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# Sensitivity Analysis for Profit Maximization with Respect to Per Unit Cost of Subsidiary Raw Materials

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**Abstract**

In this article the sensitivity analysis of an economic model is provided with detail mathematical analysis. To achieve maximum profit through sustainable way in the competitive global economy; an organization must scrutinize sensitivity analysis efficiently. In the study Cobb-Douglas production function is operated. Method of Lagrange multiplier is applied here when sensitivity analysis is investigated to obtain accurate results. The paper also uses the bordered Hessian and Jacobian to propel mathematical methods appropriately.

**Keywords:** Lagrange multiplier, economic inputs, profit maximization, sensitivity analysis

## 1. Introduction

Mathematical modeling in economics has started its journey in the $19^{th}$ century, which is considered as the application of mathematics to represent theories and analyze problems in economics (Samuelson, 1947). But at the elapse of time in the $21^{st}$ century, it becomes an essential part of economics (Carter, 2001). Profit maximization is considered as the capability of an organization to earn the maximum profit with minimum cost (Eaton & Lipsey, 1975). Every organization tries to maximize their profits; subject to their production functions, input costs, and market demand (Dixit, 1990). Cobb-Douglas production function helps the organizations to follow profit maximization strategy (Cobb & Douglas, 1928).

The method of Lagrange multiplier is a very useful and influential technique in multivariable calculus, which transfers a constrained problem to a higher dimensional unconstrained problem (Islam et al. 2010a, b). To discuss sensitivity analysis, we have started our activities with four commodity variables. Then we have included Lagrange multiplier to make the 4-dimensional constrained problem as 5-dimensional unconstrained character. Then we have preceded with the determinant of 6×6 bordered Hessian matrix and 6×6 Jacobian matrix by the use of implicit function theorem.

## 2. Literature Review

The literature review section is an introductory unit of research, which displays the works of previous researchers in the same field within the existing knowledge (Polit & Hungler, 2013). In 1928, two famous American professors; mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976), have worked on production functions which can be used to discuss profit maximization (Cobb & Douglas, 1928). Later in 1984, another two scholars of the same country; mathematician John V. Baxley and economist John C. Moorhouse have worked on the optimization (Baxley & Moorhouse, 1984). Bangladeshi famous mathematician Jamal Nazrul Islam (1939-2013) and his coauthors have discussed profit maximization for the welfare of the mathematical economics (Islam et al., 2010a, b). Abhishek Tripathi has realized that profit maximization is generally used to explain managerial decision making (Tripathi, 2019).

Pahlaj Moolio and his coworkers have considered the Cobb-Douglas production functions to analyze the mathematical structure of optimization (Moolio et al., 2009). Steven D. Levitt shows that profit maximizing behavior by firms is one of the widely applied policies in all of economics (Levitt, 2006). Lia Roy and her coauthors have performed laborious economic activities with patients (Roy et al., 2021). Jannatul Ferdous and Haradhan Kumar Mohajan have considered three inputs, such as capital, labor, and raw materials and subsidiary inputs for the mathematical analysis of the production procedures of the industry (Ferdous & Mohajan, 2022). Devajit Mohajan and Haradhan Kumar Mohajan have worked on profit maximization policies by using four variable inputs, such as capital, labor, principal raw materials, and subsidiary inputs in an industry (Mohajan & Mohajan, 2022a).

## 3. Research Methodology of the Study

Research is the procedures of systematic investigations that requires collection, interpretation and refinement of data, and ultimately prepare an acceptable article, working paper, book chapter or a thesis by the appropriate use of human knowledge. It is an essential and powerful device to the academicians for the leading in academic disciplines. It drives the global humanity through advancing to make a sustainable peaceful society (Pandey & Pandey, 2015). On the other hand, methodology is a guideline for the accomplishment of a good research (Kothari, 2008). Therefore, research methodology is the specific procedures that are used to identify, select, process, and analyze materials related to the topics. It tries to describe the types of research and the types of data, which are gathered to do the research efficiently and successfully (Somekh & Lewin, 2005).

In this study we have discussed the sensitivity analysis by the use of Cobb-Douglas production function. Throughout the study we have presented mathematical calculations in some details. We have used the determinant of 5×5 bordered Hessian and Jacobian matrices to make the study naïve to the interested readers. A researcher allows the reader to critically evaluate a study's overall validity and reliability. In the study we have tried to maintain the rules of reliability and validity. We have tried to cite the references properly throughout our research area (Mohajan, 2017b, 2018a, 2020). In this study we have depended on the profit maximization related mathematical secondary data sources. We have unsparingly consulted the published and unpublished research papers, books and handbooks of renowned authors, various research reports, internet, websites, etc. to enrich this paper (Mohajan, 2017a, 2018b).

## 4. Objective of the Study

The principal objective of this article is to discuss sensitivity analysis of major inputs with increase of cost of irregular used raw materials during profit maximization analysis of an organization. Other minor objectives are as follows:

- to show the results appropriately,
- to interpret the Cobb-Douglas production function, and
- to predict on the future production structure.

## 5. An Economic Model

Let us consider that an organization produces and distributes its products within a year using $\alpha_1$ quantity of capital, $\alpha_2$ quantity of labor, $\alpha_3$ quantity of principal raw materials, and $\alpha_4$ quantity of subsidiary raw materials. The profit can be represented by the Cobb-Douglas production function as (Moolio et al., 2009; Mohajan & Mohajan, 2022b, c),

$$P = f\left(\alpha_1, \alpha_2, \alpha_3, \alpha_4\right) = A\alpha_1^x \alpha_2^y \alpha_3^z \alpha_4^w \tag{1}$$

where $A$ is the technical process of economic system that indicates total factor productivity. Here $x$, $y$, $z$, and $w$ are parameters, where $x$ is the output of elasticity of capital measures the percentage change in $P$ for 1% change in $\alpha_1$, while $\alpha_2$, $\alpha_3$, and $\alpha_4$ are held constants. Similar properties carry the other parameters $y$, $z$, and $w$. The values of $x$, $y$, $z$, and $w$ are determined by the available technologies, and must satisfy the following four inequalities (Mohajan & Mohajan, 2022a; Mohajan et al., 2013):

$$0 < x < 1,\ 0 < y < 1,\ 0 < z < 1, \text{ and } 0 < w < 1. \tag{2}$$

A strict Cobb-Douglas production function, in which $\Lambda = x + y + z + w = 1$ indicates constant returns to scale, $\Lambda < 1$ indicates decreasing returns to scale, and $\Lambda > 1$ indicates increasing returns to scale (Moolio et al., 2009; Roy et al., 2021; Mohajan & Mohajan, 2022b, d ).

Now we consider that the budget constraint,

$$B(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = k\alpha_1 + l\alpha_2 + m\alpha_3 + n\alpha_4, \tag{3}$$

where *k* is rate of interest or services of capital per unit of capital $\alpha_1$; *l* is the wage rate per unit of labor $\alpha_2$; *m* is the cost per unit of principal raw material $\alpha_3$; and *n* is the cost of per unit of subsidiary raw materials $\alpha_4$.

Now we introduce a single Lagrange multiplier $\mu$, as a device; and by using equations (1) and (3) we can represent the Lagrangian function $U(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu)$, in a 5-dimensional unconstrained problem as (Mohajan, 2021a, b; Mohajan & Mohajan, 2022c),

$$U(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu) = A\alpha_1^x \alpha_2^y \alpha_3^z \alpha_4^w + \mu(B - k\alpha_1 - l\alpha_2 - m\alpha_3 - n\alpha_4). \tag{4}$$

where $\frac{\partial B}{\partial \alpha_1} = B_1$, $\frac{\partial B}{\partial \alpha_2} = B_2$, $\frac{\partial U}{\partial \alpha_1} = U_1$, $\frac{\partial^2 U}{\partial \alpha_1 \alpha_3} = U_{31}$, $\frac{\partial^2 U}{\partial \alpha_2^2} = U_{22}$, etc. are partial derivatives.

Let us consider the determinant of the 5×5 bordered Hessian matrix as,

$$H = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & U_{11} & U_{12} & U_{13} & U_{14} \\ -B_2 & U_{21} & U_{22} & U_{23} & U_{24} \\ -B_3 & U_{31} & U_{32} & U_{33} & U_{34} \\ -B_4 & U_{41} & U_{42} & U_{43} & U_{44} \end{vmatrix}. \tag{5}$$

Taking first-order partial differentiations of (3) we get,

$$B_1 = k, \ B_2 = l, \ B_3 = m, \text{ and } \ B_4 = n \tag{6}$$

Taking second-order and cross partial derivatives of (4) we get,

$$\begin{aligned}
U_{11} &= x(x-1)A\alpha_1^{x-2}\alpha_2^y\alpha_3^z\alpha_4^w, \\
U_{22} &= y(y-1)A\alpha_1^x\alpha_2^{y-2}\alpha_3^z\alpha_4^w, \\
U_{33} &= z(z-1)A\alpha_1^x\alpha_2^y\alpha_3^{z-2}\alpha_4^w, \\
U_{44} &= w(w-1)A\alpha_1^x\alpha_2^y\alpha_3^z\alpha_4^{w-2}, \\
U_{12} &= U_{21} = xyA\alpha_1^{x-1}\alpha_2^{y-1}\alpha_3^z\alpha_4^w, \\
U_{13} &= U_{31} = xzA\alpha_1^{x-1}\alpha_2^y\alpha_3^{z-1}\alpha_4^w, \\
U_{14} &= U_{41} = xwA\alpha_1^{x-1}\alpha_2^y\alpha_3^z\alpha_4^{w-1}, \\
U_{23} &= U_{32} = yzA\alpha_1^x\alpha_2^{y-1}\alpha_3^{z-1}\alpha_4^w, \\
U_{24} &= U_{42} = ywA\alpha_1^x\alpha_2^{y-1}\alpha_3^z\alpha_4^{w-1}, \\
U_{34} &= U_{43} = zwA\alpha_1^x\alpha_2^y\alpha_3^{z-1}\alpha_4^{w-1}.
\end{aligned} \tag{7}$$

Now we expand the Hessian (5) as,

$$\left|H\right| = \frac{A^3B^2xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2\Lambda} > 0 \tag{8}$$

where $A > 0$, $x, y, z, w >$, and budget, $B > 0$, therefore, $\left|H\right| > 0$. Hence, the profit is maximized (Islam et al., 2010a, b; Mohajan & Mohajan, 2022d).

**6. Sensitivity Analysis**

We have observed that the second order condition is satisfied, so that the determinant of (5) survive at the optimum, i.e., $\left|J\right| = \left|H\right|$; hence, we can apply the implicit function theorem. Let $\mathbf{G}$ be the vector-valued function of ten variables $\mu^*, \alpha_1^*, \alpha_2^*, \alpha_3^*, \alpha_4^*, k, l, m, n$, and $B$, and we define the function $\mathbf{G}$ for the point $\left(\mu^*, \alpha_1^*, \alpha_2^*, \alpha_3^*, \alpha_4^*, k, l, m, n, B\right) \in R^{10}$, and take the values in $R^5$. By the implicit function theorem of multivariable calculus, the equation (Mohajan & Mohajan, 2022e),

$$F\left(\mu^*, \alpha_1^*, \alpha_2^*, \alpha_3^*, \alpha_4^*, k, l, m, n, B\right) = 0, \tag{9}$$

may be solved in the form of

$$\begin{bmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \mathbf{G}\left(k, l, m, n, B\right). \tag{10}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}$; regarded as $J_G = \dfrac{\partial\left(\mu, \alpha_1, \alpha_2, \alpha_3, \alpha_4\right)}{\partial\left(k, l, m, n, B\right)}$, and is presented by;

$$J_G = \begin{bmatrix} \frac{\partial \mu}{\partial k} & \frac{\partial \mu}{\partial l} & \frac{\partial \mu}{\partial m} & \frac{\partial \mu}{\partial n} & \frac{\partial \mu}{\partial B} \\ \frac{\partial \alpha_1}{\partial k} & \frac{\partial \alpha_1}{\partial l} & \frac{\partial \alpha_1}{\partial m} & \frac{\partial \alpha_1}{\partial n} & \frac{\partial \alpha_1}{\partial B} \\ \frac{\partial \alpha_2}{\partial k} & \frac{\partial \alpha_2}{\partial l} & \frac{\partial \alpha_2}{\partial m} & \frac{\partial \alpha_2}{\partial n} & \frac{\partial \alpha_2}{\partial B} \\ \frac{\partial \alpha_3}{\partial k} & \frac{\partial \alpha_3}{\partial l} & \frac{\partial \alpha_3}{\partial m} & \frac{\partial \alpha_3}{\partial n} & \frac{\partial \alpha_3}{\partial B} \\ \frac{\partial \alpha_4}{\partial k} & \frac{\partial \alpha_4}{\partial l} & \frac{\partial \alpha_4}{\partial m} & \frac{\partial \alpha_4}{\partial n} & \frac{\partial \alpha_4}{\partial B} \end{bmatrix}. \tag{11}$$

$$= -J^{-1} \begin{bmatrix} -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 & 1 \\ -\mu & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & -\mu & 0 \end{bmatrix}$$

The inverse of Jacobian matrix is, $J^{-1} = \dfrac{1}{\left|J\right|} C^T$, where $C = \left(C_{ij}\right)$, the matrix of cofactors of $J$, where $T$ indicates transpose, then (14) becomes (Mohajan, 2021c; Moolio et al., 2009),

$$=-\frac{1}{|J|}\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 & 1 \\ -\mu & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & -\mu & 0 \end{bmatrix}$$

$$J_G=-\frac{1}{|J|}\begin{bmatrix} -\alpha_1 C_{11}-\mu C_{21} & -\alpha_2 C_{11}-\mu C_{31} & -\alpha_3 C_{11}-\mu C_{41} & -\alpha_4 C_{11}-\mu C_{51} & C_{11} \\ -\alpha_1 C_{12}-\mu C_{22} & -\alpha_2 C_{12}-\mu C_{32} & -\alpha_3 C_{12}-\mu C_{42} & -\alpha_4 C_{12}-\mu C_{52} & C_{12} \\ -\alpha_1 C_{13}-\mu C_{23} & -\alpha_2 C_{13}-\mu C_{33} & -\alpha_3 C_{13}-\mu C_{43} & -\alpha_4 C_{13}-\mu C_{53} & C_{13} \\ -\alpha_1 C_{14}-\mu C_{24} & -\alpha_2 C_{14}-\mu C_{34} & -\alpha_3 C_{14}-\mu C_{44} & -\alpha_4 C_{14}-\mu C_{54} & C_{14} \\ -\alpha_1 C_{15}-\mu C_{25} & -\alpha_2 C_{15}-\mu C_{35} & -\alpha_3 C_{15}-\mu C_{45} & -\alpha_4 C_{15}-\mu C_{55} & C_{15} \end{bmatrix}. \qquad (12)$$

In (11) total 25 comparative statics are available, and for sensitivity analysis we will try some of them to predict the economic analysis for the profit maximization (Baxley & Moorhouse, 1984; Islam et al., 2010a, b, Mohajan & Mohajan, 2020f).

Now we study the effect of capital $\alpha_1$ when per unit cost of subsidiary raw materials, *n* increases. Taking $T_{24}$ (i.e., term of 2nd row and 4th column) from both sides of (12) we get (Mohajan, 2021a, b; Wiese, 2021),

$$\frac{\partial \alpha_1}{\partial n}=\frac{\alpha_4}{|J|}[C_{12}]+\frac{\mu}{|J|}[C_{52}]$$

$$=\frac{\alpha_4}{|J|}\text{Cofactor of } C_{12}+\frac{\mu}{|J|}\text{Cofactor of } \mathrm{C}_{52}$$

$$=-\frac{\alpha_4}{|J|}\begin{vmatrix} -B_1 & U_{12} & U_{13} & U_{14} \\ -B_2 & U_{22} & U_{23} & U_{24} \\ -B_3 & U_{32} & U_{33} & U_{34} \\ -B_4 & U_{42} & U_{43} & U_{44} \end{vmatrix}-\frac{\mu}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & U_{12} & U_{13} & U_{14} \\ -B_2 & U_{22} & U_{23} & U_{24} \\ -B_3 & U_{32} & U_{33} & U_{34} \end{vmatrix}$$

$$=-\frac{\alpha_4}{|J|}\left\{-B_1\begin{vmatrix} U_{22} & U_{23} & U_{24} \\ U_{32} & U_{33} & U_{34} \\ U_{42} & U_{43} & U_{44} \end{vmatrix} -U_{12}\begin{vmatrix} -B_2 & U_{23} & U_{24} \\ -B_3 & U_{33} & U_{34} \\ -B_4 & U_{43} & U_{44} \end{vmatrix} +U_{13}\begin{vmatrix} -B_2 & U_{22} & U_{24} \\ -B_3 & U_{32} & U_{34} \\ -B_4 & U_{42} & U_{44} \end{vmatrix}\right.$$

$$\left. -U_{14}\begin{vmatrix} -B_2 & U_{22} & U_{23} \\ -B_3 & U_{32} & U_{33} \\ -B_4 & U_{42} & U_{43} \end{vmatrix}\right\}-\frac{\mu}{|J|}\left\{B_2\begin{vmatrix} -B_1 & U_{13} & U_{14} \\ -B_2 & U_{23} & U_{24} \\ -B_3 & U_{33} & U_{34} \end{vmatrix}-B_3\begin{vmatrix} -B_1 & U_{12} & U_{14} \\ -B_2 & U_{22} & U_{24} \\ -B_3 & U_{32} & U_{34} \end{vmatrix}+B_4\begin{vmatrix} -B_1 & U_{12} & U_{13} \\ -B_2 & U_{22} & U_{23} \\ -B_3 & U_{32} & U_{33} \end{vmatrix}\right\}$$

$$=-\frac{\alpha_4}{|J|}\left[-B_1\{U_{22}(U_{33}U_{44}-U_{43}U_{34})+U_{23}(U_{42}U_{34}-U_{32}U_{44})+U_{24}(U_{32}U_{43}-U_{42}U_{33})\}\right.$$

$$-U_{12}\{-B_2(U_{33}U_{44}-U_{43}U_{34})+U_{23}(-B_4U_{34}+B_3U_{44})+U_{24}(-B_3U_{43}+B_4U_{33})\}$$

$$+U_{13}\{-B_2(U_{32}U_{44}-U_{42}U_{34})+U_{22}(-B_4U_{34}+B_3U_{44})+U_{24}(-B_3U_{42}+B_4U_{32})\}$$

$$\left.-U_{14}\{-B_2(U_{32}U_{43}-U_{42}U_{33})+U_{22}(-B_4U_{33}+B_3U_{43})+U_{23}(-B_3U_{42}+B_4U_{32})\}\right]$$

$$-\frac{\mu}{|J|}\left[B_2\left\{-B_1\left(U_{23}U_{34}-U_{33}U_{24}\right)+U_{13}\left(-B_3U_{24}+B_2U_{34}\right)+U_{14}\left(-B_2U_{33}+B_3U_{23}\right)\right\}\right.$$

$$-B_3\left\{-B_1\left(U_{22}U_{34}-U_{32}U_{24}\right)+U_{12}\left(-B_3U_{24}+B_2U_{34}\right)+U_{14}\left(-B_2U_{32}+B_3U_{22}\right)\right\}$$

$$\left.+B_4\left\{-B_1\left(U_{22}U_{33}-U_{32}U_{23}\right)+U_{12}\left(-B_3U_{23}+B_2U_{33}\right)+U_{13}\left(-B_2U_{32}+B_3U_{22}\right)\right\}\right]$$

$$=-\frac{\alpha_4}{|J|}\left\{-B_1U_{22}U_{33}U_{44}+B_1U_{22}U_{34}^2-B_1U_{23}U_{42}U_{34}+B_1U_{23}^2U_{44}-B_1U_{24}U_{32}U_{43}+B_1U_{24}^2U_{33}\right.$$

$$+B_2U_{12}U_{33}U_{44}-B_2U_{12}U_{34}^2+B_4U_{12}U_{23}U_{34}-B_3U_{12}U_{23}U_{44}+B_3U_{12}U_{24}U_{43}-B_4U_{12}U_{24}U_{33}$$

$$-B_2U_{13}U_{32}U_{44}+B_2U_{13}U_{42}U_{34}-B_4U_{13}U_{22}U_{34}+B_3U_{13}U_{22}U_{44}-B_3U_{13}U_{24}^2+B_4U_{13}U_{24}U_{32}$$

$$\left.+B_2U_{14}U_{32}U_{43}-B_2U_{14}U_{42}U_{33}+B_4U_{14}U_{22}U_{33}-B_3U_{14}U_{22}U_{43}+B_3U_{14}U_{23}U_{42}-B_4U_{14}U_{23}^2\right\}$$

$$-\frac{\mu}{|J|}\left\{-B_1B_2U_{23}U_{34}+B_1B_2U_{33}U_{24}+B_2B_3U_{13}U_{24}-B_2^2U_{13}U_{34}+B_2^2U_{14}U_{33}-B_2B_3U_{14}U_{23}\right.$$

$$+B_1B_3U_{22}U_{34}-B_1B_3U_{32}U_{24}+B_3^2U_{12}U_{24}-B_2B_3U_{12}U_{34}+B_2B_3U_{14}U_{32}-B_3^2U_{14}U_{22}-B_1B_4U_{22}U_{33}$$

$$\left.+B_1B_4U_{32}U_{23}+B_3B_4U_{12}U_{23}+B_2B_4U_{12}U_{33}-B_2B_4U_{13}U_{32}+B_3B_4U_{13}U_{22}\right\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4}\left\{-k\alpha_1^2y(y-1)z(z-1)w(w-1)+k\alpha_1^2y(y-1)z^2w^2-k\alpha_1^2y^2z^2w^2\right.$$

$$+k\alpha_1^2y^2z^2w(w-1)-k\alpha_1^2y^2z^2w^2+k\alpha_1^2y^2z(z-1)w^2+l\alpha_1\alpha_2xyz(z-1)w(w-1)-l\alpha_1\alpha_2xyz^2w^2$$

$$+n\alpha_1\alpha_4xy^2z^2w-m\alpha_1\alpha_3xy^2zw(w-1)+m\alpha_1\alpha_3xy^2zw^2-n\alpha_1\alpha_4xy^2z(z-1)w-l\alpha_1\alpha_2xyz^2w(w-1)$$

$$+l\alpha_1\alpha_2xyz^2w^2-n\alpha_1\alpha_4xy(y-1)z^2w+m\alpha_1\alpha_3xy(y-1)zw(w-1)-m\alpha_1\alpha_3xy^2zw^2+n\alpha_1\alpha_4xy^2z^2w$$

$$+l\alpha_1\alpha_2xyz^2w^2-l\alpha_1\alpha_2xyz(z-1)w^2+n\alpha_1\alpha_4xy(y-1)z(z-1)w-m\alpha_1\alpha_3xy(y-1)zw^2$$

$$\left.+m\alpha_1\alpha_3xy^2zw^2-n\alpha_1\alpha_4xy^2z^2w\right\}-\frac{\mu}{|J|}\frac{A^2\alpha_1^{2x}\alpha_2^{2y}\alpha_3^{2z}\alpha_4^{2w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2}\left\{-kl\alpha_1^2\alpha_2\alpha_4yz^2w\right.$$

$$+kl\alpha_1^2\alpha_2\alpha_4yz(z-1)w+lm\alpha_1\alpha_2\alpha_3\alpha_4xyzw-l^2\alpha_1\alpha_2^2\alpha_4xz^2w+l^2\alpha_1\alpha_2^2\alpha_4xz(z-1)w$$

$$-lm\alpha_1\alpha_2\alpha_3\alpha_4xyzw+km\alpha_1^2\alpha_3\alpha_4y(y-1)zw-km\alpha_1^2\alpha_3\alpha_4y^2zw+m^2\alpha_1\alpha_3^2\alpha_4xy^2w$$

$$-lm\alpha_1\alpha_2\alpha_3\alpha_4xyzw+lm\alpha_1\alpha_2\alpha_3\alpha_4xyzw-m^2\alpha_1\alpha_3^2\alpha_4xy(y-1)w-kn\alpha_1^2\alpha_4^2y(y-1)z(z-1)$$

$$\left.+kn\alpha_1^2\alpha_4^2y^2z^2+mn\alpha_1\alpha_3\alpha_4^2xy^2z+nl\alpha_1\alpha_2\alpha_4^2xyz(z-1)-nl\alpha_1\alpha_2\alpha_4^2xyz^2+mn\alpha_1\alpha_3\alpha_4^2xy(y-1)z\right\}$$

$$
=-\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4}\quad\{-k\alpha_1^2x^{-1}(y-1)(z-1)(w-1)\quad+k\alpha_1^2x^{-1}(y-1)zw\quad-k\alpha_1^2x^{-1}yzw
$$

$$
+k\alpha_1^2x^{-1}yz(w-1)\ -k\alpha_1^2x^{-1}yzw\ +k\alpha_1^2x^{-1}y(z-1)w\ +l\alpha_1\alpha_2(z-1)(w-1)\ -l\alpha_1\alpha_2zw\ +n\alpha_1\alpha_4yz
$$

$$
-m\alpha_1\alpha_3y(w-1)\ +m\alpha_1\alpha_3yw\ -n\alpha_1\alpha_4y(z-1)\ -l\alpha_1\alpha_2z(w-1)\ +l\alpha_1\alpha_2zw\ -n\alpha_1\alpha_4(y-1)z
$$

$$
+m\alpha_1\alpha_3(y-1)(w-1)\ -m\alpha_1\alpha_3yw\ +n\alpha_1\alpha_4yz\ +l\alpha_1\alpha_2zw\ -l\alpha_1\alpha_2(z-1)w\ +n\alpha_1\alpha_4(y-1)(z-1)
$$

$$
-m\alpha_1\alpha_3(y-1)w+m\alpha_1\alpha_3yw-n\alpha_1\alpha_4yz\}-\frac{1}{|J|}\frac{A^2\alpha_1^{2x}\alpha_2^{2y}\alpha_3^{2z}\alpha_4^{2w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2}\,\frac{A\alpha_1^x\alpha_2^y\alpha_3^z\alpha_4^w\Lambda}{B}
$$

$$
\{-kl\alpha_1\alpha_2yz^2w\quad+kl\alpha_1\alpha_2yz(z-1)w\quad-l^2\alpha_2^2xz^2w\quad+l^2\alpha_2^2xz(z-1)w\quad+km\alpha_1\alpha_3y(y-1)zw
$$

$$
-km\alpha_1\alpha_3y^2zw\quad+m^2\alpha_3^2xy^2w\quad-m^2\alpha_3^2xy(y-1)w\quad-kn\alpha_1\alpha_4x(x-1)z(z-1)\quad+kn\alpha_1\alpha_4x^2z^2
$$

$$
+mn\alpha_3\alpha_4xy^2z+mn\alpha_3\alpha_4xy(y-1)z+nl\alpha_2\alpha_4xyz(z-1)-nl\alpha_2\alpha_4xyz^2\}
$$

$$
=-\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1\alpha_2^2\alpha_3^2\alpha_4}\quad\{-k\alpha_1x^{-1}(y-1)(z-1)(w-1)\quad+k\alpha_1x^{-1}(y-1)zw\quad-2k\alpha_1x^{-1}yzw
$$

$$
+k\alpha_1x^{-1}yz(w-1)\ +k\alpha_1x^{-1}y(z-1)w\ +l\alpha_2(z-1)(w-1)\ -l\alpha_2z(w-1)\ -l\alpha_2(z-1)w\ +l\alpha_2zw
$$

$$
-m\alpha_3y(w-1)\ +m\alpha_3yw\ +m\alpha_3(y-1)(w-1)\ -m\alpha_3(y-1)w\ +n\alpha_4(y-1)(z-1)\ -n\alpha_4y(z-1)
$$

$$
-n\alpha_4(y-1)z\quad+n\alpha_4yz\}\quad-\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}\Lambda}{\alpha_1\alpha_2^2\alpha_3^2\alpha_4B}\quad\{-kl\alpha_1\alpha_2yz^2w\quad+kl\alpha_1\alpha_2yz(z-1)w
$$

$$
-l^2\alpha_2^2xz^2w\qquad+l^2\alpha_2^2xz(z-1)w\qquad+km\alpha_1\alpha_3y(y-1)zw\qquad-km\alpha_1\alpha_3y^2zw\qquad+m^2\alpha_3^2xy^2w
$$

$$
-m^2\alpha_3^2xy(y-1)w\ -kn\alpha_1\alpha_4y(y-1)z(z-1)\ +kn\alpha_1\alpha_4y^2z^2\ +mn\alpha_3\alpha_4xy^2z\ +mn\alpha_3\alpha_4xy(y-1)z
$$

$$
+nl\alpha_2\alpha_4xyz(z-1)-nl\alpha_2\alpha_4xyz^2\}
$$

$$
=\frac{1}{|J|}\frac{2A^3xy^2zw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}B}{\alpha_1\alpha_2^2\alpha_3\alpha_4^2\Lambda}(1-z)>0 \tag{13}
$$

From (2) we see, $z<1$, i.e., $1-z>0$, therefore, from (13) we see that,

$$
\frac{\partial\alpha_1}{\partial n}>0. \tag{14}
$$

We study we observe that per unit cost of subsidiary raw materials, *n* increases; the amount of capital increases. For the survival of the organization, it will increase its capital structure $\alpha_1$ for paying extra amount to purchase necessary amount of subsidiary raw materials $\alpha_4$.

Now we study the effect of labor $\alpha_2$ when per unit cost of subsidiary raw materials, *n* increases. Taking $T_{34}$ (i.e., term of 3rd row and 4th column) from both sides of (12) we get (Mohajan, 2021a, b; Ferdous & Mohajan, 2022),

$$\frac{\partial \alpha_2}{\partial n} = \frac{\alpha_4}{|J|}[C_{13}] + \frac{\mu}{|J|}[C_{53}]$$

$$= \frac{\alpha_4}{|J|}\text{Cofactor of } C_{13} + \frac{\mu}{|J|}\text{Cofactor of } \mathrm{C}_{53}$$

$$= \frac{\alpha_4}{|J|}\begin{vmatrix} -B_1 & U_{11} & U_{13} & U_{14} \\ -B_2 & U_{21} & U_{23} & U_{24} \\ -B_3 & U_{31} & U_{33} & U_{34} \\ -B_4 & U_{41} & U_{43} & U_{44} \end{vmatrix} + \frac{\mu}{|J|}\begin{vmatrix} 0 & -B_1 & -B_3 & -B_4 \\ -B_1 & U_{11} & U_{13} & U_{14} \\ -B_2 & U_{21} & U_{23} & U_{24} \\ -B_3 & U_{31} & U_{33} & U_{34} \end{vmatrix}$$

$$= \frac{\alpha_4}{|J|}\left\{ -B_1\begin{vmatrix} U_{21} & U_{23} & U_{24} \\ U_{31} & U_{33} & U_{34} \\ U_{41} & U_{43} & U_{44} \end{vmatrix} - U_{11}\begin{vmatrix} -B_2 & U_{23} & U_{24} \\ -B_3 & U_{33} & U_{34} \\ -B_4 & U_{43} & U_{44} \end{vmatrix} + U_{13}\begin{vmatrix} -B_2 & U_{21} & U_{24} \\ -B_3 & U_{31} & U_{34} \\ -B_4 & U_{41} & U_{44} \end{vmatrix} \right.$$

$$\left. -U_{14}\begin{vmatrix} -B_2 & U_{21} & U_{23} \\ -B_3 & U_{31} & U_{33} \\ -B_4 & U_{41} & U_{43} \end{vmatrix} \right\} + \frac{\mu}{|J|}\left\{ B_1\begin{vmatrix} -B_1 & U_{13} & U_{14} \\ -B_2 & U_{23} & U_{24} \\ -B_3 & U_{33} & U_{34} \end{vmatrix} - B_3\begin{vmatrix} -B_1 & U_{11} & U_{14} \\ -B_2 & U_{21} & U_{24} \\ -B_3 & U_{31} & U_{34} \end{vmatrix} \right.$$

$$\left. + B_4\begin{vmatrix} -B_1 & U_{11} & U_{13} \\ -B_2 & U_{21} & U_{23} \\ -B_3 & U_{31} & U_{33} \end{vmatrix} \right\}$$

$$= \frac{\alpha_4}{|J|}\left[ -B_1\{U_{21}(U_{33}U_{44} - U_{43}U_{34}) + U_{23}(U_{41}U_{34} - U_{31}U_{44}) + U_{24}(U_{31}U_{43} - U_{41}U_{33})\}\right.$$

$$-U_{11}\{-B_2(U_{33}U_{44} - U_{43}U_{34}) + U_{23}(-B_4U_{34} + B_3U_{44}) + U_{24}(-B_3U_{43} + B_4U_{33})\}$$

$$+U_{13}\{-B_2(U_{31}U_{44} - U_{41}U_{34}) + U_{21}(-B_4U_{34} + B_3U_{44}) + U_{24}(-B_3U_{41} + B_4U_{31})\}$$

$$\left. -U_{14}\{-B_2(U_{31}U_{43} - U_{41}U_{33}) + U_{21}(-B_4U_{33} + B_3U_{43}) + U_{23}(-B_3U_{41} + B_4U_{31})\}\right]$$

$$+ \frac{\mu}{|J|}\left[ B_1\{-B_1(U_{23}U_{34} - U_{33}U_{24}) + U_{13}(-B_3U_{24} + B_2U_{34}) + U_{14}(-B_2U_{33} + B_3U_{23})\}\right.$$

$$-B_3\{-B_1(U_{21}U_{34} - U_{31}U_{24}) + U_{11}(-B_3U_{24} + B_2U_{34}) + U_{14}(-B_2U_{31} + B_3U_{21})\}$$

$$\left. +B_4\{-B_1(U_{21}U_{33} - U_{31}U_{23}) + U_{11}(-B_3U_{23} + B_2U_{33}) + U_{13}(-B_2U_{31} + B_3U_{21})\}\right]$$

$$= -\frac{\alpha_4}{|J|} \quad \{B_1U_{21}U_{33}U_{44} \quad -B_1U_{21}U_{43}U_{34} \quad +B_1U_{23}U_{41}U_{24} \quad -B_1U_{23}U_{31}U_{44} \quad +B_1U_{24}U_{31}U_{43}$$

$$-B_1U_{24}U_{41}U_{33} \quad -B_2U_{11}U_{33}U_{44} \quad +B_2U_{11}U_{43}U_{34} \quad -B_4U_{11}U_{23}U_{34} \quad +B_3U_{11}U_{23}U_{44} \quad -B_3U_{11}U_{24}U_{43}$$

$$+B_4U_{11}U_{24}U_{33} \quad +B_2U_{13}U_{31}U_{44} \quad -B_2U_{13}U_{41}U_{34} \quad +B_4U_{13}U_{21}U_{34} \quad -B_3U_{13}U_{21}U_{44} \quad +B_3U_{13}U_{24}U_{41}$$

$$-B_4U_{13}U_{24}U_{31} - B_2U_{14}U_{31}U_{43} + B_2U_{14}U_{41}U_{33} - B_4U_{14}U_{21}U_{33} + B_3U_{14}U_{21}U_{43} - B_3U_{14}U_{23}U_{41}$$

$$+ B_4U_{14}U_{23}U_{31}\} + \frac{\mu}{|J|}\{B_1^2U_{23}U_{34} - B_1^2U_{33}U_{24} + B_1B_3U_{13}U_{24} - B_1B_2U_{13}U_{34} + B_1B_2U_{14}U_{33}$$

$$- B_1B_3U_{14}U_{23} + B_1B_3U_{21}U_{34} - B_1B_3U_{31}U_{24} + B_3^2U_{11}U_{24} - B_2B_3U_{11}U_{34} + B_2B_3U_{14}U_{31} - B_3^2U_{14}U_{21}$$

$$- B_1B_4U_{21}U_{33} + B_1B_4U_{31}U_{23} - B_3B_4U_{11}U_{23} + B_2B_4U_{11}U_{33} - B_2B_4U_{13}U_{31} + B_3B_4U_{13}U_{21}\}$$

$$= -\frac{\alpha_4}{|J|} \frac{A^3\alpha_1^{3y}\alpha_2^{3x}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2} \{k\alpha_1\alpha_2xyz(z-1)w(w-1) - k\alpha_1\alpha_2xyz^2w^2 + k\alpha_1\alpha_2xyz^2w^2$$

$$- k\alpha_1\alpha_2xyz^2w(w-1) + k\alpha_1\alpha_2xyz^2w^2 - k\alpha_1\alpha_2xyz(z-1)w^2 - l\alpha_2^2x(x-1)z(z-1)w(w-1)$$

$$+ l\alpha_2^2x(x-1)z^2w^2 - n\alpha_2\alpha_4x(x-1)yz^2w + m\alpha_2\alpha_3x(x-1)yzw(w-1) - m\alpha_2\alpha_3x(x-1)yzw^2$$

$$+ n\alpha_2\alpha_4x(x-1)yz(z-1)w + l\alpha_2^2x^2z^2w(w-1) - l\alpha_2^2x^2z^2w^2 + n\alpha_2\alpha_4x^2yz^2w$$

$$- m\alpha_2\alpha_3x^2yzw(w-1) + m\alpha_2\alpha_3x^2yzw^2 - n\alpha_2\alpha_4x^2yz^2w - l\alpha_2^2x^2z^2w^2 + l\alpha_2^2x^2z(z-1)w^2$$

$$- n\alpha_2\alpha_4x^2yz(z-1)w + m\alpha_2\alpha_3x^2yzw^2 - m\alpha_2\alpha_3x^2yzw^2 + n\alpha_2\alpha_4x^2yz^2w\} + \frac{\mu}{|J|}\frac{A^2\alpha_1^{2x}\alpha_2^{2y}\alpha_3^{2z}\alpha_4^{2w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2}$$

$$\{k^2\alpha_1^2\alpha_2\alpha_4yz^2w - k^2\alpha_1^2\alpha_2\alpha_4yz(z-1)w + km\alpha_1\alpha_2\alpha_3\alpha_4xyzw - kl\alpha_1\alpha_2^2\alpha_4xz^2w$$

$$+ kl\alpha_1\alpha_2^2\alpha_4xz(z-1)w - km\alpha_1\alpha_2\alpha_3\alpha_4xyzw + km\alpha_1\alpha_2\alpha_3\alpha_4xyzw - km\alpha_1\alpha_2\alpha_3\alpha_4xyzw$$

$$+ m^2\alpha_2\alpha_3^2\alpha_4x(x-1)yw - lm\alpha_2^2\alpha_3\alpha_4x(x-1)zw + lm\alpha_2^2\alpha_3\alpha_4x^2zw - m^2\alpha_2\alpha_3^2\alpha_4x^2yw$$

$$- kn\alpha_1\alpha_2\alpha_4^2xyw(w-1) + kn\alpha_1\alpha_2\alpha_4^2xyw^2 - mn\alpha_2\alpha_3\alpha_4^2x(x-1)yz + nl\alpha_2^2\alpha_4^2x(x-1)z(z-1)$$

$$- nl\alpha_2^2\alpha_4^2x^2z^2 + mn\alpha_2\alpha_3\alpha_4^2x^2yz\}$$

$$= -\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3y}\alpha_2^{3x}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1\alpha_2\alpha_3^2\alpha_4^2}\left\{\frac{B}{\Lambda}(1-w-z)+\frac{zB}{\Lambda}+\frac{wB}{\Lambda}\right\} + \frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3y}\alpha_2^{3x}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1\alpha_2\alpha_3^2\alpha_4^2}\frac{B}{\Lambda}$$

$$= -\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3y}\alpha_2^{3x}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1\alpha_2\alpha_3^2\alpha_4^2}\frac{B}{\Lambda} + \frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3y}\alpha_2^{3x}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1\alpha_2\alpha_3^2\alpha_4^2}\frac{B}{\Lambda} = 0 \qquad (15)$$

From (15) we have realized that when per unit cost of subsidiary raw materials, *n* increases; there is no effect on labor $\alpha_2$, and the organization may increase or decrease its total labor force depending on the demand and supply.

Now we study the effect of principal raw materials $\alpha_3$ when per unit cost of subsidiary raw materials, *n* increases. Taking $T_{44}$ (i.e., term of 5th row and 4th column) from both sides of (12) we get (Mohajan, 2022; Mohajan & Mohajan, 2022a, b),

$$\frac{\partial \alpha_3}{\partial n} = \frac{\alpha_4}{|J|}[C_{14}] + \frac{\mu}{|J|}[C_{54}]$$

$$= \frac{\alpha_4}{|J|}\text{Cofactor of } C_{14} + \frac{\mu}{|J|}\text{Cofactor of } \mathrm{C}_{54}$$

$$= -\frac{\alpha_4}{|J|}\begin{vmatrix} -B_1 & U_{11} & U_{12} & U_{14} \\ -B_2 & U_{21} & U_{22} & U_{24} \\ -B_3 & U_{31} & U_{32} & U_{34} \\ -B_4 & U_{41} & U_{42} & U_{44} \end{vmatrix} - \frac{\mu}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_4 \\ -B_1 & U_{11} & U_{12} & U_{14} \\ -B_2 & U_{21} & U_{22} & U_{24} \\ -B_3 & U_{31} & U_{32} & U_{34} \end{vmatrix}$$

$$= -\frac{\alpha_4}{|J|}\left\{ -B_1\begin{vmatrix} U_{21} & U_{22} & U_{24} \\ U_{31} & U_{32} & U_{34} \\ U_{41} & U_{42} & U_{44} \end{vmatrix} - U_{11}\begin{vmatrix} -B_2 & U_{22} & U_{24} \\ -B_3 & U_{32} & U_{34} \\ -B_4 & U_{42} & U_{44} \end{vmatrix} + U_{12}\begin{vmatrix} -B_2 & U_{21} & U_{24} \\ -B_3 & U_{31} & U_{34} \\ -B_4 & U_{41} & U_{44} \end{vmatrix}\right.$$

$$\left. - U_{14}\begin{vmatrix} -B_2 & U_{21} & U_{22} \\ -B_3 & U_{31} & U_{32} \\ -B_4 & U_{41} & U_{42} \end{vmatrix}\right\} - \frac{\mu}{|J|}\left\{ B_1\begin{vmatrix} -B_1 & U_{12} & U_{14} \\ -B_2 & U_{22} & U_{24} \\ -B_3 & U_{32} & U_{34} \end{vmatrix} - B_2\begin{vmatrix} -B_1 & U_{11} & U_{14} \\ -B_2 & U_{21} & U_{24} \\ -B_3 & U_{31} & U_{34} \end{vmatrix}\right.$$

$$\left. + B_4\begin{vmatrix} -B_1 & U_{11} & U_{12} \\ -B_2 & U_{21} & U_{22} \\ -B_3 & U_{31} & U_{32} \end{vmatrix}\right\}$$

$$= -\frac{\alpha_4}{|J|}\left[ -B_1\{U_{21}(U_{32}U_{44} - U_{42}U_{34}) + U_{22}(U_{41}U_{34} - U_{31}U_{44}) + U_{24}(U_{31}U_{42} - U_{41}U_{32})\}\right.$$

$$- U_{11}\{-B_2(U_{32}U_{44} - U_{42}U_{34}) + U_{22}(-B_4U_{34} + B_3U_{44}) + U_{24}(-B_3U_{42} + B_4U_{32})\}$$

$$+ U_{12}\{-B_2(U_{31}U_{44} - U_{41}U_{34}) + U_{21}(-B_4U_{34} + B_3U_{44}) + U_{24}(-B_3U_{41} + B_4U_{31})\}$$

$$\left. - U_{14}\{-B_2(U_{31}U_{42} - U_{41}U_{32}) + U_{21}(-B_4U_{32} + B_3U_{42}) + U_{22}(-B_3U_{41} + B_4U_{31})\}\right]$$

$$- \frac{\mu}{|J|}\left[ B_1\{-B_1(U_{22}U_{34} - U_{32}U_{24}) + U_{12}(-B_3U_{24} + B_2U_{34}) + U_{14}(-B_2U_{32} + B_3U_{22})\}\right.$$

$$- B_2\{-B_1(U_{21}U_{34} - U_{31}U_{24}) + U_{11}(-B_3U_{24} + B_2U_{34}) + U_{14}(-B_2U_{31} + B_3U_{21})\}$$

$$\left. + B_4\{-B_1(U_{21}U_{32} - U_{31}U_{22}) + U_{11}(-B_3U_{22} + B_2U_{32}) + U_{12}(-B_2U_{31} + B_3U_{21})\}\right]$$

$$= -\frac{\alpha_4}{|J|}\{-B_1U_{21}U_{32}U_{44} + B_1U_{21}U_{42}U_{34} - B_1U_{22}U_{41}U_{34} + B_1U_{22}U_{31}U_{44} - B_1U_{24}U_{31}U_{42} + B_1U_{24}U_{41}U_{32}$$

$$+ B_2U_{11}U_{32}U_{44} - B_2U_{11}U_{42}U_{34} + B_4U_{11}U_{22}U_{34} - B_3U_{11}U_{22}U_{44} + B_3U_{11}U_{24}U_{42} - B_4U_{11}U_{24}U_{32}$$

$$- B_2U_{12}U_{31}U_{44} + B_2U_{12}U_{41}U_{34} - B_4U_{12}U_{21}U_{34} + B_3U_{12}U_{21}U_{44} - B_3U_{12}U_{24}U_{41} + B_4U_{12}U_{24}U_{31}$$

$$+B_2U_{14}U_{31}U_{42}-B_2U_{14}U_{41}U_{32}+B_4U_{14}U_{21}U_{32}-B_3U_{14}U_{21}U_{42}+B_3U_{14}U_{22}U_{41}-B_4U_{14}U_{22}U_{31}\}$$

$$-\frac{\mu}{|J|}\{-B_1^2U_{22}U_{34}+B_1^2U_{32}U_{24}-B_1B_3U_{12}U_{24}+B_1B_2U_{12}U_{34}-B_1B_2U_{14}U_{32}+B_1B_3U_{14}U_{22}$$

$$+B_1B_2U_{21}U_{34}-B_1B_2U_{31}U_{24}+B_2B_3U_{11}U_{24}-B_2^2U_{11}U_{34}+B_2^2U_{14}U_{31}-B_2B_3U_{14}U_{21}$$

$$-B_1B_4U_{21}U_{32}+B_1B_4U_{31}U_{22}-B_3B_4U_{11}U_{22}+B_2B_4U_{11}U_{32}-B_2B_4U_{12}U_{31}+B_3B_4U_{12}U_{21}\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4}\quad\{-k\alpha_1\alpha_3xy^2zw(w-1)\quad+k\alpha_1\alpha_3xy^2zw^2\quad-k\alpha_1\alpha_3xy(y-1)zw^2$$

$$+k\alpha_1\alpha_3xy(y-1)zw(w-1)\quad-k\alpha_1\alpha_3xy^2zw^2\quad+k\alpha_1\alpha_3xy^2zw^2\quad+l\alpha_2\alpha_3x(x-1)yzw(w-1)$$

$$-l\alpha_2\alpha_3x(x-1)yzw^2\quad+n\alpha_3\alpha_4x(x-1)y(y-1)zw\quad-m\alpha_3^2x(x-1)y(y-1)w(w-1)$$

$$+m\alpha_3^2x(x-1)y^2w^2-n\alpha_3\alpha_4x(x-1)y^2zw-l\alpha_2\alpha_3x^2yzw(w-1)+l\alpha_2\alpha_3x^2yzw^2-n\alpha_3\alpha_4x^2y^2zw$$

$$+m\alpha_3^2x^2y^2w(w-1)\quad-m\alpha_3^2x^2y^2w^2\quad+n\alpha_3\alpha_4x^2y^2zw\quad+l\alpha_2\alpha_3x^2yzw^2\quad-l\alpha_2\alpha_3x^2yzw^2$$

$$+n\alpha_3\alpha_4x^2y^2zw\quad-m\alpha_3^2x^2y^2w^2\quad+m\alpha_3^2x^2y(y-1)w^2\quad-n\alpha_3\alpha_4x^2y(y-1)zw\}$$

$$+\frac{\mu}{|J|}\frac{A^2\alpha_1^{2x}\alpha_2^{2y}\alpha_3^{2z}\alpha_4^{2w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2}\quad\{-k^2\alpha_1^2\alpha_3\alpha_4y(y-1)zw\quad+k^2\alpha_1^2\alpha_3\alpha_4y^2zw\quad-km\alpha_1\alpha_3^2\alpha_4xy^2w$$

$$+kl\alpha_1\alpha_2\alpha_3\alpha_4xyzw\quad-kl\alpha_1\alpha_2\alpha_3\alpha_4xyzw\quad+km\alpha_1\alpha_3^2\alpha_4xy(y-1)w\quad+kl\alpha_1\alpha_2\alpha_3\alpha_4xyzw$$

$$-kl\alpha_1\alpha_2\alpha_3\alpha_4xyzw\quad+lm\alpha_2\alpha_3^2\alpha_4x(x-1)yw\quad-l^2\alpha_2^2\alpha_3\alpha_4x(x-1)zw\quad+l^2\alpha_2^2\alpha_3\alpha_4x^2zw$$

$$-lm\alpha_2\alpha_3^2\alpha_4x^2yw\quad-kn\alpha_1\alpha_3\alpha_4^2xy^2z\quad+kn\alpha_1\alpha_3\alpha_4^2xy(y-1)z\quad-mn\alpha_3^2\alpha_4^2x(x-1)y(y-1)$$

$$+nl\alpha_2\alpha_3\alpha_4^2x(x-1)yz-nl\alpha_2\alpha_3\alpha_4^2x^2yz+mn\alpha_3^2\alpha_4^2x^2y^2\}$$

$$=-\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3\alpha_4}\quad\{-k\alpha_1y(w-1)\quad-k\alpha_1(y-1)w\quad+k\alpha_1(y-1)(w-1)\quad+k\alpha_1yw$$

$$+l\alpha_2(x-1)(w-1)\quad-l\alpha_2x(w-1)\quad+l\alpha_2xw\quad-l\alpha_2(x-1)w\quad-2m\alpha_3xyz^{-1}w\quad+m\alpha_3x(y-1)z^{-1}w$$

$$-m\alpha_3(x-1)(y-1)z^{-1}(w-1)\quad+m\alpha_3xyz^{-1}(w-1)\quad+m\alpha_3(x-1)yz^{-1}w\quad+n\alpha_4(x-1)(y-1)$$

$$-n\alpha_4(x-1)y\quad+n\alpha_4xy\quad-n\alpha_4x(y-1)\}\quad+\frac{1}{|J|}\frac{A^2\alpha_1^{2x}\alpha_2^{2y}\alpha_3^{2z}\alpha_4^{2w}}{\alpha_1^2\alpha_2^2\alpha_3\alpha_4}\quad\frac{A\alpha_1^x\alpha_2^y\alpha_3^z\alpha_4^w\Lambda}{B}$$

$$\{-k^2\alpha_1^2y(y-1)zw\quad+k^2\alpha_1^2y^2zw\quad-km\alpha_1\alpha_3xy^2w\quad+km\alpha_1\alpha_3xy(y-1)w\quad-lm\alpha_2\alpha_3x^2yw$$

$$+lm\alpha_2\alpha_3x(x-1)yw \quad -l^2\alpha_2^2x(x-1)zw \quad +l^2\alpha_2^2x^2zw \quad -kn\alpha_1\alpha_4xy^2z \quad +kn\alpha_1\alpha_4xy(y-1)z$$

$$+nl\alpha_2\alpha_4x(x-1)yz-nl\alpha_2\alpha_4x^2yz-mn\alpha_3\alpha_4x(x-1)y(y-1)+mn\alpha_3\alpha_4x^2y^2\Big\}$$

$$=-\frac{2}{|J|}\frac{A^3xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}B}{\alpha_1^2\alpha_2^2\alpha_3\alpha_4\Lambda}<0 \tag{16}$$

From (16) we have realized that when per unit cost of subsidiary raw materials, *n* increases; the tendency of buying of principal raw material, $\alpha_3$ decreases. This seems that two types of raw materials in this study are complementary.

Now we study the effect subsidiary raw materials $\alpha_4$ when per unit cost of subsidiary raw materials, *n* increases. Taking $T_{54}$ (i.e., term of 5th row and 4th column) from both sides of (12) we get (Islam, 2009a, b; Mohajan & Mohajan, 2022f),

$$\frac{\partial\alpha_4}{\partial n}=\frac{\alpha_4}{|J|}[C_{15}]+\frac{\mu}{|J|}[C_{55}]$$

$$=\frac{\alpha_4}{|J|}\text{Cofactor of } C_{15}+\frac{\mu}{|J|}\text{Cofactor of } \mathrm{C}_{55}$$

$$=\frac{\alpha_4}{|J|}\begin{vmatrix}-B_1 & U_{11} & U_{12} & U_{13}\\ -B_2 & U_{21} & U_{22} & U_{23}\\ -B_3 & U_{31} & U_{32} & U_{33}\\ -B_4 & U_{41} & U_{42} & U_{43}\end{vmatrix}+\frac{\mu}{|J|}\begin{vmatrix}0 & -B_1 & -B_2 & -B_3\\ -B_1 & U_{11} & U_{12} & U_{13}\\ -B_2 & U_{21} & U_{22} & U_{23}\\ -B_3 & U_{31} & U_{32} & U_{33}\end{vmatrix}$$

$$=\frac{\alpha_4}{|J|}\left\{-B_1\begin{vmatrix}U_{21} & U_{22} & U_{23}\\ U_{31} & U_{32} & U_{33}\\ U_{41} & U_{42} & U_{43}\end{vmatrix}-U_{11}\begin{vmatrix}-B_2 & U_{22} & U_{23}\\ -B_3 & U_{32} & U_{33}\\ -B_4 & U_{42} & U_{43}\end{vmatrix}+U_{12}\begin{vmatrix}-B_2 & U_{21} & U_{23}\\ -B_3 & U_{31} & U_{33}\\ -B_4 & U_{41} & U_{43}\end{vmatrix}\right.$$

$$\left.-U_{13}\begin{vmatrix}-B_2 & U_{21} & U_{22}\\ -B_3 & U_{31} & U_{32}\\ -B_4 & U_{41} & U_{42}\end{vmatrix}\right\}+\frac{\mu}{|J|}\left\{B_1\begin{vmatrix}-B_1 & U_{12} & U_{13}\\ -B_2 & U_{22} & U_{23}\\ -B_3 & U_{32} & U_{33}\end{vmatrix}-B_2\begin{vmatrix}-B_1 & U_{11} & U_{13}\\ -B_2 & U_{21} & U_{23}\\ -B_3 & U_{31} & U_{33}\end{vmatrix}\right.$$

$$\left.+B_3\begin{vmatrix}-B_1 & U_{11} & U_{12}\\ -B_2 & U_{21} & U_{22}\\ -B_3 & U_{31} & U_{32}\end{vmatrix}\right\}$$

$$=\frac{\alpha_4}{|J|}\big[-B_1\{U_{21}(U_{32}U_{43}-U_{42}U_{33})+U_{22}(U_{41}U_{33}-U_{31}U_{43})+U_{23}(U_{31}U_{42}-U_{41}U_{32})\}$$

$$-U_{11}\{-B_2(U_{32}U_{43}-U_{42}U_{33})+U_{22}(-B_4U_{33}+B_3U_{43})+U_{23}(-B_3U_{42}+B_4U_{32})\}$$

$$+U_{12}\{-B_2(U_{31}U_{43}-U_{41}U_{33})+U_{21}(-B_4U_{33}+B_3U_{43})+U_{23}(-B_3U_{41}+B_4U_{31})\}$$

$$-U_{13}\{-B_2(U_{31}U_{42}-U_{41}U_{32})+U_{21}(-B_4U_{32}+B_3U_{42})+U_{22}(-B_3U_{41}+B_4U_{31})\}]$$

$$+\frac{\mu}{|J|}[B_1\{-B_1(U_{22}U_{33}-U_{32}U_{23})+U_{12}(-B_3U_{23}+B_2U_{33})+U_{13}(-B_2U_{32}+B_3U_{22})\}$$

$$-B_2\{-B_1(U_{21}U_{33}-U_{31}U_{23})+U_{11}(-B_3U_{23}+B_2U_{33})+U_{13}(-B_2U_{31}+B_3U_{21})\}$$

$$+B_3\{-B_1(U_{21}U_{32}-U_{31}U_{22})+U_{11}(-B_3U_{22}+B_2U_{32})+U_{12}(-B_2U_{31}+B_3U_{21})\}]$$

$$=\frac{\alpha_4}{|J|}\{-B_1U_{21}U_{32}U_{43}+B_1U_{21}U_{42}U_{33}-B_1U_{22}U_{41}U_{33}+B_1U_{22}U_{31}U_{43}-B_1U_{23}U_{31}U_{42}+B_1U_{23}U_{41}U_{32}$$

$$+B_2U_{11}U_{32}U_{43}-B_2U_{11}U_{42}U_{33}-B_4U_{11}U_{22}U_{33}+B_3U_{11}U_{22}U_{43}-B_3U_{11}U_{23}U_{42}+B_4U_{11}U_{23}U_{32}$$

$$-B_2U_{12}U_{31}U_{43}+B_2U_{12}U_{41}U_{33}-B_4U_{12}U_{21}U_{33}+B_3U_{12}U_{21}U_{43}-B_3U_{12}U_{23}U_{41}+B_4U_{12}U_{23}U_{31}$$

$$+B_2U_{13}U_{31}U_{42}-B_2U_{13}U_{41}U_{32}+B_4U_{13}U_{21}U_{32}-B_3U_{13}U_{21}U_{42}+B_3U_{13}U_{22}U_{41}-B_4U_{13}U_{22}U_{31}\}$$

$$+\frac{\mu}{|J|}\{-B_1^2U_{22}U_{33}+B_1^2U_{32}U_{23}-B_1B_3U_{12}U_{23}+B_1B_2U_{12}U_{33}-B_1B_2U_{13}U_{32}+B_1B_3U_{13}U_{22}$$

$$+B_1B_2U_{21}U_{33}-B_1B_2U_{31}U_{23}+B_2B_3U_{11}U_{23}-B_2^2U_{11}U_{33}+B_2^2U_{13}U_{31}-B_2B_3U_{13}U_{21}$$

$$-B_1B_3U_{21}U_{32}+B_1B_3U_{31}U_{22}-B_3^2U_{11}U_{22}+B_2B_3U_{11}U_{32}-B_2B_3U_{12}U_{31}+B_3^2U_{12}U_{21}\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4}\quad\{-k\alpha_1\alpha_4xy^2z^2w\quad+k\alpha_1\alpha_4xy^2z(z-1)w\quad-k\alpha_1\alpha_4xy(y-1)z(z-1)w$$

$$+k\alpha_1\alpha_4xy(y-1)z(z-1)w\quad-k\alpha_1\alpha_4xy^2z^2w\quad+k\alpha_1\alpha_4xy^2z^2w\quad+l\alpha_2\alpha_4x(x-1)yz^2w$$

$$-l\alpha_2\alpha_4x(x-1)yz(z-1)w\quad-n\alpha_4^2x(x-1)y(y-1)z(z-1)\quad+m\alpha_3\alpha_4x(x-1)y(y-1)zw$$

$$-m\alpha_3\alpha_4x(x-1)y^2zw\quad-n\alpha_4^2x(x-1)y^2z^2\quad-l\alpha_2\alpha_4x^2yz^2w\quad+l\alpha_2\alpha_4x^2yz(z-1)w$$

$$-n\alpha_4^2x^2y^2z(z-1)\quad+m\alpha_3\alpha_4x^2y^2zw\quad-m\alpha_3\alpha_4x^2y^2zw\quad+n\alpha_4^2x^2y^2z^2\quad+l\alpha_2\alpha_4x^2yz^2w$$

$$-l\alpha_2\alpha_4x^2yz^2w\quad+n\alpha_4^2x^2y^2z^2\quad-m\alpha_3\alpha_4x^2y^2zw\quad+m\alpha_3\alpha_4x^2y(y-1)zw\quad-n\alpha_4^2x^2y(y-1)z^2\}$$

$$+\frac{\mu}{|J|}\frac{A^2\alpha_1^{2x}\alpha_2^{2y}\alpha_3^{2z}\alpha_4^{2w}}{\alpha_1^2\alpha_2^2\alpha_3^2\alpha_4^2}\quad\{-k^2\alpha_1^2\alpha_4^2y(y-1)z(z-1)\quad+k^2\alpha_1^2\alpha_4^2y^2z^2\quad-km\alpha_1\alpha_3\alpha_4^2xy^2z$$

$$+kl\alpha_1\alpha_2\alpha_4^2xyz(z-1)\quad-kl\alpha_1\alpha_2\alpha_4^2xyz^2\quad+km\alpha_1\alpha_3\alpha_4^2xy(y-1)z\quad+kl\alpha_1\alpha_2\alpha_4^2xyz(z-1)$$

$$-kl\alpha_1\alpha_2\alpha_4^2xyz^2+lm\alpha_2\alpha_3\alpha_4^2x(x-1)yz-l^2\alpha_2^2\alpha_4^2x(x-1)z(z-1)+l^2\alpha_2^2\alpha_4^2x^2z^2-lm\alpha_2\alpha_3\alpha_4^2x^2yz$$

$$-km\alpha_1\alpha_3\alpha_4^2xy^2z+km\alpha_1\alpha_3\alpha_4^2xy(y-1)z-m^2\alpha_3^2\alpha_4^2x(x-1)y(y-1)+lm\alpha_2\alpha_3\alpha_4^2x(x-1)yz$$

$$-lm\alpha_2\alpha_3\alpha_4^2x^2yz+m^2\alpha_3^2\alpha_4^2x^2y^2\Big\}$$

$$=-\frac{1}{|J|}\frac{A^3xyzw\alpha_1^{3x}\alpha_2^{3y}\alpha_3^{3z}\alpha_4^{3w}B}{\alpha_1^2\alpha_2^2\alpha_3^2\Lambda}(yz+z+1)<0 \qquad (17)$$

From (17) we have realized that when per unit cost of subsidiary raw materials, *n* increases; the tendency of buying of subsidiary raw material, $\alpha_4$ decreases. This seems that enormous quantities of subsidiary raw materials are not necessary to run the organization smoothly.

**7. Conclusions**

In this study we have studied sensitivity analysis of various inputs of organization and have tried to predict the effects when the price of any one is increased. We have discussed the Cobb-Douglas production function with the subject to constraint of budget. We have used Lagrange multiplier method for the mathematical analysis procedures. Throughout the paper we have tried to show mathematical calculations in some details.

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