Paradigm Academic Press
Studies in Social Science & Humanities
ISSN 2709-7862
JUL. 2023 VOL.2, NO.7

Who Contributed More on Calculus? Newton or Leibniz?

Jiachen Lu[1]

[1] Culver Academies, Indiana 46511, US

Correspondence: Jiachen Lu, Culver Academies, Indiana 46511, US.

doi:10.56397/SSSH.2023.07.04

Abstract

Modern calculus is a strong tool in mathematical models and analysis. It is greatly applied in a variety of aspects including astrology, computer science, physics, business, etc. However, its origin has been debated between two great mathematicians from 18th century: Newton and Leibniz. Newton claimed to have invented calculus first, and had used this technique in some of his papers. However, no published records could prove his discovery. Leibniz published the first paper regarding the discovery of calculus "Nova Methodus pro Maximis et Minims" (New Method for the Greatest and the Least). Newton then accused Leibniz of stealing his idea and asserted his credit for inventing calculus. In this essay, to explain the cause of the Newton-Leibniz controversy, the discovery of calculus is evaluated from both Newton's and Leibniz's aspects. Their different methods are compared to answer the question: who is the true inventor of calculus?

Keywords: math, calculus, Newton, Leibniz

1. Background

Nowadays, Newton is commonly considered the inventor of calculus. However, the fundamental theorem of calculus is called Newton-Leibniz formula instead of Newton formula. Who is the greatest contributor to the method of calculus and why Leibniz is not honored for this great mathematic invention?

2. Newton as the Inventor of Calculus

The 17th century was an age of scientific innovations. Newton, one of the greatest scientists in human history, claimed he invented the method of fluxions in 1666, but it was not published until decades later. The method of fluxions can be summarized as the solution of a mechanical problem: "The length of the Space described being continually given, to find the Velocity of the Motion at any time proposed," or "The relation of the flowing Quantities being given, to determine the relation of their Fluxions." These two problems are the same with the modern-day differential and integral calculus, and Newton's notion of fluxions is just the same of derivatives in calculus. Newton also developed his own letter systems to express fluxions. He represented the fluxions of variables x, y, z as "pointed" or "prickt" letters: $\dot{x}$, $\dot{y}$, $\dot{z}$.

In his famous work *Mathematical principles of natural philosophy*, he didn't include calculus, but his theories in the book reflected the usage of calculus. In Marquis de l'Hospital's book *Analyse des infiniment petits* (Infinitesimal analysis), he stated in its preface, about the "Principia", that "nearly all of it is of this calculus" (Schrader). This book showed Newton obviously had found calculus in advance to his book's publish year 1687 and already mastered this skill and used it in his works. Also, from "The mathematical principles underlying Newton's *Principia Mathematica*" by D.T. Whiteside, it was mentioned that "the arguments used in the central portions of the *Principia* are essentially fluxional analyses clothed in the heavy guise of traditional synthetic geometry" (Schrader). The article claimed Newton made his masterpiece so esoteric and sophisticated as he hoped to make his mathematical worldview only available for a group of tightly restricted mathematical elites to comprehend. For this reason, even though he has mastered calculus as early as he invented the method of fluxions, he disguised the usage of it with geometry and not mentioned it in his *Principia Mathematica*. As Cliffor

Truesdell's claim that the *Principia* is "a book dense with the theory and application of infinitesimal calculus", all the evidence has shown Newton as the first invention of calculus in human history.

3. Leibniz as the Inventor of Calculus

Leibniz began his working on his variant of calculus in 1674 and published his first paper employing it "Nova Methodus pro Maximis et Minims" (New Method for the Greatest and the Least) in 1684. His paper was the first in human history which recorded the usage of calculus. Instead of working on velocities and flowing quantities, Leibniz identified the infinitely small difference and sums as the primary problem. He studied the relationship between difference sequences and sums and focused on an infinitesimal version which gave him the idea of the essential features of the calculus. In 1672, Dutch mathematician Huygens gave Leibniz a problem regarding the infinite sum of the "triangular numbers": $\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\cdots+\frac{2}{n(n+1)}+\cdots$

Leibniz solved this by rewrite the sum as:

$$\left(\frac{2}{1}-\frac{2}{2}\right)+\left(\frac{2}{2}-\frac{2}{3}\right)+\left(\frac{2}{3}-\frac{2}{4}\right)+\left(\frac{2}{4}-\frac{2}{5}\right)+\cdots+\left(\frac{2}{n}-\frac{2}{n+1}\right)+\cdots$$

All the middle terms are cancelled, and the sum becomes 2-2/(n+1), which leads to 2 as n goes to infinity. This observation expressed the essence of calculus in a discrete, rather than continuous way, to Leibniz. Afterwards, Leibniz studied this phenomenon further in his harmonic triangle which led him to realize that forming difference sequences and sum of sequences were mutually inverse operations. He used this idea to identify the area of a triangle as an analogue as a summation of infinitesimal differences which formed his fundamental understanding of calculus. (Laubenbacher)

Leibniz developed a complete system of notation for his method. The modern calculus symbols are based on his invention. He used $\frac{dy}{dx}$ to express derivative and $\int$ to express integration. The German mathematician Felix Klein praised Leibniz in his type of symbolism (Klein). In addition, Leibniz was much more concerned with finding the best notations for his calculus. He went through a variety of possible symbols, asked the advice from mathematicians, and had decided to use $\frac{dy}{dx}$ for a long time before he published it. His choice of $\int$ came from the term "calculus summatorius". Therefore, from the aspect of notation, Leibniz definitely contributed much more on modern calculus.

4. Newton Leibniz Controversy

The Newton-Leibniz controversy was a quarrel between Newton and Leibniz concerning the question of who should be credited as the father of calculus. It involved mutual accusation of plagiarism, anonymous letters published, divisions between each man's supporters, and even national jealousies. After Leibniz published his method of calculus, Newton and his friends accused him of plagiarism for several times and Leibniz and his friends refuted Newton's accusations. With the intervention of the Royal Society which was basically under the influence of Newton, a report concerning Newton's authority of calculus was written, and finally the controversy came to an end with after Leibniz's death on November 14, 1716.

In 1684, Leibniz published his first paper about calculus in *Acta Eruditorum.* He made a vague reference to Newton as mentioning Newton had a method similar to his but made no claim concerning who's the first inventor. However, Newton, in his famous *Principia*, Book II, Lemma II, explained the fundamental principle of the fluxionary calculus and claimed Leibniz's method was "hardly differed from mine, except in his forms of words and symbols" (Newton). Even though Newton's accusation wasn't shown explicitly, his words might indicate Newton as recognizing himself as the first and sole inventor of calculus and accusing Leibniz's method as a second version of his. In 1688, Fatio de Duillier, a Swiss mathematician also a close friend of Newton, published a memoir claiming Newton as the sole inventor of Calculus: "I am bound to acknowledge that Newton was the first, and by many years the first, inventor of this calculus: from whom, whether Leibniz, the second inventor, borrowed anything, I preferred that the decision should lie, not with me, but with others who have had sight of the papers of Newton, and other additions to this same manuscript." Duillier's words publicly charged Leibniz of plagiarism, and since Duillier was a close friend of Newton, it could be inferred that Newton held the similar viewpoint as Duillier's. Another friend of Newton John Keill, in his letter to Edmund Halley, an English astronomer, also openly accused Leibniz of plagiarism. He wrote in his letter, "All these laws follow from that very celebrated arithmetic of fluxions which, without any doubt, Dr. Newton invented first … Yet the same arithmetic afterwards, under a changed name and method of notation, was published by Dr. Leibniz in *Acta Eruditorum*." Apart from recognizing Newton's fluxions as the first invention of calculus, Keill directly blamed Leibniz's calculus as a same method just under a changed name and notation. This open accusation showed great hostility to Leibniz's honor and integrity as a mathematician. Similarly, as a close friend of Newton's, Newton probably had the same idea as

Keill's. Although Newton never explicitly attacked Leibniz by himself, his friends actively accused Leibniz of plagiarizing Newton's method of fluxions, and that obviously outraged Leibniz.

Leibniz responded to the accusations respectively with firm standpoint of him being the first inventor of calculus without any form of inspiration from Newton's fluxions, furthermore, he also counterattacked Newton for plagiarizing his novel method of calculus. After Newton published *Opticks* which was a short essay explaining his method of fluxions in 1704, Leibniz published an anonymous review on the *Acta Eruditorum* regarding the inventor of calculus in the next year. He crowned himself in his own review as the "inventor" of calculus and described Newton's usage of fluxions in his *Principia* as an analogy of Honoratus Fabri's usage of Cavalieri's principle in his *Synopsis of Geometry*. Fabri was a notorious plagiarist, although Leibniz didn't publicly announce his authorship of the review, we can still see his accusation of Newton's plagiarism. After reading the letter of Keill, Leibniz wrote to the Royal Society, demanding the retraction of Keill's letter. However, in his response to Leibniz, Keill further stated that Leibniz must have seen something from which he had been led to his own method. This response irritated Leibniz and he requested the Royal Society for an investigation and the silence of Keill. However, since the president of the Royal Society at that time was Newton, the investigation was highly biased and leaned towards Newton's side.

A committee was appointed for the investigation in 1712. Not surprisingly, almost all of the committee members were Newton's friends. After a year's investigation, on January 8, 1713, a report was published by the Royal Society named *Commercium Epistolicum D. Johannis Collinsii et aliorum de analysi promota*. This report mainly contained a chronology of Leibniz's contact with Newtonian influence, prove of Newton's priority on the invention of calculus, and a statement that Keill's words didn't insult Leibniz to any extent. This report was shown to be highly biased, and its conclusion was basically asserting Newton as the first and sole inventor of calculus. (Schrader)

The quarrel gets to an end after Leibniz's death in 1716. It made the Royal Society investigation final sentence of the Newton-Leibniz controversy and honored Newton as the inventor of calculus after all.

5. Extent of Newton's Calculus and Extent of Leibniz's Calculus

Newton's fluxions and Leibniz's calculus were both used in a large extent. As the Newton's homeland, England adopted his method of fluxions regardless of its lack of symbolism. The Leibniz notation system was also banned in England in order to promote Newton's method of fluxions. However, for the calculus of variations and for most theoretical work, the lack of a complete notation system caused the England math scholars not able to conduct their studies efficiently. Also, since Newton preferred to use classical geometry rather than the advanced analytic geometry proposed by Descartes and Fermat in 1637, it usually required English mathematicians to spend much more labor to work out geometric demonstrations. In addition, the English refused to translate all the progresses made from Leibniz's calculus in the European continent. This caused a century of isolation and the decline of English mathematics. Finally, Newton's method of fluxions was abandoned by a group of young students called the Analytical Society in 1830. To this end, the extent of Newton's calculus was only limited to a century in England. (Schrader)

6. Conclusion

Both Newton and Leibniz should be the founder of calculus as Newton was the first inventor of calculus and Leibniz was the founder of modern calculus. Apart from their contributions on calculus, both Newton and Leibniz also made great progress on other subjects like physics and philosophy. Newton was most renowned for his law of forces and his discovery of gravity which formed the basis of modern physics. He accounted for the motion of the moon by the force of gravity and proposed there's gravitational force between any two masses in the space (Newman). He proposed the famous three laws of motion:

1) If a body is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless it is acted upon by a force.
2) The time rate of change of the momentum of a body is equal in both magnitude and direction to the force imposed on it.
3) When two bodies interact, they apply forces to one another that are equal in magnitude and opposite in direction. (Britannica)

On the other hand, Leibniz's contribution in philosophy was also significant. As a philosopher, he wrote *Discourse on Metaphysics* in 1686; *New system of the Nature and Communication of Substances* in 1695; *New Essays on Human understanding* between 1695 and 1705; and *The Monadologie* in 1714.

References

Bix, R. Alan and D'Souza, Harry Joseph, (2016, June 6). *Analytic geometry*. *Encyclopedia Britannica*. https://www.britannica.com/science/analytic-geometry.

Britannica, T. (Editors of Encyclopedia), (2021, July 23). Newton's laws of motion. *Encyclopedia Britannica*. https://www.britannica.com/science/Newtons-laws-of-motion.

Cassirer, E., (1943). Newton and Leibniz. *The Philosophical Review*, *52*(4), 366. https://doi.org/10.2307/2180670.

Felix Klein, (1932). *Elementary Mathematics from an Advanced Standpoint*, Part I, New York, p. 215.

Green, D. R., (1979). Historical Topics: The Development of Integral Calculus. *Mathematics in School*, *8*(3), 24-28. http://www.jstor.org/stable/30213467.

Laubenbacher, R., & Pengelley, D., (2013). *Mathematical Expeditions*. Springer Publishing.

Newman, E. B., (1969). Newton, Physics, and the Psychology of the Nineteenth Century. *The American Journal of Psychology*, *82*(3), 400-406. https://doi.org/10.2307/1420758.

Newton's Philosophiae Naturalis Principia Mathematica, (2007, December 20). *Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/entries/newton-principia/#Bib.

Rosenthal, A., (1951). The History of Calculus. *The American Mathematical Monthly*, *58*(2), 75-86. https://doi.org/10.2307/2308368.

Schrader, D. V., (1962). Historically Speaking: The Newton-Leibniz controversy concerning the discovery of the calculus. *The Mathematics Teacher*, *55*(5), 385-396. https://doi.org/10.5951/mt.55.5.0385.

Uzzi, B., Mukherjee, S., Stringer, M., & Jones, B., (2013). Atypical Combinations and Scientific Impact. *Science*, *342*(6157), 468-472. https://doi.org/10.1126/science.1240474.

Wikipedia contributors, (2022, April 23). *Philosophiæ Naturalis Principia Mathematica*. Wikipedia. https://en.wikipedia.org/wiki/Philosophi%C3%A6_Naturalis_Principia_Mathematica#cite_note-geomcalc-10.

Wikipedia contributors, (2022b, June 1). Gottfried Wilhelm Leibniz. Wikipedia. https://en.wikipedia.org/wiki/Gottfried_Wilhelm_Leibniz#Principles.

Copyrights