Paradigm Academic Press
Frontiers in Management Science
ISSN 2788-8592
JUN. 2023 VOL.2, NO.3

Economic Aspects of Profit Maximization if Cost of Principal Raw Material Increases

Devajit Mohajan[1] & Haradhan Kumar Mohajan[2]

[1] Department of Civil Engineering, Chittagong University of Engineering & Technology, Chittagong, Bangladesh

[2] Department of Mathematics, Premier University, Chittagong, Bangladesh

Correspondence: Haradhan Kumar Mohajan, Department of Mathematics, Premier University, Chittagong, Bangladesh.

doi:10.56397/FMS.2023.06.05

Abstract

In this study sensitivity analysis of various inputs is discussed, when per unit cost of principal raw materials is increased. The organization has its main target of achieving maximum profit through the adjustment of various inputs and outputs in future production. In this study Cobb-Douglas production function is considered as a profit function to investigate sensitivity analysis during profit maximization procedures. In this article the method of Lagrange multiplier is applied to represent higher dimensional unconstrained problem from the lower dimensional constrained problem. Moreover, the determinant of the 6×6 bordered Hessian matrix and 6×6 Jacobian are applied for the augmentation of the sensitivity analysis efficiently.

Keywords: principal raw materials, Lagrange multiplier, profit maximization, sensitivity analysis

1. Introduction

From the 19th century mathematical modeling became a popular method in social sciences (Samuelson, 1947). It supports mathematical concepts and language, and moves through an iterative process. In recent years it is using in economics and it increases welfare of the society (Eaton & Lipsey, 1975). The economists have gained enormous benefits from it. It helps the organizations for the development of their financial structure (Ferdous & Mohajan, 2022).

In this paper we have used Cobb-Douglas production function as an operation tool to discuss sensitivity analysis (Cobb & Douglas, 1928). We have used the determinant of 6×6 bordered Hessian matrix, 6×6 Jacobian, and four input variables; and have operated the overall study for sensitivity analysis. An organization not only hunts its own benefits but also sees the welfare of the society where it survives. In this study we have tried our best to give mathematical calculations more clearly in every step of our procedures.

2. Literature Review

In any kind of research, the literature review is a beginning section, which deals with a secondary research source where a researcher shows the works of previous researchers in the same field (Polit & Hungler, 2013). Two American professors, mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976) have derived the functional distribution of income between capital and labor (Cobb & Douglas, 1928). Later, another two US scholars: mathematician John V. Baxley and economist John C. Moorhouse have established a mathematical formulation for nontrivial constrained optimization problem with special reference for the application in economics (Baxley & Moorhouse, 1984).

Jamal Nazrul Islam, a famous mathematician of Bangladesh, and his dedicated coauthors have discussed the profit maximization problems by the representation and reasonable interpretation of the Lagrange multipliers

(Islam et al., 2009a, b, 2010a, b). Shaiara Husain and Md. Shahidul Islam have used the Cobb-Douglas production functions for impressive average annual growth of manufacturing sector of Bangladesh (Husain & Islam, 2016). Mathematician Pahlaj Moolio and his coauthors have also analyzed optimization problems in an organization (Moolio et al., 2009). Devajit Mohajan and Haradhan Kumar Mohajan have discussed profit maximization policies by using four variable inputs (Mohajan & Mohajan, 2022a).

3. Research Methodology of the Study

Research is an essential device to the academicians for the leading in academic area (Pandey & Pandey, 2015). Methodology is a guideline for performing a good research that helps the researchers to increase the trust of the readers (Kothari, 2008). Hence, research methodology is the collection of a set of principles for organizing, planning, designing and conducting a good research (Legesse, 2014).

We have used 6×6 bordered Hessian matrix and 6×6 Jacobian to make the profit function interesting to the readers. In this study we have depended on the optimization related mathematical secondary data sources (Mohajan, 2017b, 2018a, 2020).

4. Objective of the Study

The foremost objective of this study is to deliberate sensitivity analysis of various inputs of an organization when the cost of its principal raw materials is increased. Profit maximization is every organization's crucial objective and sensitivity analysis will help to know about the future production of the organization if cost of any one or more inputs is changed. Other insignificant but related objectives are as follows:

- to provide the predictions accurately,
- to show the mathematical representation properly, and
- to encourage the young researchers in the mathematical modeling in economics.

5. Economic Model

Let us consider that an organization uses x_1 quantity of capital, x_2 quantity of labor, x_3 quantity of principal raw materials, and x_4 quantity of irregular inputs. Now we can introduce Cobb-Douglas production function as a profit function for this study (Islam et al., 2011; Mohajan, 2018a),

$$P = f\left(x_1, x_2, x_3, x_4\right) = A x_1^p x_2^q x_3^r x_4^s \quad (1)$$

where A is the technical process of economic system that indicates total factor productivity. Here p, q, r, and s are parameters; p indicates the output of elasticity of capital measures the percentage change in P for 1% change in x_1, while x_2, x_3, and x_4 are held constants. Similar properties bear parameters q, r, and s. The organization wants to maximize the profit function (1) for the sustainability in the global economic environment. The values of p, q, r, and s are determined by the available technologies, and must satisfy the following four inequalities (Moolio et al., 2009; Mohajan, 2021a):

$$0 < p < 1,\ 0 < q < 1,\ 0 < r < 1, \text{ and } 0 < s < 1. \quad (2)$$

A strict Cobb-Douglas production function, in which $\Theta = p + q + r + s = 1$ indicates constant returns to scale, $\Theta < 1$ indicates decreasing returns to scale, and $\Theta > 1$ indicates increasing returns to scale (Roy et al., 2021; Mohajan, 2022).

Now we consider that the budget constraint,

$$B\left(x_1, x_2, x_3, x_4\right) = kx_1 + lx_2 + mx_3 + nx_4, \quad (3)$$

where k is rate of interest or services of capital per unit of capital x_1; l is the wage rate per unit of labor x_2; m is the cost per unit of principal raw material x_3; and n is the cost per unit of other inputs x_4.

Now we introduce a single Lagrange multiplier η, as a device; and by using equations (1) and (3) we can represent the Lagrangian function $W\left(x_1, x_2, x_3, x_4, \eta\right)$, in a 5-dimensional unconstrained problem as follows (Mohajan & Mohajan, 2022b; Mohajan, 2021b):

$$W\left(x_1, x_2, x_3, x_4, \eta\right) = A x_1^p x_2^q x_3^r x_4^s + \eta\left(B - kx_1 - lx_2 - mx_3 - nx_4\right), \quad (4)$$

where $\frac{\partial B}{\partial x_1} = B_1$, $\frac{\partial B}{\partial x_2} = B_2$, $\frac{\partial W}{\partial x_1} = W_1$, $\frac{\partial^2 W}{\partial x_1 \partial x_3} = W_{31}$, $\frac{\partial^2 W}{\partial x_2^2} = W_{22}$, etc. are partial derivatives. Let us consider the determinant of the 5×5 bordered Hessian matrix as,

$$H = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & W_{11} & W_{12} & W_{13} & W_{14} \\ -B_2 & W_{21} & W_{22} & W_{23} & W_{24} \\ -B_3 & W_{31} & W_{32} & W_{33} & W_{34} \\ -B_4 & W_{41} & W_{42} & W_{43} & W_{44} \end{vmatrix}. \tag{5}$$

Taking first-order partial differentiations of (3) we get,

$$B_1 = k,\ B_2 = l,\ B_3 = m, \text{ and } \quad B_4 = n. \tag{6}$$

Taking second-order and cross partial derivatives of (4) we get,

$$\begin{aligned} W_{11} &= p(p-1)Ax_1^{p-2}x_2^q x_3^r x_4^s, \\ W_{22} &= q(q-1)Ax_1^p x_2^{q-2} x_3^r x_4^s, \\ W_{33} &= r(r-1)Ax_1^p x_2^q x_3^{r-2} x_4^s, \\ u_{44} &= s(s-1)Ar_1^p r_2^q r_3^r r_4^{s-2}, \\ W_{12} = W_{21} &= pqAx_1^{p-1}x_2^{q-1}x_3^r x_4^s, \\ W_{13} = W_{31} &= prAx_1^{p-1}x_2^q x_3^{r-1} x_4^s, \\ W_{14} = W_{41} &= psAx_1^{p-1}x_2^q x_3^r x_4^{s-1}, \\ W_{23} = W_{32} &= qrAx_1^p x_2^{q-1} x_3^{r-1} x_4^s, \\ W_{24} = W_{42} &= qsAx_1^p x_2^{q-1} x_3^r x_4^{s-1}, \\ W_{34} = W_{43} &= rsAx_1^p x_2^q x_3^{r-1} x_4^{s-1}. \end{aligned} \tag{7}$$

Now we expand the bordered Hessian (5) as,

$$|H| = \frac{A^3 B^2 pqrs x_1^{3p} x_2^{3q} x_3^{3r} x_4^{3s}}{x_1^2 x_2^2 x_3^2 x_4^2 \Theta} > 0 \tag{8}$$

where $A > 0$, $a,b,c,d > 0$, and budget, $B > 0$, therefore, $|H| > 0$. Hence, the profit is maximized (Mohajan et al., 2013; Mohajan & Mohajan, 2022c).

6. Highlights on Matrix Operations

We have observed that the second order condition is satisfied, so that the determinant of (5) survives at the optimum, i.e., $|J| = |H|$; hence, we can apply the implicit function theorem. Let $\mathbf{G}$ be the vector-valued function of ten variables $\eta^*, x_1^*, x_2^*, x_3^*, x_4^*, k, l, m, n$, and *B*, and we define the function $\mathbf{G}$ for the point

$\left(\eta^*, x_1^*, x_2^*, x_3^*, x_4^*, k, l, m, n, B\right) \in R^{10}$, and take the values in R^5. By the Implicit Function Theorem of multivariable calculus, the equation,

$$F\left(\eta^*, x_1^*, x_2^*, x_3^*, x_4^*, k, l, m, n, B\right) = 0, \tag{9}$$

may be solved in the form of

$$\begin{bmatrix} \eta \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \mathbf{G}\left(k, l, m, n, B\right). \tag{10}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}$; regarded as $J_G = \dfrac{\partial\left(\eta, x_1, x_2, x_3, x_4\right)}{\partial\left(k, l, m, n, B\right)}$, and is presented by;

$$J_G = \begin{bmatrix} \frac{\partial \pi}{\partial k} & \frac{\partial \pi}{\partial l} & \frac{\partial \pi}{\partial m} & \frac{\partial \pi}{\partial n} & \frac{\partial \pi}{\partial B} \\ \frac{\partial x_1}{\partial k} & \frac{\partial x_1}{\partial l} & \frac{\partial x_1}{\partial m} & \frac{\partial x_1}{\partial n} & \frac{\partial x_1}{\partial B} \\ \frac{\partial x_2}{\partial k} & \frac{\partial x_2}{\partial l} & \frac{\partial x_2}{\partial m} & \frac{\partial x_2}{\partial n} & \frac{\partial x_2}{\partial B} \\ \frac{\partial x_3}{\partial k} & \frac{\partial x_3}{\partial l} & \frac{\partial x_3}{\partial m} & \frac{\partial x_3}{\partial n} & \frac{\partial x_3}{\partial B} \\ \frac{\partial x_4}{\partial k} & \frac{\partial x_4}{\partial l} & \frac{\partial x_4}{\partial m} & \frac{\partial x_4}{\partial n} & \frac{\partial x_4}{\partial B} \end{bmatrix}. \tag{11}$$

$$= -J^{-1} \begin{bmatrix} -x_1 & -x_2 & -x_3 & -x_4 & 1 \\ -\eta & 0 & 0 & 0 & 0 \\ 0 & -\eta & 0 & 0 & 0 \\ 0 & 0 & -\eta & 0 & 0 \\ 0 & 0 & 0 & -\eta & 0 \end{bmatrix}.$$

The inverse of Jacobian matrix is, $J^{-1} = \dfrac{1}{|J|} C^T$, where $C = \left(C_{ij}\right)$, the matrix of cofactors of J, where T indicates transpose, then (11) becomes (Mohajan, 2017a; Mohajan & Mohajan, 2022d),

$$= -\frac{1}{|J|} \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix} \begin{bmatrix} -x_1 & -x_2 & -x_3 & -x_4 & 1 \\ -\eta & 0 & 0 & 0 & 0 \\ 0 & -\eta & 0 & 0 & 0 \\ 0 & 0 & -\eta & 0 & 0 \\ 0 & 0 & 0 & -\eta & 0 \end{bmatrix}$$

$$J_G = -\frac{1}{|J|}\begin{bmatrix} -x_1C_{11}-\eta C_{21} & -x_2C_{11}-\eta C_{31} & -x_3C_{11}-\eta C_{41} & -x_4C_{11}-\eta C_{51} & C_{11} \\ -x_1C_{12}-\eta C_{22} & -x_2C_{12}-\eta C_{32} & -x_3C_{12}-\eta C_{42} & -x_4C_{12}-\eta C_{52} & C_{12} \\ -x_1C_{13}-\eta C_{23} & -x_2C_{13}-\eta C_{33} & -x_3C_{13}-\eta C_{43} & -x_4C_{13}-\eta C_{53} & C_{13} \\ -x_1C_{14}-\eta C_{24} & -x_2C_{14}-\eta C_{34} & -x_3C_{14}-\eta C_{44} & -x_4C_{14}-\eta C_{54} & C_{14} \\ -x_1C_{15}-\eta C_{25} & -x_2C_{15}-\eta C_{35} & -x_3C_{15}-\eta C_{45} & -x_4C_{15}-\eta C_{55} & C_{15} \end{bmatrix}. \tag{12}$$

In (11) there are total 25 comparative statics, and for sensitivity analysis we will try only four of these to predict the economic analysis for the profit maximization (Baxley & Moorhouse, 1984; Mohajan & Mohajan, 2022e).

7. Sensitivity Analysis

Now we analyze the effect on capital x_1 when per unit cost of principal raw material, *m* increases. Taking T_{23} (i.e., term of 2nd row and 3rd column) from both sides of (12) we get (Islam et al., 2011; Roy et al., 2021),

$$\frac{\partial x_1}{\partial m} = \frac{x_3}{|J|}[C_{12}] + \frac{\eta}{|J|}[C_{42}]$$

$$= \frac{x_3}{|J|}\text{Cofactor of } C_{12} + \frac{\eta}{|J|}\text{Cofactor of } C_{42}$$

$$= -\frac{x_3}{|J|}\begin{vmatrix} -B_1 & W_{12} & W_{13} & W_{14} \\ -B_2 & W_{22} & W_{23} & W_{24} \\ -B_3 & W_{32} & W_{33} & W_{34} \\ -B_4 & W_{42} & W_{43} & W_{44} \end{vmatrix} + \frac{\eta}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & W_{12} & W_{13} & W_{14} \\ -B_2 & W_{22} & W_{23} & W_{24} \\ -B_4 & W_{42} & W_{43} & W_{44} \end{vmatrix}$$

$$= -\frac{x_3}{|J|}\left\{ -B_1\begin{vmatrix} W_{22} & W_{23} & W_{24} \\ W_{32} & W_{33} & W_{34} \\ W_{42} & W_{43} & W_{44} \end{vmatrix} - W_{12}\begin{vmatrix} -B_2 & W_{23} & W_{24} \\ -B_3 & W_{33} & W_{34} \\ -B_4 & W_{43} & W_{44} \end{vmatrix} + W_{13}\begin{vmatrix} -B_2 & W_{22} & W_{24} \\ -B_3 & W_{32} & W_{34} \\ -B_4 & W_{42} & W_{44} \end{vmatrix} \right.$$

$$\left. -W_{14}\begin{vmatrix} -B_2 & W_{22} & W_{23} \\ -B_3 & W_{32} & W_{33} \\ -B_4 & W_{42} & W_{43} \end{vmatrix} \right\} + \frac{\eta}{|J|}\left\{ B_2\begin{vmatrix} -B_1 & W_{13} & W_{14} \\ -B_2 & W_{23} & W_{24} \\ -B_4 & W_{43} & W_{44} \end{vmatrix} - B_3\begin{vmatrix} -B_1 & W_{12} & W_{14} \\ -B_2 & W_{22} & W_{24} \\ -B_4 & W_{42} & W_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_1 & W_{12} & W_{13} \\ -B_2 & W_{22} & W_{23} \\ -B_4 & W_{42} & W_{43} \end{vmatrix} \right\}$$

$$= -\frac{x_3}{|J|}\left[-B_1\{W_{22}(W_{33}W_{44} - W_{43}W_{34}) + W_{23}(W_{42}W_{34} - W_{32}W_{44}) + W_{24}(W_{32}W_{43} - W_{42}W_{33})\} \right.$$

$$-W_{12}\{-B_2(W_{33}W_{44} - W_{43}W_{34}) + W_{23}(-B_4W_{34} + B_3W_{44}) + W_{24}(-B_3W_{43} + B_4W_{33})\}$$

$$+W_{13}\{-B_2(W_{32}W_{44} - W_{42}W_{34}) + W_{22}(-B_4W_{34} + B_3W_{44}) + W_{24}(-B_3W_{42} + B_4W_{32})\}$$

$$\left. -W_{14}\{-B_2(W_{32}W_{43} - W_{42}W_{33}) + W_{22}(-B_4W_{33} + B_3W_{43}) + W_{23}(-B_3W_{42} + B_4W_{32})\} \right]$$

$$+\frac{\eta}{|J|}\left[B_2\{-B_1(W_{23}W_{44} - W_{43}W_{24}) - W_{13}(-B_2W_{44} + B_4W_{24}) + W_{14}(-B_2W_{43} + B_4W_{23})\} \right.$$

$$-B_3\{-B_1(W_{22}W_{44} - W_{42}W_{24}) - W_{12}(-B_2W_{44} + B_4W_{24}) + W_{14}(-B_2W_{42} + B_4W_{22})\}$$

$$\left. +B_4\{-B_1(W_{22}W_{43} - W_{42}W_{23}) - W_{12}(-B_2W_{43} + B_4W_{23}) + W_{13}(-B_2W_{42} + B_4W_{22})\} \right]$$

$$=-\frac{x_3}{|J|}\{-B_1W_{22}W_{33}W_{44}\quad +B_1W_{22}W_{43}W_{34}\quad -B_1W_{23}W_{42}W_{34}\quad +B_1W_{23}W_{32}W_{44}\quad -B_1W_{24}W_{32}W_{43}$$

$$+B_1W_{24}W_{42}W_{33}$$

$$+B_2W_{12}W_{33}W_{44}-B_2W_{12}W_{43}W_{34}+B_4W_{12}W_{23}W_{34}-B_3W_{12}W_{23}W_{44}+B_3W_{12}W_{24}W_{43}-B_4W_{12}W_{24}W_{33}$$

$$-B_2W_{13}W_{32}W_{44}+B_2W_{13}W_{42}W_{34}-B_2W_{13}W_{22}W_{34}+B_3W_{13}W_{22}W_{44}-B_3W_{13}W_{24}W_{42}+B_4W_{13}W_{24}W_{32}$$

$$+B_2W_{14}W_{32}W_{43}-B_2W_{14}W_{42}W_{33}+B_4W_{14}W_{22}W_{33}-B_3W_{14}W_{22}W_{43}+B_3W_{14}W_{23}W_{42}-B_4W_{14}W_{23}W_{32}\}$$

$$+\frac{\eta}{|J|}\{-B_1B_2W_{23}W_{44}\quad +B_1B_2W_{24}W_{34}\quad +B_2^2W_{13}W_{44}\quad -B_2B_4W_{13}W_{24}\quad -B_2^2W_{14}W_{34}\quad +B_2B_4W_{14}W_{23}$$

$$+B_1B_3W_{22}W_{44}-B_1B_3W_{24}^2-B_2B_3W_{12}W_{44}+B_3B_4W_{12}W_{24}+B_2B_3W_{14}W_{24}-B_3B_4W_{14}W_{22}-B_1B_4W_{22}W_{34}$$

$$+B_1B_4W_{23}W_{24}+B_2B_4W_{12}W_{34}-B_4^2W_{12}W_{23}-B_2B_4W_{13}W_{24}+B_4^2W_{13}W_{22}\}$$

$$=-\frac{x_3}{|J|}\frac{A^3x_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2^2x_3^2x_4^2}\{-kx_1^2q(q-1)r(r-1)s(s-1)+kx_1^2q(q-1)r^2s^2-kx_1^2q^2r^2s^2+kx_1^2q^2r^2s(s-1)$$

$$-kx_1^2q^2r^2s^2\quad +kx_1^2q^2r(r-1)s^2\quad +lx_1x_2pqr(r-1)s(s-1)\quad -lx_1x_2pqr^2s^2\quad +nx_1x_4pq^2r^2s$$

$$+mx_1x_3pq^2rs^2\quad -mx_1x_3pq^2rs(s-1)\quad -nx_1x_4pq^2r(r-1)s\quad -lx_1x_2pqr^2s(s-1)\quad +lx_1x_2pqr^2s^2$$

$$-nx_1x_4pq(q-1)r^2s\quad +mx_1x_3pq(q-1)rs(s-1)\quad -mx_1x_3pq^2rs^2\quad +nx_1x_4pq^2r^2s\quad +lx_1x_2pqr^2s^2$$

$$-lx_1x_2pqr(r-1)s^2+nx_1x_4pq(q-1)r(r-1)s-mx_1x_3pq(q-1)rs^2+mx_1x_3pq^2rs^2-nx_1x_4pq^2r^2s\}$$

$$+\frac{\eta}{|J|}\frac{A^2x_1^{2p}x_2^{2q}x_3^{2r}x_4^{2s}}{x_1^2x_2^2x_3^2x_4^2}\{-klx_1^2x_2x_3qrs(s-1)+klx_1^2x_2x_3qrs^2+l^2x_1x_2^2x_3pqs(s-1)-nlx_1x_2x_3x_4pqrs$$

$$-l^2x_1x_2^2x_3x_4prs^2+nlx_1x_2x_3x_4pqrs+kmx_1^2x_2^2q(q-1)s(s-1)-kmx_1^2x_3^2q^2s^2-lmx_1x_2x_3^2pqs(s-1)$$

$$+mnx_1x_3^2x_4pq^2s\quad +lmx_1x_2x_3^2pqs^2\quad -mnx_1x_3^2x_4pq(q-1)s\quad -knx_1^2x_3x_4q(q-1)rs\quad +knx_1^2x_3x_4pq^2rs$$

$$+nlx_1x_2x_3x_4pqrs-n^2x_1x_3x_4^2pq^2r-nlx_1x_2x_3x_4pqrs+n^2x_1x_3x_4^2pq(q-1)r\}$$

$$=-\frac{x_3}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2^2x_3^2x_4^2}\{-kx_1^2p^{-1}(q-1)(r-1)(s-1)+kx_1^2p^{-1}(q-1)rs-kx_1^2p^{-1}qrs-kx_1^2p^{-1}qrs$$

$$+kr_1^2x^{-1}qr(s-1)\quad +kx_1^2p^{-1}q(r-1)s\quad +lx_1x_2(r-1)(s-1)\quad -lx_1x_2rs\quad +nx_1x_4qr\quad -mx_1x_3q(s-1)$$

$$+mx_1x_3qs\ -nx_1x_4q(r-1)-lx_1x_2r(s-1)+lx_1x_2rs-nx_1x_4(q-1)r+mx_1x_3(q-1)(s-1)-mx_1x_3qs$$

$$+nx_1x_4qr \quad +lx_1x_2rs \quad -lx_1x_2(r-1)s \quad +nx_1x_4(q-1)(r-1) \quad -mx_1x_3(q-1)s \quad +mx_1x_3qs \quad -nx_1x_4qr\}$$

$$+\frac{1}{|J|}\frac{A^2x_1^{2p}x_2^{2q}x_3^{2r}x_4^{2s}}{x_1x_2^2x_3x_4^2}\ \frac{Aqrx_1^px_2^qx_3^rx_4^s\Theta x_1x_3}{B}\ \{-klx_1x_2(s-1) \ +klx_1x_2s \ +l^2x_2^2prs(s-1) \ -l^2x_2^2prs^2$$

$$+kmx_1x_3q(q-1)s(s-1) \quad -kmx_1x_3q^2s^2 \quad -lmx_2x_3pqs(s-1) \quad +lmx_2x_3pqs^2 \quad +mnx_3x_4pq^2s$$

$$-mnx_3x_4pq(q-1)s - knx_1x_4q(q-1)rs + knx_1x_4pq^2rs - n^2x_4^2pq^2r + n^2x_4^2pq(q-1)r\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1x_2^2x_3x_4^2\Theta}\{(1-\Theta)+(p+q+r+s)\}+\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1x_2^2x_3x_4^2\Theta}$$

$$\frac{\partial x_1}{\partial m}=-\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1x_2^2x_3x_4^2\Theta}+\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1x_2^2x_3x_4^2\Theta}=0\,. \tag{13}$$

From the relation (13) we have realized that when per unit cost of principal raw material, *m* increases, there will be no effect on the level of capital x_1. Hence, from this study it seems that there is no relation between capital x_1 and principal raw materials x_3. In any organization, principal raw materials are the necessary goods and perhaps increased cost is managed from the profit of the organization or it is managed from the other sources of funds.

Now we inspect the effect on worker x_2 when per unit cost of principal raw material, *m* increases. Taking T_{33} (i.e., term of 3rd row and 3rd column) from both sides of (12) we get (Mohajan, 2022; Wiese, 2021),

$$\frac{\partial x_2}{\partial m}=\frac{x_3}{|J|}[C_{13}]+\frac{\eta}{|J|}[C_{43}]$$

$$=\frac{x_3}{|J|}\text{Cofactor of } C_{13}+\frac{\eta}{|J|}\text{Cofactor of C}_{43}$$

$$=\frac{x_3}{|J|}\begin{vmatrix}-B_1 & W_{11} & W_{13} & W_{14}\\ -B_2 & W_{21} & W_{23} & W_{24}\\ -B_3 & W_{31} & W_{33} & W_{34}\\ -B_4 & W_{41} & W_{43} & W_{44}\end{vmatrix}-\frac{\eta}{|J|}\begin{vmatrix}0 & -B_1 & -B_3 & -B_4\\ -B_1 & W_{11} & W_{13} & W_{14}\\ -B_2 & W_{21} & W_{23} & W_{24}\\ -B_4 & W_{41} & W_{43} & W_{44}\end{vmatrix}$$

$$=\frac{x_3}{|J|}\left\{-B_1\begin{vmatrix}W_{21} & W_{23} & W_{24}\\ W_{31} & W_{33} & W_{34}\\ W_{41} & W_{43} & W_{44}\end{vmatrix}-W_{11}\begin{vmatrix}-B_2 & W_{23} & W_{24}\\ -B_3 & W_{33} & W_{34}\\ -B_4 & W_{43} & W_{44}\end{vmatrix}+W_{13}\begin{vmatrix}-B_2 & W_{21} & W_{24}\\ -B_3 & W_{31} & W_{34}\\ -B_4 & W_{41} & W_{44}\end{vmatrix}\right.$$

$$\left.-W_{14}\begin{vmatrix}-B_2 & W_{21} & W_{23}\\ -B_3 & W_{31} & W_{33}\\ -B_4 & W_{41} & W_{43}\end{vmatrix}\right\}-\frac{\eta}{|J|}\left\{B_1\begin{vmatrix}-B_1 & W_{13} & W_{14}\\ -B_2 & W_{23} & W_{24}\\ -B_4 & W_{43} & W_{44}\end{vmatrix}-B_3\begin{vmatrix}-B_1 & W_{11} & W_{14}\\ -B_2 & W_{21} & W_{24}\\ -B_4 & W_{41} & W_{44}\end{vmatrix}+B_4\begin{vmatrix}-B_1 & W_{11} & W_{13}\\ -B_2 & W_{21} & W_{23}\\ -B_4 & W_{41} & W_{43}\end{vmatrix}\right\}$$

$$=\frac{x_3}{|J|}[-B_1\{W_{21}(W_{33}W_{44}-W_{43}W_{34})+W_{23}(W_{41}W_{34}-W_{31}W_{44})+W_{24}(W_{31}W_{43}-W_{41}W_{33})\}$$

$$-W_{11}\{-B_2(W_{33}W_{44}-W_{43}W_{34})+W_{23}(-B_4W_{34}+B_3W_{44})+W_{24}(-B_3W_{43}+B_4W_{33})\}$$

$$+W_{13}\{-B_2(W_{31}W_{44}-W_{41}W_{34})+W_{21}(-B_4W_{34}+B_3W_{44})+W_{24}(-B_3W_{41}+B_4W_{31})\}$$

$$-W_{14}\{-B_2(W_{31}W_{43}-W_{41}W_{33})+W_{21}(-B_4W_{33}+B_3W_{43})+W_{23}(-B_3W_{41}+B_4W_{31})\}]$$

$$-\frac{\eta}{|J|}[B_1\{-B_1(W_{23}W_{44}-W_{43}W_{24})+W_{13}(-B_4W_{24}+B_2W_{44})+W_{14}(-B_2W_{43}+B_4W_{23})\}$$

$$-B_3\{-B_1(W_{21}W_{44}-W_{41}W_{24})+W_{11}(-B_4W_{24}+B_2W_{44})+W_{14}(-B_2W_{41}+B_4W_{21})\}$$

$$+B_4\{-B_1(W_{21}W_{43}-W_{41}W_{23})+W_{11}(-B_4W_{23}+B_2W_{43})+W_{13}(-B_2W_{41}+B_4W_{21})\}]$$

$$=-\frac{x_3}{|J|}\{B_1W_{21}W_{33}W_{44}-B_1W_{21}W_{43}W_{34}+B_1W_{23}W_{41}W_{24}-B_1W_{23}W_{31}W_{44}+B_1W_{24}W_{31}W_{43}-B_1W_{24}W_{41}W_{33}$$

$$-B_2W_{11}W_{33}W_{44}+B_2W_{11}W_{43}W_{34}-B_4W_{11}W_{23}W_{34}+B_3W_{11}W_{23}W_{44}-B_3W_{11}W_{24}W_{43}+B_4W_{11}W_{24}W_{33}$$

$$+B_2W_{13}W_{31}W_{44}-B_2W_{13}W_{41}W_{34}+B_4W_{13}W_{21}W_{34}-B_3W_{13}W_{21}W_{44}+B_3W_{13}W_{24}W_{41}-B_4W_{13}W_{24}W_{31}$$

$$-B_2W_{14}W_{31}W_{43}+B_2W_{14}W_{41}W_{33}-B_4W_{14}W_{21}W_{33}+B_3W_{14}W_{21}W_{43}-B_3W_{14}W_{23}W_{41}+B_4W_{14}W_{23}W_{31}\}$$

$$-\frac{\eta}{|J|}\{-B_1^2W_{23}W_{44}+B_1^2W_{24}W_{43}-B_1B_4W_{13}W_{24}+B_1B_2W_{13}W_{44}-B_1B_2W_{14}W_{43}+B_1B_4W_{14}W_{23}$$

$$+B_1B_3W_{21}W_{44}-B_1B_3W_{41}W_{24}+B_3B_4W_{11}W_{24}-B_2B_3W_{11}W_{44}+B_2B_3W_{14}W_{41}-B_3B_4W_{14}W_{21}$$

$$-B_1B_4W_{21}W_{43}+B_1B_4W_{41}W_{23}-B_4^2W_{11}W_{23}+B_2B_4W_{11}W_{43}-B_2B_4W_{13}W_{41}+B_4^2W_{13}W_{21}\}$$

$$=-\frac{x_3}{|J|}\frac{A^3x_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2^2x_3^2x_4^2}\{kx_1x_2pqr(r-1)s(s-1)-kx_1x_2pqr^2s^2+kx_1x_2pqr^2s^2-kx_1x_2pqr^2s(s-1)$$

$$+kx_1x_2pqr^2s^2-kx_1x_2pqr(r-1)s^2-lx_2^2p(p-1)r(r-1)s(s-1)+lx_2^2p(p-1)r^2s^2$$

$$-nx_2x_4p(p-1)qr^2s+mx_2x_3p(p-1)qrs(s-1)-mx_2x_3p(p-1)qrs^2+nx_2x_4p(p-1)qr(r-1)s$$

$$+lx_2^2p^2r^2s(s-1)-lx_2^2p^2r^2s^2+nx_2x_4p^2qr^2s-mx_2x_3p^2qrs(s-1)+mx_2x_3p^2qrs^2-nx_2x_4p^2qr^2s$$

$$-lx_2^2p^2r^2s^2+lx_2^2p^2r(r-1)s^2-nx_2x_4p^2qr(r-1)s+mx_2x_3p^2qrs^2-mx_2x_3p^2qrs^2+nx_2x_4p^2qr^2s\}$$

$$-\frac{1}{|J|}\frac{A^2x_1^{2p}x_2^{2q}x_3^{2r}x_4^{2s}}{x_1^2x_2x_3x_4^2}\frac{Aqrx_1^px_2^qx_3^rx_4^s\Theta}{B}\{-k^2x_1^2x_2x_3qrs(s-1)+k^2x_1^2x_2x_3qrs^2-knx_1x_2x_3x_4pqrs$$

$$+klx_1x_2^2x_3prs(s-1)+klx_1x_2^2x_3prs^2+knx_1x_2x_3x_4pqrs+kmx_1x_2x_3^2pqs(s-1)-kmx_1x_2x_3^2pqs^2$$

$$+mnx_2x_3^2x_4p(p-1)qs\ -lmx_2^2x_3^2p(p-1)s(s-1)+lmx_2^2x_3^2p^2s^2\ -mnx_2x_3^2x_4p^2qs\ -knx_1x_2x_3x_4pqrs$$

$$+knx_1x_2x_3x_4pqrs-n^2x_2x_3x_4^2p(p-1)qr\ +nlx_2^2x_3x_4p(p-1)rs\ -nlx_2^2x_3x_4p^2rs\ +n^2x_2x_3x_4^2p^2qr\}$$

$$=-\frac{1}{|J|}\frac{pqrsA^3x_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2x_3x_4^2}\ \{kx_1(r-1)(s-1)\ -kx_1r(s-1)\ +kx_1rs\ -kx_1(r-1)s\ +lx_2pq^{-1}r(s-1)$$

$$-lx_2pq^{-1}rs\quad +lx_2pq^{-1}(r-1)s\quad -lx_2pq^{-1}rs\quad -lx_2(p-1)q^{-1}(r-1)(s-1)\quad +lx_2(p-1)q^{-1}rs$$

$$-mx_3(p-1)s\ +mx_3(p-1)(s-1)-mx_3p(s-1)+mx_3ps\ +nx_4(p-1)(r-1)-nx_4(p-1)r\ +nx_4pr$$

$$-nx_4p(r-1)\}$$

$$-\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}\Theta}{x_3^2x_2x_3x_4^2B}\{-k^2x_1^2qrs(s-1)\quad +k^2x_1^2qrs^2\quad +klx_1x_2prs(s-1)\quad +klx_1x_2prs^2$$

$$+kmx_1x_3pqs(s-1)\ -kmx_1x_3pqs^2\ +mnx_3x_4p(p-1)qs\ -mnx_3x_4p^2qs\ -lmx_2x_3p(p-1)s(s-1)$$

$$+lmx_2x_3p^2s^2\ +nlx_2x_4p(p-1)rs-nlx_2x_4p^2rs-n^2x_4^2p(p-1)qr\ +n^2x_4^2p^2qr\}$$

$$\frac{\partial x_2}{\partial m}=-\frac{1}{|J|}\frac{A^3Bpqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_3^2x_2x_3x_4^2\Theta}+\frac{1}{|J|}\frac{A^3Bpqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_3^2x_2x_3x_4^2\Theta}=0. \tag{14}$$

From the equation (14) we have realized that when per unit cost of principal raw material, *m* increases, there will be no effect on the labor x_2. Hence, there is no relation between labor x_2 and principal raw materials x_3. It seems that increase or decrease of one of them will not affect the other.

Now we test the effect on principal raw material x_3 when per unit cost of it increases. Taking T_{43} (i.e., term of 4th row and 3rd column) from both sides of (12) we get (Roy et al., 2021; Mohajan & Mohajan, 2022f),

$$\frac{\partial x_3}{\partial m}=\frac{x_3}{|J|}[C_{14}]+\frac{\eta}{|J|}[C_{44}]$$

$$=\frac{x_3}{|J|}\text{Cofactor of }C_{14}+\frac{\eta}{|J|}\text{Cofactor of C}_{44}$$

$$=-\frac{x_3}{|J|}\begin{vmatrix}-B_1 & W_{11} & W_{12} & W_{14}\\ -B_2 & W_{21} & W_{22} & W_{24}\\ -B_3 & W_{31} & W_{32} & W_{34}\\ -B_4 & W_{41} & W_{42} & W_{44}\end{vmatrix}+\frac{\eta}{|J|}\begin{vmatrix}0 & -B_1 & -B_2 & -B_4\\ -B_1 & W_{11} & W_{12} & W_{14}\\ -B_2 & W_{21} & W_{22} & W_{24}\\ -B_4 & W_{41} & W_{42} & W_{44}\end{vmatrix}$$

$$=-\frac{x_3}{|J|}\left\{-B_1\begin{vmatrix}W_{21} & W_{22} & W_{24}\\ W_{31} & W_{32} & W_{34}\\ W_{41} & W_{42} & W_{44}\end{vmatrix}-W_{11}\begin{vmatrix}-B_2 & W_{22} & W_{24}\\ -B_3 & W_{32} & W_{34}\\ -B_4 & W_{42} & W_{44}\end{vmatrix}+W_{12}\begin{vmatrix}-B_2 & W_{21} & W_{24}\\ -B_3 & W_{31} & W_{34}\\ -B_4 & W_{41} & W_{44}\end{vmatrix}-W_{14}\begin{vmatrix}-B_2 & W_{21} & W_{22}\\ -B_3 & W_{31} & W_{32}\\ -B_4 & W_{41} & W_{42}\end{vmatrix}\right\}$$

$$+\frac{\eta}{|J|}\left\{B_1\begin{vmatrix}-B_1 & W_{12} & W_{14}\\ -B_2 & W_{22} & W_{24}\\ -B_4 & W_{42} & W_{44}\end{vmatrix}-B_2\begin{vmatrix}-B_1 & W_{11} & W_{14}\\ -B_2 & W_{21} & W_{24}\\ -B_4 & W_{41} & W_{44}\end{vmatrix}+B_4\begin{vmatrix}-B_1 & W_{11} & W_{12}\\ -B_2 & W_{21} & W_{22}\\ -B_4 & W_{41} & W_{42}\end{vmatrix}\right\}$$

$$=-\frac{x_3}{|J|}\left[-B_1\{W_{21}(W_{32}W_{44}-W_{42}W_{34})+W_{22}(W_{41}W_{34}-W_{31}W_{44})+W_{24}(W_{31}W_{42}-W_{41}W_{32})\}\right.$$

$$-W_{11}\{-B_2(W_{32}W_{44}-W_{42}W_{34})+W_{22}(-B_4W_{34}+B_3W_{44})+W_{24}(-B_3W_{42}+B_4W_{32})\}$$

$$+W_{12}\{-B_2(W_{31}W_{44}-W_{41}W_{34})+W_{21}(-B_4W_{34}+B_3W_{44})+W_{24}(-B_3W_{41}+B_4W_{31})\}$$

$$\left.-W_{14}\{-B_2(W_{31}W_{42}-W_{41}W_{32})+W_{21}(-B_4W_{32}+B_3W_{42})+W_{22}(-B_3W_{41}+B_4W_{31})\}\right]$$

$$+\frac{\eta}{|J|}\left[B_1\{-B_1(W_{22}W_{44}-W_{42}W_{24})+W_{12}(-B_4W_{24}+B_2W_{44})+W_{14}(-B_2W_{42}+B_4W_{22})\}\right.$$

$$-B_2\{-B_1(W_{21}W_{44}-W_{41}W_{24})+W_{11}(-B_4W_{24}+B_2W_{44})+W_{14}(-B_2W_{41}+B_4W_{21})\}$$

$$\left.+B_4\{-B_1(W_{21}W_{42}-W_{41}W_{22})+W_{11}(-B_4W_{22}+B_2W_{42})+W_{12}(-B_2W_{41}+B_4W_{21})\}\right]$$

$$=-\frac{x_3}{|J|}\{-B_1W_{21}W_{32}W_{44}+B_1W_{21}W_{42}W_{34}-B_1W_{22}W_{41}W_{34}+B_1W_{22}W_{31}W_{44}-B_1W_{24}W_{31}W_{42}$$

$$+B_1W_{24}W_{41}W_{32}+B_2W_{11}W_{32}W_{44}-B_2W_{11}W_{42}W_{34}+B_4W_{11}W_{22}W_{34}-B_3W_{11}W_{22}W_{44}+B_3W_{11}W_{24}W_{42}$$

$$-B_4W_{11}W_{24}W_{32}-B_2W_{12}W_{31}W_{44}+B_2W_{12}W_{41}W_{34}-B_4W_{12}W_{21}W_{34}+B_3W_{12}W_{21}W_{44}-B_3W_{12}W_{24}W_{41}$$

$$+B_4W_{12}W_{24}W_{31}+B_2W_{14}W_{31}W_{42}-B_2W_{14}W_{41}W_{32}+B_4W_{14}W_{21}W_{32}-B_3W_{14}W_{21}W_{42}+B_3W_{14}W_{22}W_{41}$$

$$-B_4W_{14}W_{22}W_{31}\}+\frac{\eta}{|J|}\{-B_1^2W_{22}W_{44}+B_1^2W_{42}W_{24}-B_1B_4W_{12}W_{24}+B_1B_2W_{12}W_{44}-B_1B_2W_{14}W_{42}$$

$$+B_1B_4W_{14}W_{22}+B_1B_2W_{21}W_{44}-B_1B_2W_{41}W_{24}+B_2B_4W_{11}W_{24}-B_2^2W_{11}W_{44}+B_2^2W_{14}W_{41}-B_2B_4W_{14}W_{21}$$

$$-B_1B_4W_{21}W_{42}+B_1B_4W_{41}W_{22}-B_4^2W_{11}W_{22}+B_2B_4W_{11}W_{42}-B_2B_4W_{12}W_{41}+B_4^2W_{12}W_{21}\}$$

$$=-\frac{1}{|J|}\frac{A^3x_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2^2x_3x_4^2}\{-kx_1x_3pq^2rs(s-1)+kx_1x_3pq^2rs^2-kx_1x_3pq(q-1)rs^2$$

$$+kx_1x_3pq(q-1)rs(s-1)-kx_1x_3pq^2rs^2+kx_1x_3pq^2rs^2+lx_2x_3p(p-1)qrs(s-1)$$

$$-lx_2x_3p(p-1)qrs^2+nx_3x_4p(p-1)q(q-1)rs-mx_3^2p(p-1)q(q-1)s(s-1)+mx_3^2p(p-1)q^2s^2$$

$$-nx_3x_4p(p-1)q^2rs-lx_2x_3p^2qrs(s-1)+lx_2x_3p^2qrs^2-nx_3x_4p^2q^2rs+mx_3^2p^2q^2s(s-1)$$

$$-mx_3^2p^2q^2s^2 \quad +nx_3x_4p^2q^2rs \quad +lx_2x_3p^2qrs^2 \quad -lx_2x_3p^2qrs^2 \quad +nx_3x_4p^2q^2rs \quad -mx_3^2p^2q^2s^2$$

$$+mx_3^2p^2q(q-1)s^2 - nx_3x_4p^2q(q-1)rs\Big\}$$

$$+\frac{1}{|J|}\frac{A^2x_1^{2p}x_2^{2q}x_3^{2r}x_4^{2s}}{x_1^2x_2^2x_3^2x_4^2}\frac{Ax_1^px_2^qx_3^rx_4^s\Theta}{B}\Big\{-k^2x_1^2x_3^2q(q-1)s(s-1) \quad +k^2x_1^2x_3^2q^2s^2 \quad -knx_1x_3^2x_4pq^2s$$

$$+klx_1x_2x_3^2pqs(s-1) \; -klx_1x_2x_3^2pqs^2 \; +knx_1x_3^2x_4pq(q-1)s \; +klx_1x_2x_3^2pqs(s-1) \; -klx_1x_2x_3^2pqs^2$$

$$+nlx_2x_3^2x_4p(p-1)qs \; -l^2x_2^2x_3^2p(p-1)s(s-1) \; +l^2x_2^2x_3^2p^2s^2 \; -nlx_2x_3^2x_4p^2qs \; -knx_1x_3^2x_4pq^2s$$

$$+knx_1x_3^2x_4pq(q-1)s \quad -n^2x_3^2x_4^2p(p-1)q(q-1) \quad +nlx_2x_3^2x_4p(p-1)qs \quad -nlx_2x_3^2x_4p^2qs$$

$$+n^2x_3^2x_4^2p^2q^2\Big\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2^2x_4^2}\;\{-kx_1q(s-1) - kx_1(q-1)s \; +kx_1(q-1)(s-1) + kx_1qs \; +lx_2(p-1)(s-1)$$

$$-lx_2p(s-1) \; +lx_2ps \; -lx_2(p-1)s \; -2mx_3pqr^{-1}s \; +mx_3p(q-1)r^{-1}s \; -mx_3(p-1)(q-1)r^{-1}(s-1)$$

$$+mx_3pqr^{-1}(s-1) + mx_3(p-1)qr^{-1}s \; +nx_4(p-1)(q-1) - nx_4(p-1)q \; +nx_4pq \; -nx_4p(q-1)\}$$

$$+\frac{1}{|J|}\frac{A^3x_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_4^2\Theta}\Big\{-k^2x_1^2q(q-1)s(s-1) \quad +k^2x_1^2q^2s^2 \quad -2knx_1x_4pq^2s \quad +2knx_1x_4pq(q-1)s$$

$$+2klx_1x_2pqs(s-1) \quad -2klx_1x_2pqs^2 \quad +nlx_2x_4p(p-1)qs \quad -l^2x_2^2p(p-1)s(s-1) \quad +l^2x_2^2p^2s^2$$

$$-nlx_2x_4p^2qs + nlx_2x_4p(p-1)qs - nlx_2x_3^2x_4p^2qs - n^2x_4^2p(p-1)q(q-1) + n^2x_4^2p^2q^2\Big\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_4^2\Theta}\quad\{-pq(s-1) \quad +p(q-1)(s-1) \quad +(p-1)q(s-1) \quad +pqs \quad -(p-1)qs$$

$$-(p-1)(q-1)(s-1) \quad +(p-1)(q-1)s \quad -p(q-1)s\} \quad +\frac{1}{|J|}\frac{A^3pqsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_4^2\Theta}\{-p(q-1)(s-1)$$

$$-3pqs + 2p(q-1)s + 2pq(s-1) + 2(p-1)qs - (p-1)q(s-1) - (p-1)(q-1)s\}$$

$$\frac{\partial x_3}{\partial m} = -\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_4^2\Theta} - \frac{1}{|J|}\frac{A^3pqsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_4^2\Theta}(p+q+s)$$

$$\frac{\partial x_3}{\partial m} = -\frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_4^2} < 0. \tag{15}$$

The relation (15) indicates that per unit cost of principal raw material, m increases; the tendency of purchasing principal raw material x_3 decreases. In this state we observe that the organization may face decreasing returns to scale (Mohajan; 2021c, Mohajan & Mohajan, 2022f).

Now we study the effects on irregular inputs x_4 when per unit cost of principal raw material, *m* increases. Taking T_{53} (i.e., term of 5th row and 3rd column) from both sides of (12) we get (Islam et al., 2010a; Roy et al., 2021),

$$\frac{\partial x_4}{\partial m} = \frac{x_3}{|J|}[C_{15}] + \frac{\eta}{|J|}[C_{45}]$$

$$= \frac{x_3}{|J|}\text{Cofactor of } C_{15} + \frac{\eta}{|J|}\text{Cofactor of } C_{45}$$

$$= \frac{x_3}{|J|}\begin{vmatrix} -B_1 & W_{11} & W_{12} & W_{13} \\ -B_2 & W_{21} & W_{22} & W_{23} \\ -B_3 & W_{31} & W_{32} & W_{33} \\ -B_4 & W_{41} & W_{42} & W_{43} \end{vmatrix} + \frac{\eta}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 \\ -B_1 & W_{11} & W_{12} & W_{13} \\ -B_2 & W_{21} & W_{22} & W_{23} \\ -B_4 & W_{41} & W_{42} & W_{43} \end{vmatrix}$$

$$= \frac{x_3}{|J|}\left\{ -B_1\begin{vmatrix} W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \\ W_{41} & W_{42} & W_{43} \end{vmatrix} - W_{11}\begin{vmatrix} -B_2 & W_{22} & W_{23} \\ -B_3 & W_{32} & W_{33} \\ -B_4 & W_{42} & W_{43} \end{vmatrix} + W_{12}\begin{vmatrix} -B_2 & W_{21} & W_{23} \\ -B_3 & W_{31} & W_{33} \\ -B_4 & W_{41} & W_{43} \end{vmatrix} - W_{13}\begin{vmatrix} -B_2 & W_{21} & W_{22} \\ -B_3 & W_{31} & W_{32} \\ -B_4 & W_{41} & W_{42} \end{vmatrix} \right\}$$

$$+ \frac{\eta}{|J|}\left\{ B_1\begin{vmatrix} -B_1 & W_{12} & W_{13} \\ -B_2 & W_{22} & W_{23} \\ -B_4 & W_{42} & W_{43} \end{vmatrix} - B_2\begin{vmatrix} -B_1 & W_{11} & W_{13} \\ -B_2 & W_{21} & W_{23} \\ -B_4 & W_{41} & W_{43} \end{vmatrix} + B_3\begin{vmatrix} -B_1 & W_{11} & W_{12} \\ -B_2 & W_{21} & W_{22} \\ -B_4 & W_{41} & W_{42} \end{vmatrix} \right\}$$

$$= \frac{x_3}{|J|}\left[-B_1\{W_{21}(W_{32}W_{43} - W_{42}W_{33}) + W_{22}(W_{41}W_{33} - W_{31}W_{43}) + W_{23}(W_{31}W_{42} - W_{41}W_{32})\} \right.$$

$$- W_{11}\{-B_2(W_{32}W_{43} - W_{42}W_{33}) + W_{22}(-B_4W_{33} + B_3W_{43}) + W_{23}(-B_3W_{42} + B_4W_{32})\}$$

$$+ W_{12}\{-B_2(W_{31}W_{43} - W_{41}W_{33}) + W_{21}(-B_4W_{33} + B_3W_{43}) + W_{23}(-B_3W_{41} + B_4W_{31})\}$$

$$\left. - W_{13}\{-B_2(W_{31}W_{42} - W_{41}W_{32}) + W_{21}(-B_4W_{32} + B_3W_{42}) + W_{22}(-B_3W_{41} + B_4W_{31})\}\right]$$

$$+ \frac{\eta}{|J|}\left[B_1\{-B_1(W_{22}W_{43} - W_{42}W_{23}) + W_{12}(-B_4W_{23} + B_2W_{43}) + W_{13}(-B_2W_{42} + B_4W_{22})\} \right.$$

$$- B_2\{-B_1(W_{21}W_{43} - W_{41}W_{23}) + W_{11}(-B_4W_{23} + B_2W_{43}) + W_{13}(-B_2W_{41} + B_4W_{21})\}$$

$$\left. + B_3\{-B_1(W_{21}W_{42} - W_{41}W_{22}) + W_{11}(-B_4W_{22} + B_2W_{42}) + W_{12}(-B_2W_{41} + B_4W_{21})\}\right]$$

$$- W_{13}\{-B_2(W_{31}W_{42} - W_{41}W_{32}) + W_{21}(-B_4W_{32} + B_3W_{42}) + W_{22}(-B_3W_{41} + B_4W_{31})\}]$$

$$= \frac{x_3}{|J|}\{-B_1W_{21}W_{32}W_{43} + B_1W_{21}W_{42}W_{33} - B_1W_{22}W_{41}W_{33} + B_1W_{22}W_{31}W_{43} - B_1W_{23}W_{31}W_{42} + B_1W_{23}W_{41}W_{32}$$

$$+ B_2W_{11}W_{32}W_{43} - B_2W_{11}W_{42}W_{33} + B_4W_{11}W_{22}W_{33} - B_3W_{11}W_{22}W_{43} + B_3W_{11}W_{23}W_{42} - B_4W_{11}W_{23}W_{32}$$

$$- B_2W_{12}W_{31}W_{43} + B_2W_{12}W_{41}W_{33} - B_4W_{12}W_{21}W_{33} + B_3W_{12}W_{21}W_{43} - B_3W_{12}W_{23}W_{41} + B_4W_{12}W_{23}W_{31}$$

$$+ B_2W_{13}W_{31}W_{42} - B_2W_{13}W_{41}W_{32} + B_4W_{13}W_{21}W_{32} - B_3W_{13}W_{21}W_{42} + B_3W_{13}W_{22}W_{41} - B_4W_{13}W_{22}W_{31}\}$$

$$+ \frac{\eta}{|J|}\{- B_1^2W_{22}W_{43} + B_1^2W_{42}W_{23} - B_1B_4W_{12}W_{23} + B_1B_2W_{12}W_{43} - B_1B_2W_{13}W_{42} + B_1B_4W_{13}W_{22}$$

$$+ B_1B_2W_{21}W_{43} - B_1B_2W_{41}W_{23} + B_2B_4W_{11}W_{23} - B_2^2W_{11}W_{43} + B_2^2W_{13}W_{41} - B_2B_4W_{13}W_{21} - B_1B_3W_{21}W_{42}$$

$$+ B_1B_3W_{41}W_{22} - B_3B_4W_{11}W_{22} + B_2B_3W_{11}W_{42} - B_2B_3W_{12}W_{41} + B_3B_4W_{12}W_{21}\}$$

$$= \frac{x_3}{|J|}\frac{A^3x_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}}{x_1^2x_2^2x_3^2x_4^2}\{- kx_1x_4pq^2r^2s + kx_1x_4pq^2r(r-1)s - kx_1x_4pq(q-1)r(r-1)s$$

$$+ kx_1x_4pq(q-1)r^2s - kx_1x_4pq^2r^2s + kx_1x_4pq^2r^2s + lx_2x_4p(p-1)qr^2s - lx_2x_4p(p-1)qr(r-1)s$$

$$+ nx_4^2p(p-1)q(q-1)r(r-1) - mx_3x_4p(p-1)q(q-1)rs + mx_3x_4p(p-1)q^2rs - nx_4^2p(p-1)q^2r^2$$

$$- lx_2x_4p^2qr^2s + lx_2x_4p^2qr(r-1)s - nx_4^2p^2q^2r(r-1) + mx_3x_4p^2q^2rs - mx_3x_4p^2q^2rs + nx_4^2p^2q^2r^2$$

$$+ lx_2x_4p^2qr^2s - lx_2x_4p^2qr^2s + nx_4^2p^2q^2r^2 - mx_3x_4p^2q^2rs + mx_3x_4p^2q(q-1)rs$$

$$- nx_4^2p^2q(q-1)r^2\}$$

$$+ \frac{1}{|J|}\frac{A^2x_1^{2p}x_2^{2q}x_3^{2r}x_4^{2s}}{x_1x_2^2x_3^2x_4^2}\frac{Ax_1^px_2^qx_3^rx_4^s\Theta}{B}\{- k^2x_1^2x_3x_4q(q-1)rs + k^2x_1^2x_3x_4q^2rs - knx_1x_3x_4^2pq^2r$$

$$+ klx_1x_2x_3x_4pqrs - klx_1x_2x_3x_4pqrs + knx_1x_3x_4^2pq(q-1)r + klx_1x_2x_3x_4pqrs - klx_1x_2x_3x_4pqrs$$

$$+ nlx_2x_3x_4^2p(p-1)qr - l^2x_2^2x_3x_4p(p-1)rs + l^2x_2^2x_3x_4p^2rs - nlx_2x_3x_4^2p^2qr - kmx_1x_3^2x_4pq^2s$$

$$+ kmx_1x_3^2x_4pq(q-1)s - mnx_3^2x_4^2p(p-1)q(q-1) + lmx_2x_3^2x_4p(p-1)qs - lmx_2x_3^2x_4p^2qs$$

$$+ mnx_3^2x_4^2p^2q^2\}$$

$$= \frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_3x_4\Theta}\{pq(r-1) - pqr - p(q-1)(r-1) + p(q-1)r + (p-1)qr - (p-1)q(r-1)$$

$$+ (p-1)(q-1)(r-1) - (p-1)(q-1)r\} + \frac{1}{|J|}\frac{A^3pqrsx_1^{3p}x_2^{3q}x_3^{3r}x_4^{3s}B}{x_1^2x_2^2x_3x_4\Theta}\{p(q-1) - pq - (p-1)(q-1)$$

$$+ (p-1)q\}$$

$$\frac{\partial x_4}{\partial m} = -\frac{2}{|J|}\frac{A^3pqrsx_1^{3x}x_2^{3y}x_3^{3z}x_4^{3w}B}{x_1^2x_2^2x_3x_4\Theta} < 0. \quad (16)$$

From the equation (16) we have observed that if per unit cost of principal raw material increases; the tendency of purchasing irregular inputs x_4 decreases. It seems that irregular input is complementary to principal raw material. When the price of x_3 goes up, citizens of the country buy less of it, and consequently level of consumption of x_4 also decreases, because complementary goods are used together; for example, tea and sugar (Mohajan, 2022; Ferdous & Mohajan, 2022).

8. Conclusions

In this study we have tried to discuss sensitivity analysis of various inputs when per unit cost of principal raw material increases. Every organization's principal target is profit maximization, and sensitivity analysis is a conducible step in this respect to predict about the future production. In this article we have used Lagrange multiplier method to make the 4-dimensional constrained problem to a higher dimensional unconstrained problem. We have used budget constraint to operate Cobb-Douglas production function as our profit function.

References

Baxley, J. V., & Moorhouse, J. C. (1984). Lagrange Multiplier Problems in Economics. *The American Mathematical Monthly*, *91*(7), 404-412.

Cobb, C. W., & Douglass, P. H. (1928). A Theory of Production. *American Economics Review*, *18*(1), 139-165.

Eaton, B., & Lipsey, R. (1975). The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition. *Review of Economic Studies*, *42*(1), 27-49.

Ferdous, J., & Mohajan, H. K. (2022). Maximum Profit Ensured for Industry Sustainability. *Annals of Spiru Haret University. Economic Series*, *22*(3), 317-337.

Hussain, W. (2012). Corporations, Profit Maximization and the Personal Sphere. *Economics and Philosophy*, *28*(3), 311-331.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009a). Preference of Social Choice in Mathematical Economics. *Indus Journal of Management & Social Sciences*, *3*(1), 17-38.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009b). Political Economy and Social Welfare with Voting Procedure. *KASBIT Business Journal*, *2*(1), 42-66.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2010a). Utility Maximization Subject to Multiple Constraints. *Indus Journal of Management & Social Sciences*, *4*(1), 15-29.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2010b). Utility Maximization Subject to Multiple Constraints. *Indus Journal of Management & Social Sciences*, *4*(1), 15-29.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2011). Output Maximization Subject to a Nonlinear Constraint. *KASBIT Business Journal*, *4*(1), 116-128.

Kothari, C. R. (2008). *Research Methodology: Methods and Techniques* (2nd Ed.). New Delhi: New Age International (P) Ltd.

Legesse, B. (2014). *Research Methods in Agribusiness and Value Chains*. School of Agricultural Economics and Agribusiness, Haramaya University.

Mohajan, D., & Mohajan, H. K. (2022a). Profit Maximization Strategy in an Industry: A Sustainable Procedure. *Law and Economy*, *1*(3), 17-43. https://doi:10.56397/LE.2022.10.02.

Mohajan, D., & Mohajan, H. K. (2022b). Utility Maximization Analysis of an Organization: A Mathematical Economic Procedure. *Law and Economy*, Manuscript Submitted.

Mohajan, D., & Mohajan, H. K. (2022c). Utility Maximization Investigation: A Bordered Hessian Method. *Annals of Spiru Haret University. Economic Series*, Manuscript Submitted.

Mohajan, D. & Mohajan, H. K. (2022d). Sensitivity Analysis among Commodities and Prices: Utility Maximization Perspectives (Unpublished Manuscript).

Mohajan, D. & Mohajan, H. K. (2022e). Sensitivity Analysis among Commodities and Coupons during Utility Maximization. *Frontiers in Management Science, 1*(3), 13-28.

Mohajan, D., & Mohajan, H. K. (2022f). Importance of Total Coupon in Utility Maximization: A Sensitivity Analysis. *Law and Economy*, *1*(5), 65-67.

Mohajan, H. K. (2017a). Optimization Models in Mathematical Economics. *Journal of Scientific Achievements*, *2*(5), 30-42.

Mohajan, H. K. (2017b). Two Criteria for Good Measurements in Research: Validity and Reliability. *Annals of*

Spiru Haret University Economic Series, *17*(3), 58-82.

Mohajan, H. K. (2018a). Qualitative Research Methodology in Social Sciences and Related Subjects. *Journal of Economic Development, Environment and People*, *7*(1), 23-48.

Mohajan, H. K. (2018b). *Aspects of Mathematical Economics, Social Choice and Game Theory*. PhD Dissertation, Jamal Nazrul Islam Research Centre for Mathematical and Physical Sciences (JNIRCMPS), University of Chittagong, Chittagong, Bangladesh.

Mohajan, H. K. (2020). Quantitative Research: A Successful Investigation in Natural and Social Sciences. *Journal of Economic Development, Environment and People*, *9*(4), 52-79.

Mohajan, H. K. (2021a). Estimation of Cost Minimization of Garments Sector by Cobb-Douglass Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, *21*(2), 267-299. doi: https://doi.org/10.26458/21212.

Mohajan, H. K. (2021b). Product Maximization Techniques of a Factory of Bangladesh: A Sustainable Procedure. *American Journal of Economics, Finance and Management*, *5*(2), 23-44.

Mohajan, H. K. (2021c). Utility Maximization of Bangladeshi Consumers within Their Budget: A Mathematical Procedure. *Journal of Economic Development, Environment and People*, *10*(3), 60-85.

Mohajan, H. K. (2022). Cost Minimization Analysis of a Running Firm with Economic Policy. *Annals of Spiru Haret University. Economic Series*, *22*(3), 171-181.

Mohajan, H. K., Islam, J. N., & Moolio, P. (2013). *Optimization and Social Welfare in Economics*. Lambert Academic Publishing, Germany.

Moolio, P., Islam, J. N., & Mohajan, H. K. (2009). Output Maximization of an Agency. *Indus Journal of Management and Social Sciences*, *3*(1), 39-51.

Pandey, P., & Pandey, M. M. (2015). *Research Methodology: Tools and Techniques*. Bridge Center, Romania, European Union.

Polit, D. F., & Hungler, B. P. (2013). *Essentials of Nursing Research: Methods, Appraisal, and Utilization* (8th Ed.). Philadelphia: Wolters Kluwer/Lippincott Williams and Wilkins.

Roy, L., Molla, R., & Mohajan, H. K. (2021). Cost Minimization is Essential for the Sustainability of an Industry: A Mathematical Economic Model Approach. *Annals of Spiru Haret University Economic Series*, *21*(1), 37-69.

Samuelson, P. A. (1947). *Foundations of Economic Analysis*. Harvard University Press, Cambridge, MA.

Wiese, H. (2021). Cost Minimization and Profit Maximization. In Advanced Microeconomics. Springer Gabler, Wiesbaden. https://doi.org/10.1007/978-3-658-34959-2_9.