partial \ell_2} = L_{12} = L_{21}$, $\frac{\partial^2 L}{\partial \ell_2^2} = L_{22}$, etc. indicate cross-partial, second order differentiations of multivariate Lagrangian function, respectively, etc.

Now we expand the Hessian (9) as $|H| > 0$ (Moolio et al., 2009; Mohajan et al., 2013; Mohajan & Mohajan, 2023c),

$$|H| = \frac{A^3 abcdA\ell_1^{3a} \ell_2^{3b} \ell_3^{3c} \ell_4^{3d} B^2}{\ell_1^2 \ell_2^2 \ell_3^2 \ell_4^2 \Delta^2} (a+b+c+d)(d+3) > 0, \tag{12}$$

where efficiency parameter, $\Delta > 0$, and budget of the firm, $B > 0$; ℓ_1, ℓ_2, ℓ_3, and ℓ_4 are four different types of inputs; and consequently, $\ell_1, \ell_2, \ell_3, \ell_4 > 0$. Parameters, $a, b, c, d > 0$; also in the model either $0 < \Delta = a+b+c+d < 1$, $\Delta = 1$ or $\Delta > 1$. Hence, equation (12) gives; $|H| > 0$ (Islam et al., 2010; Mohajan & Mohajan, 2022c, 2023a).

8. Lagrange Multiplier λ

Now using the necessary values from (8) in (7a) we get (Roy et al., 2021; Mohajan & Mohajan, 2023b),

$$B = \frac{aA\ell_1^a \ell_2^b \ell_3^c \ell_4^d}{\lambda} + \frac{bA\ell_1^a \ell_2^b \ell_3^c \ell_4^d}{\lambda} + \frac{cA\ell_1^a \ell_2^b \ell_3^c \ell_4^d}{\lambda} + \frac{dA\ell_1^a \ell_2^b \ell_3^c \ell_4^d}{\lambda}$$

$$\lambda = \frac{A\ell_1^a \ell_2^b \ell_3^c \ell_4^d \Delta}{B}. \tag{13}$$

9. Jacobian

We have observed that the second-order condition is satisfied, so that the determinant of (5) survives at the optimum, i.e., $|J| = |H|$; and hence, we can apply the implicit function theorem. Now we compute twenty-five partial derivatives, such as $\frac{\partial \lambda}{\partial k}$, $\frac{\partial \ell_1}{\partial k}$, $\frac{\partial \ell_3}{\partial l}$, $\frac{\partial \ell_4}{\partial B}$, etc. that are referred to as the comparative statics of the model (Chiang, 1984; Mohajan & Mohajan, 2022c).

Let $\mathbf{G}$ be the vector-valued function of ten variables $\lambda^*, \ell_1^*, \ell_2^*, \ell_3^*, \ell_4^*, k, l, m, n$, and *B*, and we define the function $\mathbf{G}$ for the point $(\lambda^*, \ell_1^*, \ell_2^*, \ell_3^*, \ell_4^*, k, l, m, n, B) \in R^{10}$, and take the values in R^5. By the Implicit Function Theorem of multivariable calculus, the equation (Mohajan, 2021b; Mohajan & Mohajan, 2022d, 2023d),

$$F(\lambda^*, \ell_1^*, \ell_2^*, \ell_3^*, \ell_4^*, k, l, m, n, B) = 0, \tag{14}$$

may be solved in the form of

$$\begin{bmatrix} \lambda \\ \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{bmatrix} = \mathbf{G}(k, l, m, n, B). \tag{15}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}(k, l, m, n, B)$; regarded as $J_G = \dfrac{\partial(\lambda, \ell_1, \ell_2, \ell_3, \ell_4)}{\partial(k, l, m, n_0, B)}$, and is represented by;

$$J_G = \begin{bmatrix} \frac{\partial\lambda}{\partial k} & \frac{\partial\lambda}{\partial l} & \frac{\partial\lambda}{\partial m} & \frac{\partial\lambda}{\partial n_0} & \frac{\partial\lambda}{\partial B} \\ \frac{\partial\ell_1}{\partial k} & \frac{\partial\ell_1}{\partial l} & \frac{\partial\ell_1}{\partial m} & \frac{\partial\ell_1}{\partial n_0} & \frac{\partial\ell_1}{\partial B} \\ \frac{\partial\ell_2}{\partial k} & \frac{\partial\ell_2}{\partial l} & \frac{\partial\ell_2}{\partial m} & \frac{\partial\ell_2}{\partial n_0} & \frac{\partial\ell_2}{\partial B} \\ \frac{\partial\ell_3}{\partial k} & \frac{\partial\ell_3}{\partial l} & \frac{\partial\ell_3}{\partial m} & \frac{\partial\ell_3}{\partial n_0} & \frac{\partial\ell_3}{\partial B} \\ \frac{\partial\ell_4}{\partial k} & \frac{\partial\ell_4}{\partial l} & \frac{\partial\ell_4}{\partial m} & \frac{\partial\ell_4}{\partial n_0} & \frac{\partial\ell_4}{\partial B} \end{bmatrix}. \tag{16}$$

$$= -J^{-1} \begin{bmatrix} -\ell_1 & -\ell_2 & -\ell_3 & -\ell_4^2 + \ell_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda\ell_4 + \lambda & 0 \end{bmatrix}. \tag{17}$$

The inverse of Jacobian matrix is, $J^{-1} = \dfrac{1}{|J|} C^T$, where $C = (C_{ij})$, the matrix of cofactors of J, where T for transpose, then (17) becomes (Moolio et al., 2009; Roy et al., 2021; Mohajan, 2021a),

$$= -\frac{1}{|J|} \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix} \begin{bmatrix} -\ell_1 & -\ell_2 & -\ell_3 & -\ell_4^2 + \ell_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda\ell_4 + \lambda & 0 \end{bmatrix}. \tag{18}$$

$$J_G = -\frac{1}{|J|}\begin{bmatrix} -\ell_1C_{11}-\lambda C_{21} & -\ell_2C_{11}-\lambda C_{31} & -\ell_3C_{11}-\lambda C_{41} & -\ell_4^2C_{11}+\ell_4C_{11}-2\lambda\ell_4C_{51}+\lambda C_{51} & C_{11} \\ -\ell_1C_{12}-\lambda C_{22} & -\ell_2C_{12}-\lambda C_{32} & -\ell_3C_{12}-\lambda C_{42} & -\ell_4^2C_{12}+\ell_4C_{12}-2\lambda\ell_4C_{52}+\lambda C_{52} & C_{12} \\ -\ell_1C_{13}-\lambda C_{23} & -\ell_2C_{13}-\lambda C_{33} & -\ell_3C_{13}-\lambda C_{43} & -\ell_4^2C_{13}+\ell_4C_{13}-2\lambda\ell_4C_{53}+\lambda C_{53} & C_{13} \\ -\ell_1C_{14}-\lambda C_{24} & -\ell_2C_{14}-\lambda C_{34} & -\ell_3C_{14}-\lambda C_{44} & -\ell_4^2C_{14}+\ell_4C_{14}-2\lambda\ell_4C_{54}+\lambda C_{54} & C_{14} \\ -\ell_1C_{15}-\lambda C_{25} & -\ell_2C_{15}-\lambda C_{35} & -\ell_3C_{15}-\lambda C_{45} & -\ell_4^2C_{15}+\ell_4C_{15}-2\lambda\ell_4C_{55}+\lambda C_{55} & C_{15} \end{bmatrix}. \quad (19)$$

In (19) there are total 25 comparative statics, and in this study we shall deal only with five of them. We shall study the economic analysis of Lagrange multiplier when per unit costs of various inputs are increased. Now we consider that the organization always attempts for the profit maximization production (Baxley & Moorhouse, 1984; Islam et al., 2011).

10. Comparative Statics

Now we analyze the economic effects on Lagrange multiplier λ when the budget of the organization increases. Taking T_{15} from both sides of (19) we get (Moolio et al., 2009; Islam et al., 2011; Roy et al., 2021),

$$\frac{\partial\lambda}{\partial B} = -\frac{1}{J}\left[C_{11}\right]$$

$$= -\frac{1}{|J|}\text{Cofactor of } C_{11}$$

$$= -\frac{1}{|J|}\begin{vmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{vmatrix}$$

$$= -\frac{1}{|J|}\left\{ L_{11}\begin{vmatrix} L_{22} & L_{23} & L_{24} \\ L_{32} & L_{33} & L_{34} \\ L_{42} & L_{43} & L_{44} \end{vmatrix} - L_{12}\begin{vmatrix} L_{21} & L_{23} & L_{24} \\ L_{31} & L_{33} & L_{34} \\ L_{41} & L_{43} & L_{44} \end{vmatrix} + L_{13}\begin{vmatrix} L_{21} & L_{22} & L_{24} \\ L_{31} & L_{32} & L_{34} \\ L_{41} & L_{42} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \\ L_{41} & L_{42} & L_{43} \end{vmatrix} \right\}$$

$$= \frac{1}{|J|}\left[-L_{11}\left\{ L_{22}\left(L_{33}L_{44} - L_{43}L_{34}\right) + L_{23}\left(L_{42}L_{34} - L_{32}L_{44}\right) + L_{24}\left(L_{32}L_{43} - L_{42}L_{33}\right)\right\}\right.$$

$$- L_{12}\left\{ -L_{21}\left(L_{33}L_{44} - L_{43}L_{34}\right) + L_{23}\left(-L_{41}L_{34} + L_{31}L_{44}\right) + L_{24}\left(-L_{31}L_{43} + L_{41}L_{33}\right)\right\}$$

$$+ L_{13}\left\{ -L_{21}\left(L_{32}L_{44} - L_{42}L_{34}\right) + L_{22}\left(-L_{41}L_{34} + L_{31}L_{44}\right) + L_{24}\left(-L_{31}L_{42} + L_{41}L_{32}\right)\right\}$$

$$\left. - L_{14}\left\{ -L_{21}\left(L_{32}L_{43} - L_{42}L_{33}\right) + L_{22}\left(-L_{41}L_{33} + L_{31}L_{43}\right) + L_{23}\left(-L_{31}L_{42} + L_{41}L_{32}\right)\right\}\right]$$

$$= \frac{1}{|J|}\left\{ -L_{11}L_{22}L_{33}L_{44} + L_{11}L_{24}L_{42}L_{33} - L_{11}L_{24}L_{32}L_{43} + L_{11}L_{23}L_{32}L_{44} - L_{11}L_{23}L_{42}L_{34} + L_{11}L_{22}L_{43}L_{34}\right.$$

$$+ L_{12}L_{21}L_{33}L_{44} - L_{12}L_{21}L_{43}L_{34} + L_{12}L_{23}L_{41}L_{34} - L_{12}L_{23}L_{31}L_{44} + L_{12}L_{24}L_{31}L_{43} - L_{12}L_{24}L_{41}L_{33}$$

$$-L_{13}L_{21}L_{32}L_{44} + L_{13}L_{21}L_{42}L_{34} - L_{13}L_{22}L_{41}L_{34} + L_{13}L_{22}L_{31}L_{44} - L_{13}L_{24}L_{31}L_{42} + L_{13}L_{24}L_{41}L_{32}$$

$$+ L_{14}L_{21}L_{32}L_{43} - L_{14}L_{21}L_{42}L_{33} + L_{14}L_{22}L_{41}L_{33} - L_{14}L_{22}L_{31}L_{43} + L_{14}L_{23}L_{31}L_{42} - L_{14}L_{23}L_{41}L_{32}\}$$

$$= \frac{1}{|J|}\frac{A^4 \ell_1^{4a}\ell_2^{4b}\ell_3^{4c}\ell_4^{4d}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2}\{-a(a-1)b(b-1)c(c-1)d(d-1) + a(a-1)b^2c(c-1)d^2 - a(a-1)b^2c^2d^2$$

$$+a(a-1)b^2c^2d(d-1) - a(a-1)b^2c^2d^2 + a(a-1)b(b-1)c^2d^2 + a^2b^2c(c-1)d(d-1) - a^2b^2c^2d^2$$

$$+a^2b^2c^2d^2 - a^2b^2c^2d(d-1) + a^2b^2c^2d^2 - a^2b^2c(c-1)d^2 - a^2b^2c^2d(d-1) + a^2b^2c^2d^2$$

$$-a^2b(b-1)c^2d^2 + a^2b(b-1)c^2d(d-1) - a^2b^2c^2d^2 + a^2b^2c^2d^2 + a^2b^2c^2d^2 - a^2b^2c(c-1)d^2$$

$$-a^2b(b-1)c^2d^2 + a^2b^2c^2d^2 - a^2b^2c^2d^2\}$$

$$= \frac{1}{|J|}\frac{A^4 abcd\,\ell_1^{4a}\ell_2^{4b}\ell_3^{4c}\ell_4^{4d}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2}\{-(a-1)(b-1)(c-1)(d-1) + (a-1)bc(d-1) + ab(c-1)(d-1)$$

$$-2abc(d-1) + a(b-1)c(d-1) - 2ab(c-1)d + 3abcd - 2(a-1)bcd + (a-1)(b-1)cd$$

$$+(a-1)b(c-1)d - 3a(b-1)cd\}$$

$$\frac{\partial\lambda}{\partial B} = \frac{1}{|J|}\frac{A^4 abcd\,\ell_1^{4a}\ell_2^{4b}\ell_3^{4c}\ell_4^{4d}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2}\left(-2abcd + 2acd + abd + bd + a + b + c - 1\right). \tag{20}$$

Now we consider $a = b = c = d = \frac{1}{2}$ then we get, $\Delta = 2$, i.e., for increasing returns to scale, in (20) we get,

$$\frac{\partial\lambda}{\partial B} = \frac{A^4}{2^4|J|} > 0. \tag{21}$$

From the relation (21) we see that when budget of the organization increases, the level of Lagrange multiplier, i.e., marginal profit also increases. Hence, for increasing returns to scale profit maximization attempts may be successful, and organization may be sustainable.

Now we consider $a = b = c = d = \frac{1}{4}$ then we get, $\Delta = 1$, i.e., for constant returns to scale, in (20) we get,

$$\frac{\partial\lambda}{\partial B} = -\frac{19A^4}{2^{13}\ell_1\ell_2\ell_3\ell_4|J|} < 0. \tag{22}$$

From the relation (22) we see that when budget of the organization increases, the level of Lagrange multiplier decreases. Consequently, the organization faces difficulties on the way of sustainability. Hence, in this situation constant return to scale is not suitable for the organization.

Now we consider $a = b = c = \frac{1}{8}$ and $d = \frac{1}{2}$ then we get, $\Delta = \frac{7}{8}$, i.e., for decreasing returns to scale, in (20) we get,

$$\frac{\partial \lambda}{\partial B} = -\frac{277A^4}{2^7|J|} < 0. \tag{23}$$

From the relation (23) we see that it provides same property as in (22). Hence, both constant and decreasing returns to scale are not suitable for the sustainable environment of the organization when budget of the organization increases.

Now we analyze the economic effects of capital when budget of the organization increases. Taking T_{25} from both sides of (19) we get (Islam et al., 2011; Roy et al., 2021, Mohajan & Mohajan, 2022a, 2023b),

$$\frac{\partial \ell_1}{\partial B} = -\frac{1}{J}[C_{12}]$$

$$= -\frac{1}{|J|}\text{Cofactor of } C_{12}$$

$$= \frac{1}{|J|}\begin{vmatrix} -B_1 & L_{12} & L_{13} & L_{14} \\ -B_2 & L_{22} & L_{23} & L_{24} \\ -B_3 & L_{32} & L_{33} & L_{34} \\ -B_4 & L_{42} & L_{43} & L_{44} \end{vmatrix}$$

$$= \frac{1}{|J|}\left\{ -B_1\begin{vmatrix} L_{22} & L_{23} & L_{24} \\ L_{32} & L_{33} & L_{34} \\ L_{42} & L_{43} & L_{44} \end{vmatrix} - L_{12}\begin{vmatrix} -B_2 & L_{23} & L_{24} \\ -B_3 & L_{33} & L_{34} \\ -B_4 & L_{43} & L_{44} \end{vmatrix} + L_{13}\begin{vmatrix} -B_2 & L_{22} & L_{24} \\ -B_3 & L_{32} & L_{34} \\ -B_4 & L_{42} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} -B_2 & L_{22} & L_{23} \\ -B_3 & L_{32} & L_{33} \\ -B_4 & L_{42} & L_{43} \end{vmatrix}\right\}$$

$$= \frac{1}{|J|}\left[-B_1\{L_{22}(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(L_{42}L_{34} - L_{32}L_{44}) + L_{24}(L_{32}L_{43} - L_{42}L_{33})\}\right.$$

$$-L_{12}\{-B_2(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{43} + B_4L_{33})\}$$

$$+L_{13}\{-B_2(L_{32}L_{44} - L_{42}L_{34}) + L_{22}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{42} + B_4L_{32})\}$$

$$\left. -L_{14}\{-B_2(L_{32}L_{43} - L_{42}L_{33}) + L_{22}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{42} + B_4L_{32})\}\right]$$

$$= \frac{1}{|J|}\{-B_1L_{22}L_{33}L_{44} + B_1L_{22}L_{43}L_{34} - B_1L_{23}L_{42}L_{34} + B_1L_{23}L_{32}L_{44} - B_1L_{24}L_{32}L_{43} + B_1L_{24}L_{42}L_{33}$$

$$+B_2L_{12}L_{33}L_{44} - B_2L_{12}L_{43}L_{34} + B_4L_{12}L_{23}L_{34} - B_3L_{12}L_{22}L_{44} + B_3L_{12}L_{24}L_{43} - B_4L_{12}L_{24}L_{33} - B_2L_{13}L_{32}L_{44}$$

$$+B_2L_{13}L_{42}L_{34} - B_4L_{13}L_{22}L_{34} + B_3L_{13}L_{22}L_{44} - B_3L_{13}L_{24}L_{42} + B_4L_{13}L_{24}L_{32} + B_2L_{14}L_{32}L_{43} - B_2L_{14}L_{42}L_{33}$$

$$+ B_4L_{14}L_{22}L_{33} - B_3L_{14}L_{22}L_{43} + B_3L_{14}L_{23}L_{42} - B_4L_{14}L_{23}L_{32}\}$$

$$= \frac{1}{|J|}\frac{A^3\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2}\ \{-k\ell_1^2b(b-1)c(c-1)d(d-1) + k\ell_1^2b(b-1)c^2d^2 - k\ell_1^2b^2c^2d^2 + k\ell_1^2b^2c^2d(d-1)$$

$$-k\ell_1^2b^2c^2d^2 + k\ell_1^2b^2c(c-1)d^2 + l\ell_1\ell_2abc(c-1)d(d-1) - l\ell_1\ell_2abc^2d^2 + n\ell_1\ell_4ab^2c^2d$$

$$-m\ell_1\ell_3ab^2cd(d-1) + m\ell_1\ell_3ab^2cd^2 - n\ell_1\ell_4ab^2c(c-1)d - l\ell_1\ell_2abc^2d(d-1) + l\ell_1\ell_2abc^2d^2$$

$$-n\ell_1\ell_4ab(b-1)c^2d + m\ell_1\ell_3ab(b-1)cd(d-1) - m\ell_1\ell_3ab^2cd^2 + n\ell_1\ell_4ab^2c^2d + l\ell_1\ell_2abc^2d^2$$

$$-l\ell_1\ell_2abc(c-1)d^2 + n\ell_1\ell_4ab(b-1)c(c-1)d - m\ell_1\ell_3ab(b-1)cd^2 + m\ell_1\ell_3ab^2cd^2 - n\ell_1\ell_4ab^2c^2d\}$$

$$= \frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}B}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2\Delta}\ \{-(b-1)(c-1)(d-1) + b(c-1)(d-1) - bc(d-1) + (b-1)c(d-1)$$

$$+2(b-1)cd + (2\ell_4-1)bcd - (2\ell_4-1)bc(d-1) - (2\ell_4-1)(b-1)cd + (2\ell_4-1)(b-1)(c-1)d\}$$

$$\frac{\partial \ell_1}{\partial B} = \frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}B}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2\Delta}\ \{2\ell_4(bc-bd+d)+(bd-2d-3bc-2cd+1)\}. \tag{24}$$

Now we consider $a=b=c=d=\frac{1}{2}$ then we get, $\Delta = 2$, i.e., for increasing returns to scale, in (24) we get,

$$\frac{\partial \ell_1}{\partial B} = \frac{A^4B}{2^9|J|\sqrt{\ell_1\ell_2\ell_3\ell_4}}(\ell_4-2). \tag{25}$$

In (25) if $\ell_4 > 2$ we get,

$$\frac{\partial \ell_1}{\partial B} > 0. \tag{26}$$

From the inequality (26) we see that when budget of the organization increases, the capital of the organization also increases. We believe that for increasing returns to scale profit maximization is possible for this organization, and we think that the organization is in sustainable position.

In (25) if $\ell_4 < 2$ we get,

$$\frac{\partial \ell_1}{\partial B} < 0. \tag{27}$$

From the inequality (27) we see that when budget of the organization increases, the capital of the organization decreases. At this situation, the organization is not in profit maximization position. It should take future production decisions very carefully.

In (25) if $\ell_4 = 2$ we get,

$$\frac{\partial \ell_1}{\partial B} = 0. \tag{28}$$

From the equation (28) we see that when budget of the organization increases, there is no effect on the level of capital. Hence, capital and budget are mutually indifferent for this organization for $\ell_4 = 2$.

Now we consider $a = b = c = d = \frac{1}{4}$ then we get, $\Delta = 1$, i.e., for constant returns to scale, in (24) we get,

$$\frac{\partial \ell_1}{\partial B} = \frac{A^4 B}{2^9 |J| \ell_1^{\frac{5}{4}} \ell_2^{\frac{5}{4}} \ell_3^{\frac{5}{4}} \ell_4^{\frac{5}{4}}} (\ell_4 - 1). \tag{29}$$

In (29) if $\ell_4 > 1$ we get,

$$\frac{\partial \ell_1}{\partial B} > 0. \tag{30}$$

Inequality (30) provides the same result as in the inequality (26). In this situation, the organization may run to the profit maximization and it seems that the organization is in sustainable position.

In (29) if $\ell_4 < 1$ we get,

$$\frac{\partial \ell_1}{\partial B} < 0. \tag{31}$$

Inequality (31) gives the same result as the inequality (27). It seems that in both cases the organization is in unsustainable condition.

In (29) if $\ell_4 = 1$ we get,

$$\frac{\partial \ell_1}{\partial B} = 0. \tag{32}$$

Properties of equations (32) and (28) are same. In both cases there is no relation between budget and capital for this organization.

Now we consider $a = b = c = \frac{1}{8}$ and $d = \frac{1}{2}$ then we get, $\Delta = \frac{7}{8} < 1$, i.e., for decreasing returns to scale, in (24) we get,

$$\frac{\partial \ell_1}{\partial B} = \frac{A^4 B}{7 \times 2^{11} |J| \ell_1^{\frac{13}{8}} \ell_2^{\frac{13}{8}} \ell_3^{\frac{13}{8}} \ell_4^{\frac{5}{4}}} (5 - 14\ell_4). \tag{33}$$

In (33) if $\ell_4 > \frac{5}{14}$ we get,

$$\frac{\partial \ell_1}{\partial B} < 0. \tag{34}$$

From the inequality (34) we see that when the budget of the organization increases, the amount of capital decreases. In this situation the organization is not in profit maximization production procedure and increase of budget will not be beneficial for this organization.

In (33) if $\ell_4 < \frac{5}{14}$ we get,

$$\frac{\partial \ell_1}{\partial B} > 0. \tag{35}$$

From the inequality (35) we see that when the budget of the organization increases, the amount of capital also increases. We see that the organization may proceed to the profit maximization and the organization is in sustainable atmosphere.

In (33) if $\ell_4 = \frac{5}{14}$ we get,

$$\frac{\partial \ell_1}{\partial B} = 0 . \tag{36}$$

The equation (36) shows that budget and capital of the organization have no relation for $\ell_4 = \frac{5}{14}$.

Now we analyze the economic effects of wage rate when the budget of the organization increases. Taking T_{35} from both sides of (19) we get (Moolio et al., 2009; Roy et al., 2021, Mohajan, 2022a; Mohajan & Mohajan, 2022c, 2023b),

$$\frac{\partial \ell_2}{\partial B} = -\frac{1}{J}[C_{13}]$$

$$= -\frac{1}{|J|}\text{Cofactor of } C_{13}$$

$$= -\frac{1}{|J|}\begin{vmatrix} -B_1 & L_{11} & L_{13} & L_{14} \\ -B_2 & L_{21} & L_{23} & L_{24} \\ -B_3 & L_{31} & L_{33} & L_{34} \\ -B_4 & L_{41} & L_{43} & L_{44} \end{vmatrix}$$

$$= -\frac{1}{|J|}\left\{ -B_1\begin{vmatrix} L_{21} & L_{23} & L_{24} \\ L_{31} & L_{33} & L_{34} \\ L_{41} & L_{43} & L_{44} \end{vmatrix} - L_{11}\begin{vmatrix} -B_2 & L_{23} & L_{24} \\ -B_3 & L_{33} & L_{34} \\ -B_4 & L_{43} & L_{44} \end{vmatrix} + L_{13}\begin{vmatrix} -B_2 & L_{21} & L_{24} \\ -B_3 & L_{31} & L_{34} \\ -B_4 & L_{41} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} -B_2 & L_{21} & L_{23} \\ -B_3 & L_{31} & L_{33} \\ -B_4 & L_{41} & L_{43} \end{vmatrix} \right\}$$

$$= -\frac{1}{|J|}\left[-B_1\{L_{21}(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(L_{41}L_{34} - L_{31}L_{44}) + L_{24}(L_{31}L_{43} - L_{41}L_{33})\}\right.$$

$$- L_{11}\{-B_2(L_{33}L_{44} - L_{43}L_{34}) + L_{23}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{43} + B_4L_{33})\}$$

$$+ L_{13}\{-B_2(L_{31}L_{44} - L_{41}L_{34}) + L_{21}(-B_4L_{34} + B_3L_{44}) + L_{24}(-B_3L_{41} + B_4L_{31})\}$$

$$\left. - L_{14}\{-B_2(L_{31}L_{43} - L_{41}L_{33}) + L_{21}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{41} + B_4L_{31})\}\right]$$

$$= -\frac{1}{|J|}\{B_1L_{21}L_{33}L_{44} - B_1L_{21}L_{43}L_{34} + B_1L_{23}L_{41}L_{24} - B_1L_{23}L_{31}L_{44} + B_1L_{24}L_{31}L_{43} - B_1L_{24}L_{41}L_{33}$$

$$- B_2L_{11}L_{33}L_{44} + B_2L_{11}L_{43}L_{34} - B_4L_{11}L_{23}L_{34} + B_3L_{11}L_{23}L_{44} - B_3L_{11}L_{24}L_{43} + B_4L_{11}L_{24}L_{33} + B_2L_{13}L_{31}L_{44}$$

$$- B_2L_{13}L_{41}L_{34} + B_4L_{13}L_{21}L_{34} - B_3L_{13}L_{21}L_{44} + B_3L_{13}L_{24}L_{41} - B_4L_{13}L_{24}L_{31} - B_2L_{14}L_{31}L_{43} + B_2L_{14}L_{41}L_{33}$$

$$- B_4L_{14}L_{21}L_{33} + B_3L_{14}L_{21}L_{43} - B_3L_{14}L_{23}L_{41} + B_4L_{14}L_{23}L_{31}\}$$

$$= -\frac{1}{|J|}\frac{A^3\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2}\ \{k\ell_1\ell_2abc(c-1)d(d-1) - k\ell_1\ell_2abc^2d^2 + k\ell_1\ell_2abcd^2 - k\ell_1\ell_2abc^2d(d-1)$$

$$+k\ell_1\ell_2abcd^2 \quad -k\ell_1\ell_2abc(c-1)d^2 \quad -l\ell_2^2a(a-1)c(c-1)d(d-1) \quad +l\ell_2^2a(a-1)c^2d^2$$

$$-n\ell_2\ell_4a(a-1)bc^2d \ + m\ell_2\ell_3a(a-1)bcd(d-1) \ - m\ell_2\ell_3a(a-1)bcd^2 \ + n\ell_2\ell_4a(a-1)bc(c-1)d$$

$$+l\ell_2^2a^2c^2d(d-1) \ -l\ell_2^2a^2c^2d^2 \ +n\ell_2\ell_4a^2bc^2d \ -m\ell_2\ell_3a(a-1)bcd(d-1) \ +m\ell_2\ell_3a^2bcd^2$$

$$-n\ell_2\ell_4a^2bc^2d \quad -l\ell_2^2a^2c^2d^2 \quad +l\ell_2^2a^2c(c-1)d^2 \quad -n\ell_2\ell_4a^2bc(c-1)d \quad +m\ell_2\ell_3a^2bcd^2$$

$$-m\ell_2\ell_3a^2bcd^2 + n\ell_2\ell_4a^2bc^2d\}$$

$$= -\frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}}{\ell_1^2\ell_2\ell_3^2\ell_4^2}\ \{k\ell_1(c-1)(d-1) - k\ell_1c(d-1) + k\ell_1cd - k\ell_1(c-1)d + l\ell_2ab^{-1}c(d-1)$$

$$-2l\ell_2ab^{-1}cd \ + l\ell_2ab^{-1}(c-1)d \ - l\ell_2(a-1)b^{-1}(c-1)(d-1) + l_2\ell_2(a-1)b^{-1}cd \ + m\ell_3(a-1)(d-1)$$

$$+2m\ell_3ad \ -2m\ell_3a(a-1) + n\ell_4(a-1)(c-1) - n\ell_4(a-1)c - n\ell_4a(c-1) + n\ell_4ac\}$$

$$= -\frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}B}{\ell_1^2\ell_2\ell_3^2\ell_4^2\Delta}\ \{a(c-1)(d-1) \ -(a-1)(c-1)(d-1) \ +(a-1)c(d-1) \ -2ac(d-1)$$

$$+acd \quad +(a-1)cd \quad +(2\ell_4-1)(a-1)(c-1)d \quad -(2\ell_4-1)(a-1)cd \quad -(2\ell_4-1)a(c-1)d$$

$$+(2\ell_4-1)acd\}$$

$$\frac{\partial\ell_2}{\partial B} = -\frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}B}{\ell_1^2\ell_2\ell_3^2\ell_4^2\Delta}\left(2\ell_4d - acd + ac - cd - 2d + 1\right). \tag{37}$$

Now we consider $a = b = c = d = \frac{1}{2}$ then we get, $\Delta = 2$, i.e., for increasing returns to scale, in (37) we get,

$$\frac{\partial\ell_2}{\partial B} = \frac{A^3B\sqrt{\ell_2}}{2^8|J|\sqrt{\ell_1\ell_3\ell_4}}\left(1 - 8\ell_4\right). \tag{38}$$

In (38) if $\ell_4 < \frac{1}{8}$ we get,

$$\frac{\partial\ell_2}{\partial B} > 0. \tag{39}$$

From the inequality (39) we see that when budget of the organization increases, the wage rate also increases. We believe that for increasing returns to scale profit maximization is possible for this organization, and we think that the organization is in sustainable position.

In (38) if $\ell_4 > \frac{1}{8}$ we get,

$$\frac{\partial \ell_2}{\partial B} < 0. \tag{40}$$

From the inequality (40) we see that when budget of the organization increases, the wage rate decreases. We see that for the decreased wage rate the workers may leave the organization and consequently profit maximization may not be possible.

In (38) if $\ell_4 = \frac{1}{8}$ we get,

$$\frac{\partial \ell_2}{\partial B} = 0. \tag{41}$$

From the inequality (41) we see that when budget of the organization increases, there is no change of wage rate. We have realized that in this circumstance there is no relation between budget and wage rate.

Now we consider $a = b = c = d = \frac{1}{4}$ then we get, $\Delta = 1$, i.e., for constant returns to scale, in (37) we get,

$$\frac{\partial \ell_2}{\partial B} = -\frac{A^3 B}{2^{14}|J|\ell_1^{\frac{5}{4}}\ell_2^{\frac{1}{4}}\ell_3^{\frac{5}{4}}\ell_4^{\frac{5}{4}}}(32\ell_4 + 63) < 0. \tag{42}$$

From (42) we see that when the budget of the organization increases, the wage rate of the laborers decreases. Hence, organization faces various difficulties for the increased budget and it compel to decrease the wage rate for the sustainability in the local and global economic markets.

Now we consider $a = b = c = \frac{1}{8}$ and $d = \frac{1}{2}$ then we get, $\Delta = \frac{7}{8}$, i.e., for decreasing returns to scale, in (37) we get,

$$\frac{\partial \ell_2}{\partial B} = \frac{A^3 B}{2^{10}|J|\ell_1^{\frac{13}{8}}\ell_2^{\frac{5}{8}}\ell_3^{\frac{13}{8}}\ell_4^{\frac{1}{2}}}(7 - 128\ell_4). \tag{43}$$

In (43) if $\ell_4 < \frac{7}{128}$ we get,

$$\frac{\partial \ell_2}{\partial B} > 0. \tag{44}$$

From the inequality (44) we see that when budget of the organization increases, the wage rate increases too. Therefore, wage rate and budget of the organization are positively correlated. It seems that the organization should increase both of them for the profit maximization.

In (43) if $\ell_4 > \frac{7}{128}$ we get,

$$\frac{\partial \ell_2}{\partial B} < 0. \tag{45}$$

From (45) we see that when budget of the organization increases, the wage rate decreases. This is not happy news for the organization. In this situation the organization may proceed in production process very carefully with patient, otherwise it cannot sustain in the competitive global economy.

In (43) if $\ell_4 = \frac{7}{128}$ we get,

$$\frac{\partial \ell_2}{\partial B} = 0. \tag{46}$$

From (46) we see that when budget of the organization increases, there are no effects in wage rate. Hence, we observe that there is no relation between wage rate and budget.

Now we analyze the economic effects of principal raw material, when budget of the organization increases. Taking T_{45} from both sides of (19) we get (Islam et al., 2011; Roy et al., 2021; Wiese, 2021; Mohajan & Mohajan, 2023a),

$$\frac{\partial \ell_3}{\partial B} = -\frac{1}{J}\left[C_{14}\right]$$

$$= -\frac{1}{|J|}\text{Cofactor of } C_{14}$$

$$= \frac{1}{|J|}\begin{vmatrix} -B_1 & L_{11} & L_{12} & L_{14} \\ -B_2 & L_{21} & L_{22} & L_{24} \\ -B_3 & L_{31} & L_{32} & L_{34} \\ -B_4 & L_{41} & L_{42} & L_{44} \end{vmatrix}$$

$$= \frac{1}{|J|}\left\{ -B_1\begin{vmatrix} L_{21} & L_{22} & L_{24} \\ L_{31} & L_{32} & L_{34} \\ L_{41} & L_{42} & L_{44} \end{vmatrix} - L_{11}\begin{vmatrix} -B_2 & L_{22} & L_{24} \\ -B_3 & L_{32} & L_{34} \\ -B_4 & L_{42} & L_{44} \end{vmatrix} + L_{12}\begin{vmatrix} -B_2 & L_{21} & L_{24} \\ -B_3 & L_{31} & L_{34} \\ -B_4 & L_{41} & L_{44} \end{vmatrix} - L_{14}\begin{vmatrix} -B_2 & L_{21} & L_{22} \\ -B_3 & L_{31} & L_{32} \\ -B_4 & L_{41} & L_{42} \end{vmatrix}\right\}$$

$$= \frac{1}{|J|}\left[-B_1\left\{L_{21}\left(L_{32}L_{44} - L_{42}L_{34}\right) + L_{22}\left(L_{41}L_{34} - L_{31}L_{44}\right) + L_{24}\left(L_{31}L_{42} - L_{41}L_{32}\right)\right\}\right.$$

$$- L_{11}\left\{-B_2\left(L_{32}L_{44} - L_{42}L_{34}\right) + L_{22}\left(-B_4L_{34} + B_3L_{44}\right) + L_{24}\left(-B_3L_{42} + B_4L_{32}\right)\right\}$$

$$+ L_{12}\left\{-B_2\left(L_{31}L_{44} - L_{41}L_{34}\right) + L_{21}\left(-B_4L_{34} + B_3L_{44}\right) + L_{24}\left(-B_3L_{41} + B_4L_{31}\right)\right\}$$

$$\left. - L_{14}\left\{-B_2\left(L_{31}L_{42} - L_{41}L_{32}\right) + L_{21}\left(-B_4L_{32} + B_3L_{42}\right) + L_{22}\left(-B_3L_{41} + B_4L_{31}\right)\right\}\right]$$

$$= \frac{1}{|J|}\left\{ -B_1L_{21}L_{32}L_{44} + B_1L_{21}L_{42}L_{34} - B_1L_{22}L_{41}L_{34} + B_1L_{22}L_{31}L_{44} - B_1L_{24}L_{31}L_{42} + B_1L_{24}L_{41}L_{32}\right.$$

$$+ B_2L_{11}L_{32}L_{44} - B_2L_{11}L_{42}L_{34} + B_4L_{11}L_{22}L_{34} - B_3L_{11}L_{22}L_{44} + B_3L_{11}L_{24}L_{42} - B_4L_{11}L_{24}L_{32}$$

$$- B_2L_{12}L_{31}L_{44} + B_2L_{12}L_{41}L_{34} - B_4L_{12}L_{21}L_{34} + B_3L_{12}L_{21}L_{44} - B_3L_{12}L_{24}L_{41} + B_4L_{12}L_{24}L_{31}$$

$$\left. + B_2L_{14}L_{31}L_{42} - B_2L_{14}L_{41}L_{32} + B_4L_{14}L_{21}L_{32} - B_3L_{14}L_{21}L_{42} + B_3L_{14}L_{22}L_{41} - B_4L_{14}L_{22}L_{31}\right\}$$

$$= \frac{1}{|J|}\frac{A^3\ell_1^{3\alpha}\ell_2^{3\beta}\ell_3^{3\gamma}\ell_4^{3\delta}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2}\left\{ -k\ell_1\ell_3ab^2cd(d-1) + k\ell_1\ell_3ab^2cd^2 - k\ell_1\ell_3ab(b-1)cd^2 - k\ell_1\ell_3ab^2cd^2\right.$$

$$+ k\ell_1\ell_3ab(b-1)cd(d-1) + k\ell_1\ell_3ab^2cd^2 + l\ell_2\ell_3a(a-1)bcd(d-1) - l\ell_2\ell_3a(a-1)bcd^2$$

$$+n\ell_3\ell_4 a(a-1)b(b-1)cd \;-m\ell_3^2 a(a-1)b(b-1)d(d-1) \;+m\ell_3^2 a(a-1)b^2d^2 \;-n\ell_3\ell_4 a(a-1)b^2cd$$

$$-l\ell_2\ell_3 a^2bcd(d-1) \;+l\ell_2\ell_3 a^2bcd^2 \;-n\ell_3\ell_4 a^2b^2cd \;+m\ell_3^2 a^2b^2d(d-1) \;-m\ell_3^2 a^2b^2d^2$$

$$+n\ell_3\ell_4 a^2b^2cd \;+l\ell_2\ell_3 a^2bcd^2 \;-l\ell_2\ell_3 a^2bcd^2 \;+n\ell_3\ell_4 a^2b^2cd \;-m\ell_3^2 a^2b^2d^2 \;+m\ell_3^2 a^2b(b-1)d^2$$

$$-n\ell_3\ell_4 a^2b(b-1)cd\}$$

$$=\frac{1}{|J|}\frac{A^3abcd\ell_1^{3\alpha}\ell_2^{3\beta}\ell_3^{3\gamma}\ell_4^{3\delta}B}{\ell_1^2\ell_2^2\ell_3\ell_4^2\Delta}\;\{-ab(d-1)\;+a(b-1)(d-1)\;+(a-1)b(d-1)\;-(a-1)(b-1)(d-1)$$

$$+(2\ell_4-1)(a-1)(b-1)d\;-(2\ell_4-1)(a-1)bd\;+(2\ell_4-1)abd\;-(2\ell_4-1)a(b-1)d\}$$

$$\frac{\partial\ell_3}{\partial B}=\frac{1}{|J|}\frac{A^3abcd\ell_1^{3\alpha}\ell_2^{3\beta}\ell_3^{3\gamma}\ell_4^{3\delta}B}{\ell_1^2\ell_2^2\ell_3\ell_4^2\Delta}\left(2\ell_4 d-2d+1\right). \tag{47}$$

Now we consider $a=b=c=d=\frac{1}{2}$ then we get, $\Delta=2$, i.e., for increasing returns to scale, in (47) we get,

$$\frac{\partial\ell_3}{\partial B}=\frac{A^3B\sqrt{\ell_3\ell_4}}{2^5|J|\sqrt{\ell_1\ell_2}}>0. \tag{48}$$

From the relation (48) we see that when budget of the organization increases, the level of principal raw material also increases. Hence, it seems that the organization is in extreme position of sustainability, and can continue its production without any tension.

Now we consider $a=b=c=d=\frac{1}{4}$ then we get, $\Delta=1$, i.e., for constant returns to scale, in (47) we get,

$$\frac{\partial\ell_3}{\partial B}=\frac{A^3B}{2^8\ell_1^{\frac{5}{4}}\ell_2^{\frac{5}{4}}\ell_3^{\frac{1}{4}}\ell_4^{\frac{5}{4}}|J|}\left(\ell_4+1\right)>0. \tag{49}$$

From the relation (49) we see that when budget of the organization increases, the amount of principal raw material also increases. It seems that the organization is in profit maximization and can easily achieve sustainable atmosphere.

Now we consider $a=b=c=\frac{1}{8}$ and $d=\frac{1}{2}$, then we get, $\Delta=\frac{7}{8}<1$, i.e., for decreasing returns to scale, in (47) we get,

$$\frac{\partial\ell_3}{\partial B}=\frac{A^3B}{7\times 2^7\ell_1^{\frac{13}{8}}\ell_2^{\frac{13}{8}}\ell_3^{\frac{5}{8}}\ell_4^{\frac{5}{8}}|J|}>0. \tag{50}$$

From the relation (50) we see have obtained the same result as there in (48) and (49). Hence, the organization is in sustainable profit maximization stage at any situation.

Now we analyze the economic effects irregular raw material when the budget of the organization increases. Taking T_{55} from both sides of (19) we get (Wiese, 2021; Mohajan & Mohajan, 2023a),

$$\frac{\partial \ell_4}{\partial B} = -\frac{1}{J}[C_{15}]$$

$$= -\frac{1}{|J|}\text{Cofactor of } C_{15}$$

$$= -\frac{1}{|J|}\begin{vmatrix} -B_1 & L_{11} & L_{12} & L_{13} \\ -B_2 & L_{21} & L_{22} & L_{23} \\ -B_3 & L_{31} & L_{32} & L_{33} \\ -B_4 & L_{41} & L_{42} & L_{43} \end{vmatrix}$$

$$= -\frac{1}{|J|}\left\{ -B_1\begin{vmatrix} L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \\ L_{41} & L_{42} & L_{43} \end{vmatrix} - L_{11}\begin{vmatrix} -B_2 & L_{22} & L_{23} \\ -B_3 & L_{32} & L_{33} \\ -B_4 & L_{42} & L_{43} \end{vmatrix} + L_{12}\begin{vmatrix} -B_2 & L_{21} & L_{23} \\ -B_3 & L_{31} & L_{33} \\ -B_4 & L_{41} & L_{43} \end{vmatrix} - L_{13}\begin{vmatrix} -B_2 & L_{21} & L_{22} \\ -B_3 & L_{31} & L_{32} \\ -B_4 & L_{41} & L_{42} \end{vmatrix} \right\}$$

$$= -\frac{1}{|J|}\left[-B_1\{L_{21}(L_{32}L_{43} - L_{42}L_{33}) + L_{22}(L_{41}L_{33} - L_{31}L_{43}) + L_{23}(L_{31}L_{42} - L_{41}L_{32})\} \right.$$

$$- L_{11}\{-B_2(L_{32}L_{43} - L_{42}L_{33}) + L_{22}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{42} + B_4L_{32})\}$$

$$+ L_{12}\{-B_2(L_{31}L_{43} - L_{41}L_{33}) + L_{21}(-B_4L_{33} + B_3L_{43}) + L_{23}(-B_3L_{41} + B_4L_{31})\}$$

$$\left. - L_{13}\{-B_2(L_{31}L_{42} - L_{41}L_{32}) + L_{21}(-B_4L_{32} + B_3L_{42}) + L_{22}(-B_3L_{41} + B_4L_{31})\}\right]$$

$$= -\frac{1}{|J|}\{ -B_1L_{21}L_{32}L_{43} + B_1L_{21}L_{42}L_{33} - B_1L_{22}L_{41}L_{33} + B_1L_{22}L_{31}L_{43} - B_1L_{23}L_{31}L_{42} + B_1L_{23}L_{41}L_{32}$$

$$+ B_2L_{11}L_{32}L_{43} - B_2L_{11}L_{42}L_{33} + B_4L_{11}L_{22}L_{33} - B_3L_{11}L_{22}L_{43} + B_3L_{11}L_{23}L_{42} - B_4L_{11}L_{23}L_{32}$$

$$- B_2L_{12}L_{31}L_{43} + B_2L_{12}L_{41}L_{33} - B_4L_{12}L_{21}L_{33} + B_3L_{12}L_{21}L_{43} - B_3L_{12}L_{23}L_{41} + B_4L_{12}L_{23}L_{31}$$

$$+ B_2L_{13}L_{31}L_{42} - B_2L_{13}L_{41}L_{32} + B_4L_{13}L_{21}L_{32} - B_3L_{13}L_{21}L_{42} + B_3L_{13}L_{22}L_{41} - B_4L_{13}L_{22}L_{31}\}$$

$$= -\frac{1}{|J|}\frac{A^3\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2} \quad \{-k\ell_1\ell_4ab^2c^2d \quad + k\ell_1\ell_4ab^2c(c-1)d \quad - k\ell_1\ell_4ab(b-1)c(c-1)d$$

$$+ k\ell_1\ell_4ab(b-1)c(c-1)d \; - k\ell_1\ell_4ab^2c^2d \; + k\ell_1\ell_4ab^2c^2d \; + l\ell_2\ell_4a(a-1)bc^2d \; + l\ell_2\ell_4a^2bc(c-1)d$$

$$+ n\ell_4^2a(a-1)b(b-1)c(c-1) \; - m\ell_3\ell_4a(a-1)b(b-1)cd \; + m\ell_3\ell_4a(a-1)b^2cd \; - n\ell_4^2a(a-1)b^2c^2$$

$$- l\ell_2\ell_4a^2bc^2d \quad + l\ell_2\ell_4a^2bc(c-1)d \quad - n\ell_4^2a^2b^2c(c-1) \quad + m\ell_3\ell_4a^2b^2cd \quad - m\ell_3\ell_4a^2b^2cd$$

$$+n\ell_4^2a^2b^2c^2 \quad +l\ell_2\ell_4a^2bc^2d \quad -l\ell_2\ell_4a^2bc^2d \quad +n\ell_4^2a^2b^2c^2 \quad -m\ell_3\ell_4a^2b^2cd \quad +m\ell_3\ell_4a^2b(b-1)cd$$

$$-na^2b(b-1)c^2\Big\}$$

$$=-\frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}B}{\ell_1^2\ell_2^2\ell_3^2\ell_4\Delta}\ \{-3abc \quad +2(a-1)bc \quad +3ab(c-1) \quad -(a-1)(b-1)c \quad +a(b-1)c$$

$$-(2\ell_4-1)(a-1)bc \qquad +(2\ell_4-1)(a-1)(b-1)(c-1) \qquad -(2\ell_4-1)ab(c-1) \qquad +2(2\ell_4-1)abc$$

$$-(2\ell_4-1)a(b-1)c\}$$

$$\frac{\partial\ell_4}{\partial B}=-\frac{1}{|J|}\frac{A^3abcd\ell_1^{3a}\ell_2^{3b}\ell_3^{3c}\ell_4^{3d}B}{\ell_1^2\ell_2^2\ell_3^2\ell_4\Delta}\{2\ell_4(a+b+c-1)+(2abc-3ab-bc-a-b-2c+1)\}. \quad (51)$$

Now using $a=b=c=d=\frac{1}{4}$ then we get, $\Delta=1$, i.e., for constant returns to scale, in (51) we get,

$$\frac{\partial\ell_4}{\partial B}=\frac{A^3B}{2^{12}\ell_1^{\frac{5}{4}}\ell_2^{\frac{5}{4}}\ell_3^{\frac{5}{4}}\ell_4^{\frac{1}{4}}|J|}(8\ell_4+7)>0. \quad (52)$$

The inequality (52) indicates that if the budget of the organization increases; the amount of irregular input increases too. It seems that irregular input is an essential element, and the organization increases purchasing capacity in parallel to the increase of the budget.

Now we consider $a=b=c=\frac{1}{8}$ and $d=\frac{1}{2}$, then we get, $\Delta=\frac{7}{8}<1$, i.e., for decreasing returns to scale, in (51) we get,

$$\frac{\partial\ell_4}{\partial B}=\frac{A^3B\ell_4^{\frac{1}{2}}}{7\times2^{12}\ell_1^{\frac{13}{8}}\ell_2^{\frac{13}{8}}\ell_3^{\frac{13}{8}}|J|}(320\ell_4-113). \quad (53)$$

In equation (53) if $\ell_4>\frac{113}{320}$ we get,

$$\frac{\partial\ell_4}{\partial B}>0. \quad (54)$$

The inequality (54) provides the same result as in (52) for decreasing returns to scale. Hence, in this situation it seems that the organization is in better position for the economic sustainability.

In equation (53) if $\ell_4<\frac{113}{320}$ we get,

$$\frac{\partial\ell_4}{\partial B}<0. \quad (55)$$

The inequality (55) shows that when the budget of the organization increases the level of purchasing irregular input decreases. It seems that the irregular input may be not essential material or the organization is not in profit maximization condition.

In equation (53) if $\ell_4 = \frac{113}{320}$ we get,

$$\frac{\partial \ell_4}{\partial B} = 0. \tag{56}$$

The equation (56) provides that when the budget of the organization increases, there is no effect on the irregular input.

Now using $a = b = c = d = \frac{1}{2}$ then we get, $\Delta = 2$, i.e., for increasing returns to scale, in (51) we get,

$$\frac{\partial \ell_4}{\partial B} = \frac{A^3 B \ell_4^{\frac{51}{2}}}{2^7 \ell_1^{\frac{5}{4}} \ell_2^{\frac{5}{4}} \ell_3^{\frac{5}{4}} |J|} (4\ell_4 - 7). \tag{57}$$

In equation (53) if $\ell_4 > \frac{7}{4}$ we get,

$$\frac{\partial \ell_4}{\partial B} > 0. \tag{58}$$

The inequality (58) indicates that if the budget of the organization increases; irregular input responses positively. It seems that the organization has no headache to operate irregular input for profit maximization during increase of budget.

In equation (53) if $\ell_4 < \frac{7}{4}$ we get,

$$\frac{\partial \ell_4}{\partial B} < 0. \tag{59}$$

The inequality (59) shows that if the budget of the organization increases; purchasing power of irregular input decreases. It is bad information for the organization. In this situation the organization has no alternate except the decrease of production level.

In equation (53) if $\ell_4 = \frac{7}{4}$ we get,

$$\frac{\partial \ell_4}{\partial B} = 0. \tag{56}$$

The inequality (59) shows that there is no relation between the budget and irregular input. Because, the irregular input has no change after increase the budget of the organization.

11. Conclusions

In this study we have consulted the economic effects of various inputs when the budget of the organization increases. We have started our study with Cobb-Douglas production function as profit function. Moreover, we have allowed 5×5 bordered Hessian matrix and 5×5 Jacobian to show economic predictions more confidently. In the study we have tried to give a sustainable environment to the organization through the mathematical analysis by considering nonlinear budget constraint.

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