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Effects of Various Inputs for Increased Interest Rate of Capital: A Nonlinear Budget Constraint Consideration

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Abstract

This article attempts to discuss economic effects of various inputs when the interest rate of capital is increased during profit maximization analysis. In this paper Cobb-Douglas production function, 6×6 bordered Hessian matrix, and 6×6 Jacobian are operated to make the study meaningful and interesting. Every firm wants to achieve a maximum profit, but achievement of profit maximization is not an easy process. A firm must be watchdog in every step of production, inventory, distribution, and management for attaining optimum result. In the twenty first century global economy faces serious complexities due to political unrest and war among the nations, also for abnormal natural calamities due to global warming. Therefore, efficient and wise decisions are necessary for the sustainability of the industrial firms.

Keywords: profit maximization, nonlinear budget constraint, interest rate of capital

1. Introduction

In the 21st century mathematical modeling becomes an inevitable practice in economics (Samuelson, 1947). It covers many fields of social sciences, such as economics, psychology, sociology, political science, etc. (Carter, 2001). In mathematical economics profit maximization is a boon for every firm (Eaton & Lipsey, 1975).

Cobb-Douglas production function plays a dominant role to analyze profit maximization policy successfully (Cobb & Douglas, 1928). In multivariable calculus, method of Lagrange multiplier plays a crucial role that transforms a constrained problem to a higher dimensional unconstrained problem (Baxley & Moorhouse, 1984; Islam et al., 2010, 2011). In this paper we have used determinant of 6×6 bordered Hessian matrix, and 6×6 Jacobian matrix to act with Implicit Function Theorem. Ultimately, we have discussed the economic predictions of profit maximization subject to nonlinear budget constraints when interest of capital of the firm increases.

2. Literature Review

In every type of research, literature review section is an introductory portion. It provides the preliminary concepts of the works of previous researchers (Polit & Hungler, 2013). Charles W. Cobb (1875-1949) and Paul H. Douglas (1892-1976) have given the idea of production functions that is known as Cobb-Douglas production function (Cobb & Douglas, 1928). John V. Baxley and John C. Moorhouse have developed the profit maximization strategy in mathematical economics (Baxley & Moorhouse, 1984).

Professor Jamal Nazrul Islam (1939-2013) is a famous mathematician in Bangladesh. He and his coauthors have worked on profit maximization problems. They have discussed social welfare and optimization in economics elaborately (Islam et al., 2010, 2011). Professor Pahlaj Moolio and his coworkers have considered the Cobb-Douglas production functions in the optimization problems (Moolio et al., 2009). Jannatul Ferdous and

Haradhan Kumar Mohajan have described briefly a profit maximization problem (Ferdous & Mohajan, 2022). Devajit Mohajan and Haradhan Kumar Mohajan have also worked on profit maximization in addition with the sensitivity analysis (Mohajan & Mohajan, 2022a-f, 2023a-g).

3. Research Methodology of the Study

An academician cannot develop himself/herself without quantitative, qualitative or other types of research(s) (Pandey & Pandey, 2015). Methodology is a guideline to complete a meaningful research successfully (Kothari, 2008). It describes the types of research and the types of data (Ojo, 2003; Somekh & Lewin, 2005). Hence, research methodology is the specific procedures that are used to identify, select, process, and analyze materials related to the topics (Somekh & Lewin, 2005). It is the science and philosophy behind all researches for organizing, planning, designing, and conducting a good research (Remenyi et al., 1998; Legesse, 2014).

In this study we have discussed effects of various inputs in an industrial firm to achieve profit maximization environment (Mohajan, 2020, 2018b; Islam et al., 2009a, b). Depending on the secondary data sources we have advanced the present research activities (Mohajan, 2017b, 2018a). We have consulted many papers and books of profit maximization to develop this paper (Mohajan, 2017a, 2018b; Islam et al., 2009a, b).

4. Objective of the Study

The core objective of this study is to discuss the effects of various inputs when interest rate of the capital is increased. Capital is a main factor of industrial firms. We have studied a profit maximization activity with subject to nonlinear budget constraint. Other non-core objectives of the study are as follows:

- to show the mathematical calculations elaborately, and
- to give the economic predictions precisely.

5. Lagrangian Function

Let us consider that an industrial firm is willing to make a maximum profit from its selling products. Let the firm uses ε_1 amount of capital, ε_2 quantity of labor, ε_3 quantity of principal raw materials, and ε_4 quantity of irregular input for its annual production. Let us consider the Cobb-Douglas production function $f(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ as a profit function for our model (Cobb & Douglas, 1928; Islam et al., 2010; Mohajan, 2021a),

$$P(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = f(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = A\varepsilon_1^p \varepsilon_2^q \varepsilon_3^r \varepsilon_4^s, \quad (1)$$

where *A* is the efficiency parameter that reflects the level of technology, i.e., technical process, economic system, etc., which represents total factor productivity. Moreover, *A* reflects the skill and efficient level of the workforce. Here *p*, *q*, *r*, and *s* are parameters; *p* indicates the output of elasticity of capital measures the percentage change in $P(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ for 1% change in ε_1, while ε_2, ε_3, and ε_4 are held constants. Similarly, *q* indicates the output of elasticity of labor, *r* indicates the output of elasticity of principal raw materials, and *s* indicates the output of elasticity of irregular input. Now these four parameters *p*, *q*, *r*, and *s* must satisfy the following four inequalities (Mohajan, 2021c; Mohajan & Mohajan, 2023a):

$$0 < p < 1, \ 0 < q < 1, \ 0 < r < 1, \text{ and } 0 < s < 1. \quad (2)$$

A strict Cobb-Douglas production function, in which $\nabla = p + q + r + s < 1$ indicates decreasing returns to scale, $\nabla = 1$ indicates constant returns to scale, and $\nabla > 1$ indicates increasing returns to scale. Now we consider that the profit function is subject to a nonlinear budget constraint,

$$B(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = k\varepsilon_1 + l\varepsilon_2 + m\varepsilon_3 + n(\varepsilon_4)\varepsilon_4, \quad (3)$$

where *k* is rate of interest or services of capital per unit of capital ε_1; *l* is the wage rate per unit of labor ε_2; *m* is the cost per unit of principal raw material ε_3; and *n* is the cost per unit of irregular input ε_4. In nonlinear budget equation (3) we consider (Mohajan & Mohajan, 2022b, 2023c),

$$n(\varepsilon_4) = n_0\varepsilon_4 - n_0, \quad (4)$$

where n_0 being the discounted price of the irregular input ε_4. Therefore, the nonlinear budget constraint (3) takes the form (Mohajan, 2021b);

$$B(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = k\varepsilon_1 + l\varepsilon_2 + m\varepsilon_3 + n_0\varepsilon_4^2 - n_0\varepsilon_4 . \tag{5}$$

We now formulate the maximization problem for the profit function (1) in terms of single Lagrange multiplier λ by defining the Lagrangian function $K(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \lambda)$ as (Mohajan & Mohajan, 2022e, 2023f),

$$K(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \lambda) = A\varepsilon_1^p \varepsilon_2^q \varepsilon_3^r \varepsilon_4^s + \lambda\{B(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) - k\varepsilon_1 - l\varepsilon_2 - m\varepsilon_3 - n_0\varepsilon_4^2 + n_0\varepsilon_4\}. \tag{6}$$

Relation (6) is a 5-dimensional unconstrained problem that is obtained from (1) and 4-dimensional constrained problem (3), where Lagrange multiplier λ, is considered as a device in our model.

6. Analysis on Four Inputs

For maximization, first order differentiation equals to zero; then from (6) we can write (Mohajan, 2022; Mohajan & Mohajan, 2023g),

$$K_\lambda = B - k\varepsilon_1 - l\varepsilon_2 - m\varepsilon_3 - n_0\varepsilon_4^2 + n_0\varepsilon_4 = 0 , \tag{7a}$$

$$K_1 = pA\varepsilon_1^{p-1} \varepsilon_2^q \varepsilon_3^r \varepsilon_4^s - \lambda k = 0 , \tag{7b}$$

$$K_2 = qA\varepsilon_1^p \varepsilon_2^{q-1} \varepsilon_3^r \varepsilon_4^s - \lambda l = 0 , \tag{7c}$$

$$K_3 = rA\varepsilon_1^p \varepsilon_2^q \varepsilon_3^{r-1} \varepsilon_4^s - \lambda m = 0 , \tag{7d}$$

$$K_4 = sA\varepsilon_1^p \varepsilon_2^q \varepsilon_3^r \varepsilon_4^{s-1} - \lambda n_0(2\varepsilon_4 - 1) = 0 , \tag{7e}$$

where, $\frac{\partial K}{\partial \lambda} = K_\lambda$, $\frac{\partial K}{\partial \varepsilon_1} = K_1$, $\frac{\partial K}{\partial \varepsilon_2} = K_2$, etc. indicate first-order partial differentiations of multivariate Lagrangian function.

Using equations (2) to (7) we can determine the values of ε_1, ε_2, ε_3, and ε_4 as follows (Mohajan & Mohajan, 2022d):

$$\varepsilon_1 = \frac{pB}{k\nabla} , \tag{8a}$$

$$\varepsilon_2 = \frac{qB}{l\nabla} , \tag{8b}$$

$$\varepsilon_3 = \frac{rB}{m\nabla} , \tag{8c}$$

$$\varepsilon_4 = \frac{sB}{n\nabla} . \tag{8d}$$

7. Bordered Hessian Matrix Analysis

Let us consider the determinant of the 5×5 bordered Hessian matrix as (Mohajan & Mohajan, 2022f),

$$\left|H\right| = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & K_{11} & K_{12} & K_{13} & K_{14} \\ -B_2 & K_{21} & K_{22} & K_{23} & K_{24} \\ -B_3 & K_{31} & K_{32} & K_{33} & K_{34} \\ -B_4 & K_{41} & K_{42} & K_{43} & K_{44} \end{vmatrix}. \tag{9}$$

Taking first-order partial differentiations of (5) we get,

$$B_1 = k\,,\ \ B_2 = l\,,\ \ B_3 = m\text{, and}\qquad B_4 = 2n_0\varepsilon_4 - n_0\,. \tag{10}$$

Taking second-order and cross partial derivatives of (6) we get (Roy et al., 2021; Mohajan & Mohajan, 2022e, 2023e),

$$\begin{aligned}
K_{11} &= p(p-1)A\varepsilon_1^{p-2}\varepsilon_2^{q}\varepsilon_3^{r}\varepsilon_4^{s}\,,\\
K_{22} &= q(q-1)A\varepsilon_1^{p}\varepsilon_2^{q-2}\varepsilon_3^{r}\varepsilon_4^{s}\,,\\
K_{33} &= r(r-1)A\varepsilon_1^{p}\varepsilon_2^{q}\varepsilon_3^{r-2}\varepsilon_4^{s}\,,\\
K_{44} &= s(s-1)A\varepsilon_1^{p}\varepsilon_2^{q}\varepsilon_3^{r}\varepsilon_4^{s-2}\,,\\
K_{12} = K_{21} &= pqA\varepsilon_1^{p-1}\varepsilon_2^{q-1}\varepsilon_3^{r}\varepsilon_4^{s}\,,\\
K_{13} = K_{31} &= prA\varepsilon_1^{p-1}\varepsilon_2^{q}\varepsilon_3^{r-1}\varepsilon_4^{s}\,,\\
K_{14} = K_{41} &= psA\varepsilon_1^{p-1}\varepsilon_2^{q}\varepsilon_3^{r}\varepsilon_4^{s-1}\,,\\
K_{23} = K_{32} &= qrA\varepsilon_1^{p}\varepsilon_2^{q-1}\varepsilon_3^{r-1}\varepsilon_4^{s}\,,\\
K_{24} = K_{42} &= qsA\varepsilon_1^{p}\varepsilon_2^{q-1}\varepsilon_3^{r}\varepsilon_4^{s-1}\,,\\
K_{34} = K_{43} &= rsA\varepsilon_1^{p}\varepsilon_2^{q}\varepsilon_3^{r-1}\varepsilon_4^{s-1}\,.
\end{aligned} \tag{11}$$

where $\frac{\partial^2 K}{\partial\varepsilon_1\partial\varepsilon_2} = K_{12} = K_{21}$, $\frac{\partial^2 K}{\partial\varepsilon_2^2} = K_{22}$, etc. indicate cross-partial, second order differentiations of multivariate Lagrangian function, respectively, etc.

Now we expand the Hessian (8) as $\left|H\right| > 0$ (Mohajan et al., 2013; Mohajan & Mohajan, 2022d),

$$\left|H\right| = \frac{A^3 pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B^2}{\varepsilon_1^2\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla^2}(p+q+r)(s+3) > 0\,, \tag{12}$$

where efficiency parameter, $A > 0$, and budget of the firm, $B > 0$; ε_1, ε_2, ε_3, and ε_4 are four different types of inputs; and consequently, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 > 0$. Parameters, $p, q, r, s > 0$; also in the model either $0 < \nabla = p + q + r + s < 1$, $\nabla = 1$ or $\nabla > 1$. Hence, equation (12) gives; $\left|H\right| > 0$ (Islam et al., 2011; Mohajan & Mohajan, 2022d).

8. Determination of Lagrange Multiplier λ

Now using the necessary values from (8), in (7a) we get (Moolio et al., 2009; Mohajan & Mohajan, 2023a),

$$B=\frac{pA\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s}{\lambda}+\frac{qA\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s}{\lambda}+\frac{rA\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s}{\lambda}+\frac{sA\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s}{\lambda}$$

$$\lambda=\frac{A\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s\nabla}{B}. \tag{13}$$

9. Jacobian Matrix Analysis

We have observed that the second order condition is satisfied, so that the determinant of (5) survives at the optimum, i.e., $|J|=|H|$; and hence, we can apply the implicit function theorem. Now we compute twenty-five partial derivatives, such as $\frac{\partial\lambda}{\partial k}$, $\frac{\partial\varepsilon_1}{\partial k}$, $\frac{\partial\varepsilon_4}{\partial B}$, etc. that are referred to as the comparative statics of the model (Chiang, 1984).

Let $\mathbf{G}$ be the vector-valued function of ten variables $\lambda^*,\varepsilon_1^*,\varepsilon_2^*,\varepsilon_3^*,\varepsilon_4^*,k,l,m,n$, and *B*, and we define the function $\mathbf{G}$ for the point $\left(\lambda^*,\varepsilon_1^*,\varepsilon_2^*,\varepsilon_3^*,\varepsilon_4^*,k,l,m,n,B\right)\in R^{10}$, and take the values in R^5. By the Implicit Function Theorem of multivariable calculus, the equation (Mohajan & Mohajan, 2022e),

$$F\left(\lambda^*,\varepsilon_1^*,\varepsilon_2^*,\varepsilon_3^*,\varepsilon_4^*,k,l,m,n,B\right)=0, \tag{14}$$

may be solved in the form of

$$\begin{bmatrix}\lambda\\ \varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\end{bmatrix}=\mathbf{G}(k,l,m,n,B). \tag{15}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}(k,l,m,n,B)$; regarded as $J_G=\frac{\partial(\lambda,\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)}{\partial(k,l,m,n_0,B)}$, and is represented by;

$$J_G=\begin{bmatrix}\frac{\partial\lambda}{\partial k}&\frac{\partial\lambda}{\partial l}&\frac{\partial\lambda}{\partial m}&\frac{\partial\lambda}{\partial n_0}&\frac{\partial\lambda}{\partial B}\\ \frac{\partial\varepsilon_1}{\partial k}&\frac{\partial\varepsilon_1}{\partial l}&\frac{\partial\varepsilon_1}{\partial m}&\frac{\partial\varepsilon_1}{\partial n_0}&\frac{\partial\varepsilon_1}{\partial B}\\ \frac{\partial\varepsilon_2}{\partial k}&\frac{\partial\varepsilon_2}{\partial l}&\frac{\partial\varepsilon_2}{\partial m}&\frac{\partial\varepsilon_2}{\partial n_0}&\frac{\partial\varepsilon_2}{\partial B}\\ \frac{\partial\varepsilon_3}{\partial k}&\frac{\partial\varepsilon_3}{\partial l}&\frac{\partial\varepsilon_3}{\partial m}&\frac{\partial\varepsilon_3}{\partial n_0}&\frac{\partial\varepsilon_3}{\partial B}\\ \frac{\partial\varepsilon_4}{\partial k}&\frac{\partial\varepsilon_4}{\partial l}&\frac{\partial\varepsilon_4}{\partial m}&\frac{\partial\varepsilon_4}{\partial n_0}&\frac{\partial\varepsilon_4}{\partial B}\end{bmatrix}. \tag{16}$$

$$=-J^{-1}\begin{bmatrix}-\varepsilon_1&-\varepsilon_2&-\varepsilon_3&-\varepsilon_4^2+\varepsilon_4&1\\ -\lambda&0&0&0&0\\ 0&-\lambda&0&0&0\\ 0&0&-\lambda&0&0\\ 0&0&0&-2\lambda\varepsilon_4+\lambda&0\end{bmatrix}.$$

The inverse of Jacobian matrix is, $J^{-1}=\frac{1}{|J|}C^{T}$, where $C=(C_{ij})$, the matrix of cofactors of J, where T for transpose, then (16) becomes (Mohajan, 2021c; Moolio et al., 2009),

$$=-\frac{1}{|J|}\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 & -\varepsilon_4^2+\varepsilon_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda\varepsilon_4+\lambda & 0 \end{bmatrix}$$

$$J_G=-\frac{1}{|J|}\begin{bmatrix} -\varepsilon_1C_{11}-\lambda C_{21} & -\varepsilon_2C_{11}-\lambda C_{31} & -\varepsilon_3C_{11}-\lambda C_{41} & -\varepsilon_4^2C_{11}+\varepsilon_4C_{11}-2\lambda\varepsilon_4C_{51}+\lambda C_{51} & C_{11} \\ -\varepsilon_1C_{12}-\lambda C_{22} & -\varepsilon_2C_{12}-\lambda C_{32} & -\varepsilon_3C_{12}-\lambda C_{42} & -\varepsilon_4^2C_{12}+\varepsilon_4C_{12}-2\lambda\varepsilon_4C_{52}+\lambda C_{52} & C_{12} \\ -\varepsilon_1C_{13}-\lambda C_{23} & -\varepsilon_2C_{13}-\lambda C_{33} & -\varepsilon_3C_{13}-\lambda C_{43} & -\varepsilon_4^2C_{13}+\varepsilon_4C_{13}-2\lambda\varepsilon_4C_{53}+\lambda C_{53} & C_{13} \\ -\varepsilon_1C_{14}-\lambda C_{24} & -\varepsilon_2C_{14}-\lambda C_{34} & -\varepsilon_3C_{14}-\lambda C_{44} & -\varepsilon_4^2C_{14}+\varepsilon_4C_{14}-2\lambda\varepsilon_4C_{54}+\lambda C_{54} & C_{14} \\ -\varepsilon_1C_{15}-\lambda C_{25} & -\varepsilon_2C_{15}-\lambda C_{35} & -\varepsilon_3C_{15}-\lambda C_{45} & -\varepsilon_4^2C_{15}+\varepsilon_4C_{15}-2\lambda\varepsilon_4C_{55}+\lambda C_{55} & C_{15} \end{bmatrix}. \quad (17)$$

In (17) total 25 comparative statics are available, and in this study, we deal only with four of them when interest rate of capital is increased. The firm always attempts for the profit maximization production (Baxley & Moorhouse, 1984; Islam et al., 2010; Mohajan & Mohajan, 2020f).

10. Sensitivity Analysis

Now we study the effect of capital ε_1 when its interest rate, *k* increases. Taking T_{21} (i.e., term of 2nd row and 1st column) from both sides of (17) we get (Mohajan, 2021a, b; Wiese, 2021),

$$\frac{\partial\varepsilon_1}{\partial k}=\frac{\varepsilon_1}{|J|}[C_{12}]+\frac{\lambda}{|J|}[C_{22}]$$

$$=\frac{\varepsilon_1}{|J|}\text{Cofactor of } C_{12}+\frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{22}$$

$$=-\frac{\varepsilon_1}{|J|}\begin{vmatrix} -B_1 & K_{12} & K_{13} & K_{14} \\ -B_2 & K_{22} & K_{23} & K_{24} \\ -B_3 & K_{32} & K_{33} & K_{34} \\ -B_4 & K_{42} & K_{43} & K_{44} \end{vmatrix}+\frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_2 & K_{22} & K_{23} & K_{24} \\ -B_3 & K_{32} & K_{33} & K_{34} \\ -B_4 & K_{42} & K_{43} & K_{44} \end{vmatrix}$$

$$=-\frac{\varepsilon_1}{|J|}\left\{-B_1\begin{vmatrix} K_{22} & K_{23} & K_{24} \\ K_{32} & K_{33} & K_{34} \\ K_{42} & K_{43} & K_{44} \end{vmatrix}-K_{12}\begin{vmatrix} -B_2 & K_{23} & K_{24} \\ -B_3 & K_{33} & K_{34} \\ -B_4 & K_{43} & K_{44} \end{vmatrix}+K_{13}\begin{vmatrix} -B_2 & K_{22} & K_{24} \\ -B_3 & K_{32} & K_{34} \\ -B_4 & K_{42} & K_{44} \end{vmatrix}\right.$$

$$\left.-K_{14}\begin{vmatrix} -B_2 & K_{22} & K_{23} \\ -B_3 & K_{32} & K_{33} \\ -B_4 & K_{42} & K_{43} \end{vmatrix}\right\}+\frac{\lambda}{|J|}\left\{B_2\begin{vmatrix} -B_2 & K_{23} & K_{24} \\ -B_3 & K_{33} & K_{34} \\ -B_4 & K_{43} & K_{44} \end{vmatrix}-B_3\begin{vmatrix} -B_2 & K_{22} & K_{24} \\ -B_3 & K_{32} & K_{34} \\ -B_4 & K_{42} & K_{44} \end{vmatrix}\right.$$

$$\left.+B_4\begin{vmatrix} -B_2 & K_{22} & K_{23} \\ -B_3 & K_{32} & K_{33} \\ -B_4 & K_{42} & K_{43} \end{vmatrix}\right\}$$

$$=-\frac{\varepsilon_1}{|J|}\left[-B_1\{K_{22}(K_{33}K_{44}-K_{43}K_{34})+K_{23}(K_{42}K_{34}-K_{32}K_{44})+K_{24}(K_{32}K_{43}-K_{42}K_{33})\}\right.$$

$$-K_{12}\{-B_2(K_{33}K_{44}-K_{43}K_{34})+K_{23}(-B_4K_{34}+B_3K_{44})+K_{24}(-B_3K_{43}+B_4K_{33})\}$$

$$+K_{13}\{-B_2(K_{32}K_{44}-K_{42}K_{34})+K_{22}(-B_4K_{34}+B_3K_{44})+K_{24}(-B_3K_{42}+B_4K_{32})\}$$

$$\left.-K_{14}\{-B_2(K_{32}K_{43}-K_{42}K_{33})+K_{22}(-B_4K_{33}+B_3K_{43})+K_{23}(-B_3K_{42}+B_4K_{32})\}\right]$$

$$+\frac{\lambda}{|J|}\left[B_2\{-B_2(K_{33}K_{44}-K_{43}K_{34})+K_{23}(-B_4K_{34}+B_3K_{44})+K_{24}(-B_3K_{43}+B_4K_{33})\}\right.$$

$$-B_3\{-B_2(K_{32}K_{44}-K_{42}K_{34})+K_{22}(-B_4K_{34}+B_3K_{44})+K_{24}(-B_3K_{42}+B_4K_{32})\}$$

$$\left.+B_4\{-B_2(K_{32}K_{43}-K_{42}K_{33})+K_{22}(-B_4K_{33}+B_3K_{43})+K_{23}(-B_3K_{42}+B_4K_{32})\}\right]$$

$$=-\frac{\varepsilon_1}{|J|}\{-B_1K_{22}K_{33}K_{44}\quad +B_1K_{22}K_{43}K_{34}\quad -B_1K_{23}K_{42}K_{34}\quad +B_1K_{23}K_{32}K_{44}\quad -B_1K_{24}K_{32}K_{43}$$

$$+B_1K_{24}K_{42}K_{33}\ +B_2K_{12}K_{33}K_{44}\ -B_2K_{12}K_{43}K_{34}\ +B_4K_{12}K_{23}K_{34}\ -B_3K_{12}K_{23}K_{44}\ +B_3K_{12}K_{24}K_{43}$$

$$-B_4K_{12}K_{24}K_{33}\ -B_2K_{13}K_{32}K_{44}\ +B_2K_{13}K_{42}K_{34}\ -B_4K_{13}K_{22}K_{34}\ +B_3K_{13}K_{22}K_{44}\ -B_3K_{13}K_{24}K_{42}$$

$$+B_4K_{13}K_{24}K_{32}\ +B_2K_{14}K_{32}K_{43}\ -B_2K_{14}K_{42}K_{33}\ +B_4K_{14}K_{22}K_{33}\ -B_3K_{14}K_{22}K_{43}\ +B_3K_{14}K_{23}K_{42}$$

$$-B_4K_{14}K_{23}K_{32}\}\ +\frac{\lambda}{|J|}\{-B_2^2K_{33}K_{44}\quad +B_2^2K_{43}K_{34}\quad -B_2B_4K_{23}K_{34}\quad +B_2B_3K_{23}K_{44}\quad -B_2BK_{24}K_{43}$$

$$+B_2B_4K_{24}K_{33}+B_2B_3L_{32}L_{44}-B_2B_3L_{42}L_{34}+B_3B_4L_{22}L_{34}-B_3^2L_{22}L_{44}+B_3^2L_{24}L_{42}-B_3B_4L_{24}L_{32}$$

$$-B_2B_4K_{32}K_{43}+B_2B_4K_{42}K_{33}-B_4^2K_{22}K_{33}+B_3B_4K_{22}K_{43}-B_3B_4K_{23}K_{42}+B_4^2K_{23}K_{32}\}$$

$$=-\frac{1}{|J|}\frac{A^3\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\ \{-k\varepsilon_1q(q-1)r(r-1)s(s-1)+k\varepsilon_1q(q-1)r^2s^2-k\varepsilon_1q^2r^2s^2+k\varepsilon_1q^2r^2s(s-1)$$

$$-k\varepsilon_1q^2r^2s^2\qquad +k\varepsilon_1q^2r(r-1)s^2\qquad +l\varepsilon_2pqr(r-1)s(s-1)\qquad -l\varepsilon_2pqr^2s^2\qquad +n\varepsilon_4pq^2r^2s$$

$$-m\varepsilon_3pq^2rs(s-1)\qquad +m\varepsilon_3pq^2rs^2\qquad -n\varepsilon_4pq^2r(r-1)s\qquad -l\varepsilon_2pqr^2s(s-1)\qquad +l\varepsilon_2pqr^2s^2$$

$$-n\varepsilon_4pq(q-1)r^2s\qquad +m\varepsilon_3pq(q-1)rs(s-1)\qquad -m\varepsilon_3pq^2s^2\qquad +n\varepsilon_4pq^2r^2s\qquad +l\varepsilon_2pqr^2s^2$$

$$-l\varepsilon_2pqr(r-1)s^2\quad +n\varepsilon_4pq(q-1)r(r-1)s\quad -m\varepsilon_3pq(q-1)rs^2\quad +m\varepsilon_3pq^2rs^2\quad -n\varepsilon_4pq^2r^2s\}$$

$$+\frac{\lambda}{|J|}\frac{A^2\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\quad \{-l^2\varepsilon_2^2r(r-1)s(s-1)\quad +l^2\varepsilon_2^2r^2s^2\quad -nl\varepsilon_2\varepsilon_4qr^2s\quad +lm\varepsilon_2\varepsilon_3qrs(s-1)$$

$$-lm\varepsilon_2\varepsilon_3qrs^2 \quad +nl\varepsilon_2\varepsilon_4qr(r-1)s \quad +lm\varepsilon_2\varepsilon_3qrs(s-1) \quad -lm\varepsilon_2\varepsilon_3qrs^2 \quad +nl\varepsilon_3\varepsilon_4q(q-1)rs$$

$$-m^2\varepsilon_3^2q(q-1)s(s-1) \quad +m^2\varepsilon_3^2q^2s^2 \quad -nl\varepsilon_3\varepsilon_4q^2rs \quad -nl\varepsilon_2\varepsilon_4qr^2s \quad +nl\varepsilon_2\varepsilon_4qr(r-1)s$$

$$-n^2\varepsilon_4^2q(q-1)r(r-1)+mn\varepsilon_3\varepsilon_4q(q-1)rs-mn\varepsilon_3\varepsilon_4q^2rs+n^2\varepsilon_4^2q^2s^2\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2} \quad \{-k\varepsilon_1p^{-1}(q-1)(r-1)(s-1) \quad +k\varepsilon_1p^{-1}(q-1)rs \quad -k\varepsilon_1p^{-1}qrs$$

$$+k\varepsilon_1p^{-1}qr(s-1)-k\varepsilon_1p^{-1}qrs+k\varepsilon_1p^{-1}q(r-1)s+l\varepsilon_2(r-1)(s-1)-l\varepsilon_2rs-l\varepsilon_2r(s-1)+l\varepsilon_2rs$$

$$+l\varepsilon_2zw-l\varepsilon_2(r-1)s-m\varepsilon_3q(s-1)+m\varepsilon_3qs+m\varepsilon_3(q-1)(s-1)-m\varepsilon_3qs-m\varepsilon_3(q-1)s+m\varepsilon_3qs$$

$$+n\varepsilon_4qr \quad +n\varepsilon_4(q-1)(r-1) \quad -n\varepsilon_4q(r-1) \quad -n\varepsilon_4(q-1)r \quad +n\varepsilon_4qr \quad -n\varepsilon_4qr\}$$

$$+\frac{\lambda}{|J|}\frac{A^2qrs\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\{-l^2\varepsilon_2^2q^{-1}(r-1)(s-1)+q^{-1}l^2\varepsilon_2^2rs-r^{-1}m^2\varepsilon_3^2(q-1)(s-1)+r^{-1}m^2\varepsilon_3^2qs$$

$$-s^{-1}n^2\varepsilon_4^2(q-1)(r-1)+s^{-1}n^2\varepsilon_4^2qr\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2} \quad \{-k\varepsilon_1p^{-1}(q-1)(r-1)(s-1) \quad +k\varepsilon_1p^{-1}(q-1)rs \quad -2k\varepsilon_1p^{-1}qrs$$

$$+k\varepsilon_1p^{-1}qr(s-1) \quad +k\varepsilon_1p^{-1}q(r-1)s \quad +l\varepsilon_2(r-1)(s-1) \quad -l\varepsilon_2r(s-1) \quad +l\varepsilon_2rs \quad -l\varepsilon_2(r-1)s$$

$$-m\varepsilon_3q(s-1)+m\varepsilon_3(q-1)(s-1)-m\varepsilon_3(q-1)s+m\varepsilon_3qs+n\varepsilon_4qr+n\varepsilon_4(q-1)(r-1)-n\varepsilon_4q(r-1)$$

$$-n\varepsilon_4(q-1)r\} \quad +\frac{\lambda}{|J|}\frac{A^2qrs\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2} \quad \{-l^2\varepsilon_2^2q^{-1}(r-1)(s-1) \quad +q^{-1}l^2\varepsilon_2^2rs$$

$$-r^{-1}m^2\varepsilon_3^2(q-1)(s-1)+r^{-1}m^2\varepsilon_3^2qs-s^{-1}n^2\varepsilon_4^2(q-1)(r-1)+s^{-1}n^2\varepsilon_4^2qr\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla}\{-(q-1)(r-1)(s-1) \quad +q(r-1)(s-1) \quad -qr(s-1) \quad +(q-1)r(s-1)$$

$$+(2\varepsilon_4-1)qrs+(2\varepsilon_4-1)(q-1)(r-1)s-(2\varepsilon_4-1)q(r-1)s-(2\varepsilon_4-1)(q-1)rs\}$$

$$+\frac{1}{|J|}\frac{A^2qrs\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}B^2}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla^2} \quad \frac{A\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s\nabla}{B} \quad \{-q(r-1)(s-1) \quad +2qrs \quad -(2\varepsilon_4-1)^2(q-1)(r-1)s$$

$$-(q-1)r(s-1)+(2\varepsilon_4-1)^2qrs\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla}\quad\{-(q-1)(r-1)(s-1)\ +q(r-1)(s-1)\ -qr(s-1)\ +(q-1)r(s-1)$$

$$+(2\varepsilon_4-1)qrs\ +(2\varepsilon_4-1)(q-1)(r-1)s-(2\varepsilon_4-1)q(r-1)s-(2\varepsilon_4-1)(q-1)rs\}$$

$$+\frac{1}{|J|}\frac{A^3qrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla}\quad\{-q(r-1)(s-1)\ +2qrs\ -(q-1)r(s-1)\ -(2\varepsilon_4-1)^2(q-1)(r-1)s$$

$$+(2\varepsilon_4-1)^2qrs\}. \tag{18}$$

For convenient we consider $p=\frac{1}{2}$ in equation (18) then we get,

$$\frac{\partial\varepsilon_1}{\partial k}=-\frac{1}{|J|}\frac{A^3qrs\varepsilon_1^{3/2}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla}\left[\{2(qs+qr+rs)-(q+r+s)\}-\left(s\varepsilon_4-s+\frac{1}{2}\right)-4\varepsilon_4(qs+rs-s)\right.$$

$$\left.+4\varepsilon_4^2(qs+rs-s)\right]. \tag{19}$$

We consider, $q=\frac{1}{4}$, $r=\frac{1}{4}$, $s=\frac{1}{4}$, in equation (19), then for increasing returns to scale, $\nabla=\frac{5}{4}>1$ we get;

$$\frac{\partial\varepsilon_1}{\partial k}=\frac{1}{|J|}\frac{A^3B\varepsilon_1^{3/2}}{512\nabla\varepsilon_2^{5/4}\varepsilon_3^{5/4}\varepsilon_4^{5/4}}\left\{\left(2\varepsilon_4-\frac{1}{2}\right)^2+\frac{19}{4}\right\}>0,\forall\varepsilon_4>0\,. \tag{20}$$

The relation (20) identifies that if the interest rate of the capital ε_1 increases, the level of capital ε_1 also increases. It seems that the firm faces increasing returns to scale, and it may increase both capital and worker forces for maximum production to achieve maximum profit (Moolio et al., 2009; Mohajan & Mohajan, 2022a).

Now we study the effect of wage ε_2 when interest rate, *k* of the capital ε_1 increases. Taking T_{31} (i.e., term of 3rd row and 1st column) from both sides of (17) we get (Mohajan, 2021a, b; Wiese, 2021),

$$\frac{\partial\varepsilon_2}{\partial k}=\frac{\varepsilon_1}{|J|}[C_{13}]+\frac{\lambda}{|J|}[C_{23}]$$

$$=\frac{\varepsilon_1}{|J|}\text{Cofactor of }C_{13}+\frac{\lambda}{|J|}\text{Cofactor of C}_{23}$$

$$=\frac{\varepsilon_1}{|J|}\begin{vmatrix}-B_1 & K_{11} & K_{13} & K_{14}\\ -B_2 & K_{21} & K_{23} & K_{24}\\ -B_3 & K_{31} & K_{33} & K_{34}\\ -B_4 & K_{41} & K_{43} & K_{44}\end{vmatrix}-\frac{\lambda}{|J|}\begin{vmatrix}0 & -B_1 & -B_3 & -B_4\\ -B_2 & K_{21} & K_{23} & K_{24}\\ -B_3 & K_{31} & K_{33} & K_{34}\\ -B_4 & K_{41} & K_{43} & K_{44}\end{vmatrix}$$

$$=\frac{\varepsilon_1}{|J|}\left\{-B_1\begin{vmatrix}K_{21} & K_{23} & K_{24}\\ K_{31} & K_{33} & K_{34}\\ K_{41} & K_{43} & K_{44}\end{vmatrix}-K_{11}\begin{vmatrix}-B_2 & K_{23} & K_{24}\\ -B_3 & K_{33} & K_{34}\\ -B_4 & K_{43} & K_{44}\end{vmatrix}+K_{13}\begin{vmatrix}-B_2 & K_{21} & K_{24}\\ -B_3 & K_{31} & K_{34}\\ -B_4 & K_{41} & K_{44}\end{vmatrix}-K_{14}\begin{vmatrix}-B_2 & K_{21} & K_{23}\\ -B_3 & K_{31} & K_{33}\\ -B_4 & K_{41} & K_{43}\end{vmatrix}\right\}$$

$$-\frac{\lambda}{|J|}\left\{B_1\begin{vmatrix}-B_2 & K_{23} & K_{24}\\ -B_3 & K_{33} & K_{34}\\ -B_4 & K_{43} & K_{44}\end{vmatrix}-B_3\begin{vmatrix}-B_2 & K_{21} & K_{24}\\ -B_3 & K_{31} & K_{34}\\ -B_4 & K_{41} & K_{44}\end{vmatrix}+B_4\begin{vmatrix}-B_2 & K_{21} & K_{23}\\ -B_3 & K_{31} & K_{33}\\ -B_4 & K_{41} & K_{43}\end{vmatrix}\right\}$$

$$=-\frac{\varepsilon_1}{|J|}\left[-B_1\left\{K_{21}\left(K_{33}K_{44}-K_{43}K_{34}\right)+K_{23}\left(K_{41}K_{34}-K_{31}K_{44}\right)+K_{24}\left(K_{31}K_{43}-K_{41}K_{33}\right)\right\}\right.$$

$$-K_{11}\left\{-B_2\left(K_{33}K_{44}-K_{43}K_{34}\right)+K_{23}\left(-B_4K_{34}+B_3K_{44}\right)+K_{24}\left(-B_3K_{43}+B_4K_{33}\right)\right\}$$

$$+K_{13}\left\{-B_2\left(K_{31}K_{44}-K_{41}K_{34}\right)+K_{21}\left(-B_4K_{34}+B_3K_{44}\right)+K_{24}\left(-B_3K_{41}+B_4K_{31}\right)\right\}$$

$$\left.-K_{14}\left\{-B_2\left(K_{31}K_{43}-K_{41}K_{33}\right)+K_{21}\left(-B_4K_{33}+B_3K_{43}\right)+K_{23}\left(-B_3K_{41}+B_4K_{31}\right)\right\}\right]$$

$$-\frac{\lambda}{|J|}\left[B_1\left\{-B_2\left(K_{33}K_{44}-K_{43}K_{34}\right)+K_{23}\left(-B_4K_{34}+B_3K_{44}\right)+K_{24}\left(-B_3K_{43}+B_4K_{33}\right)\right\}\right.$$

$$-B_3\left\{-B_2\left(K_{31}K_{44}-K_{41}K_{34}\right)+K_{21}\left(-B_4K_{34}+B_3K_{44}\right)+K_{24}\left(-B_3K_{41}+B_4K_{31}\right)\right\}$$

$$\left.+B_4\left\{-B_2\left(K_{31}K_{43}-K_{41}K_{33}\right)+K_{21}\left(-B_4K_{33}+B_3K_{43}\right)+K_{23}\left(-B_3K_{41}+B_4K_{31}\right)\right\}\right]$$

$$=-\frac{\varepsilon_1}{|J|}\{B_1K_{21}K_{33}K_{44}\quad -B_1K_{21}K_{43}K_{34}\quad +B_1K_{23}K_{41}K_{24}\quad -B_1K_{23}K_{31}K_{44}\quad +B_1K_{24}K_{31}K_{43}$$

$$-B_1K_{24}K_{41}K_{33}\ -B_2K_{11}K_{33}K_{44}\ +B_2K_{11}K_{43}K_{34}\ -B_4K_{11}K_{23}K_{34}\ +B_3K_{11}K_{23}K_{44}\ -B_3K_{11}K_{24}K_{43}$$

$$+B_4K_{11}K_{24}K_{33}\ +B_2K_{13}K_{31}K_{44}\ -B_2K_{13}K_{41}K_{34}\ +B_4K_{13}K_{21}K_{34}\ -B_3K_{13}K_{21}K_{44}\ +B_3K_{13}K_{24}K_{41}$$

$$-B_4K_{13}K_{24}K_{31}\ -B_2K_{14}K_{31}K_{43}\ +B_2K_{14}K_{41}K_{33}\ -B_4K_{14}K_{21}K_{33}\ +B_3K_{14}K_{21}K_{43}\ -B_3K_{14}K_{23}K_{41}$$

$$+B_4K_{14}K_{23}K_{31}\}\quad +\frac{\lambda}{|J|}\{B_1B_2K_{33}K_{44}\quad -B_1B_2K_{34}^2\quad +B_1B_4K_{23}K_{34}\quad -B_4B_3K_{23}K_{44}\quad +B_1B_3K_{24}K_{43}$$

$$-B_1B_4K_{24}K_{33}\quad +B_2B_3K_{31}K_{44}\quad -B_2B_3K_{41}K_{34}\quad +B_3B_4K_{21}K_{34}\quad -B_3^2K_{21}K_{44}\quad +B_3^2K_{24}K_{41}$$

$$-B_3B_4K_{24}K_{31}\quad -B_2B_4K_{31}K_{43}\quad +B_2B_4K_{41}K_{33}\quad -B_4^2K_{21}K_{33}\quad +B_3B_4K_{21}K_{43}\quad -B_3B_4K_{23}K_{41}$$

$$+B_4^2K_{23}K_{31}\}$$

$$=-\frac{\varepsilon_1}{|J|}\{B_1K_{21}K_{33}K_{44}\ -B_1K_{21}K_{34}^2\ +B_1K_{23}K_{41}K_{34}\ -B_1K_{23}K_{31}K_{44}\ +B_1K_{24}K_{31}K_{43}\ -B_1K_{24}K_{41}K_{33}$$

$$-B_2K_{11}K_{33}K_{44}\quad +B_2K_{11}K_{34}^2\quad -B_4K_{11}K_{23}K_{34}\quad +B_3K_{11}K_{23}K_{44}\quad -B_3K_{11}K_{24}K_{43}\quad +B_4K_{11}K_{24}K_{33}$$

$$+B_2K_{13}^2K_{44}\quad -B_2K_{13}K_{41}K_{34}\quad +B_4K_{13}K_{21}K_{34}\quad -B_3K_{13}K_{21}K_{44}\quad +B_3K_{13}K_{24}K_{41}\quad -B_4K_{13}^2K_{24}$$

$$-B_2K_{14}K_{31}K_{43} \quad +B_2K_{14}^2K_{33} \quad -B_4K_{14}K_{21}K_{33} \quad +B_3K_{14}K_{21}K_{43} \quad -B_3K_{14}^2K_{23} \quad +B_4K_{14}K_{23}K_{31}\}$$

$$+\frac{\lambda}{|J|}\{B_1B_2K_{33}K_{44} \quad -B_1B_2K_{34}^2 \quad +B_1B_4K_{23}K_{34} \quad -B_1B_3K_{23}K_{44} \quad +B_1B_3K_{24}K_{43} \quad -B_1B_4K_{24}K_{33}$$

$$+B_2B_3K_{31}K_{44} \quad -B_2B_3K_{41}K_{34} \quad +B_3B_4K_{24}K_{34} \quad -B_3^2K_{21}K_{44} \quad +B_3^2K_{24}K_{41} \quad -B_3B_4K_{24}K_{31}$$

$$-B_2B_4K_{31}K_{43}+B_2B_4K_{41}K_{33}-B_4^2K_{21}K_{33}+B_3B_4K_{21}K_{43}-B_3B_4K_{23}K_{41}+B_4^2K_{23}K_{31}\}$$

$$=-\frac{\varepsilon_1}{|J|}\frac{A^3\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_1^2\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2} \quad \{k\varepsilon_1\varepsilon_2pqr(r-1)s(s-1) \quad -k\varepsilon_1\varepsilon_2pqr^2s^2 \quad +k\varepsilon_1\varepsilon_2pqr^2s^2$$

$$-k\varepsilon_1\varepsilon_2pqr^2s(s-1) \quad +k\varepsilon_1\varepsilon_2pqr^2s^2 \quad -k\varepsilon_1\varepsilon_2pqr(r-1)s^2 \quad -l\varepsilon_2^2p(p-1)r(r-1)s(s-1)$$

$$+l\varepsilon_2^2p(p-1)r^2s^2 \quad -n\varepsilon_2\varepsilon_4p(p-1)pr^2s \quad +m\varepsilon_2\varepsilon_3p(p-1)qrs(s-1) \quad -m\varepsilon_2\varepsilon_3p(p-1)qrs^2$$

$$+n\varepsilon_2\varepsilon_4p(p-1)qr(r-1)s \quad +l\varepsilon_2^2p^2r^2s(s-1) \quad -l\varepsilon_2^2p^2r^2s^2 \quad +n\varepsilon_2\varepsilon_4p^2qr^2s \quad -m\varepsilon_2\varepsilon_3p^2qrs(s-1)$$

$$+m\varepsilon_2\varepsilon_3p^2qrs^2 \quad -n\varepsilon_2\varepsilon_4p^2qr^2s \quad -l\varepsilon_2^2p^2r^2s^2 \quad +l\varepsilon_2^2p^2r(r-1)s^2 \quad -n\varepsilon_2\varepsilon_4p^2qr(r-1)s$$

$$+m\varepsilon_2\varepsilon_3p^2qrs^2 \quad -m\varepsilon_2\varepsilon_3p^2qrs^2 \quad +n\varepsilon_2\varepsilon_4p^2qr^2s\} \quad +\frac{\lambda}{|J|}\frac{A^2\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_1^2\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2} \quad \{kl\varepsilon_1^2\varepsilon_2^2r(r-1)s(s-1)$$

$$-kl\varepsilon_1^2\varepsilon_2^2r^2s^2 \quad +kn\varepsilon_1^2\varepsilon_2\varepsilon_4qr^2s \quad -km\varepsilon_1^2\varepsilon_2\varepsilon_4qrs(s-1) \quad +km\varepsilon_1^2\varepsilon_2\varepsilon_3qrs^2 \quad -kn\varepsilon_1^2\varepsilon_2\varepsilon_4qr(r-1)s$$

$$+lm\varepsilon_1^2\varepsilon_2\varepsilon_3prs(s-1) \quad -lm\varepsilon_1^2\varepsilon_2\varepsilon_3prs^2 \quad +mn\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs \quad -m^2\varepsilon_1\varepsilon_2\varepsilon_3^2pqs(s-1) \quad +m^2\varepsilon_3^2pqs^2$$

$$-mn\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs \quad -nl\varepsilon_1\varepsilon_2^2\varepsilon_4pr^2s \quad +nl\varepsilon_1\varepsilon_2^2\varepsilon_4pr(r-1)s \quad -n^2\varepsilon_1\varepsilon_2\varepsilon_4^2pqr(r-1)$$

$$+mn\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs-mn\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs+n^2a\varepsilon_1\varepsilon_2\varepsilon_4^2pqr^2\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2\varepsilon_3^2\varepsilon_4^2\nabla}\ \{p(r-1)(s-1)-pr(s-1)-(p-1)(r-1)(s-1)-(2\varepsilon_4-1)(p-1)rs$$

$$+(p-1)r(s-1) \quad +(2\varepsilon_4-1)(p-1)(r-1)s \quad +prs \quad -(2\varepsilon_4-1)p(r-1)s\}$$

$$+\frac{1}{|J|}\frac{A^2pqrs\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_1\varepsilon_2\varepsilon_3^2\varepsilon_4^2}\frac{A\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s\nabla}{B}\ \{(r-1)(s-1)+rs-(2\varepsilon_4-1)^2(r-1)s+(2\varepsilon_4-1)^2rs\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2\varepsilon_3^2\varepsilon_4^2\nabla}\ \{p(r-1)(s-1)-pr(s-1)-(p-1)(r-1)(s-1)-(2\varepsilon_4-1)(p-1)rs$$

$$+(p-1)r(s-1)+(2\varepsilon_4-1)(p-1)(r-1)s+prs-(2\varepsilon_4-1)p(r-1)s\}+\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2\varepsilon_3^2\varepsilon_4^2\nabla}$$

$$\left\{1-s-(2\varepsilon_4-1)^2(r-1)s+(2\varepsilon_4-1)^2 rs\right\}$$

$$\frac{\partial\varepsilon_2}{\partial k}=\frac{1}{|J|}\frac{4A^3 pqrs^2\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2\varepsilon_3^2\varepsilon_4^2\nabla}\left(\varepsilon_4+\varepsilon_4^2\right)>0\,. \tag{21}$$

From the relation (21) we see that when the wage rate increases, the firm increases the level of capital ε_1. It seems that after the increase of wage rate, workers spend more working hours due to substitution effects. Consequently, the firm increases its capital structure for more production to achieve maximum profit (Islam et al., 2011; Moolio et al., 2009; Mohajan & Mohajan, 2023b).

Now we inspect the effect on principal raw material ε_3 when interest rate of capital increases. Taking T_{41} (i.e., term of 4th row and 1st column) from both sides of (17) we get (Mohajan, 2021a),

$$\frac{\partial\varepsilon_3}{\partial k}=\frac{\varepsilon_1}{|J|}[C_{14}]+\frac{\lambda}{|J|}[C_{24}]$$

$$=\frac{\varepsilon_1}{|J|}\text{Cofactor of }C_{14}+\frac{\lambda}{|J|}\text{Cofactor of }\mathrm{C}_{24}$$

$$=-\frac{\varepsilon_1}{|J|}\begin{vmatrix}-B_1 & K_{11} & K_{12} & K_{14}\\ -B_2 & K_{21} & K_{22} & K_{24}\\ -B_3 & K_{31} & K_{32} & K_{34}\\ -B_4 & K_{41} & K_{42} & K_{44}\end{vmatrix}+\frac{\lambda}{|J|}\begin{vmatrix}0 & -B_1 & -B_2 & -B_4\\ -B_2 & K_{21} & K_{22} & K_{24}\\ -B_3 & K_{31} & K_{32} & K_{34}\\ -B_4 & K_{41} & K_{42} & K_{44}\end{vmatrix}$$

$$=-\frac{\varepsilon_1}{|J|}\left\{-B_1\begin{vmatrix}K_{21} & K_{22} & K_{24}\\ K_{31} & K_{32} & K_{34}\\ K_{41} & K_{42} & K_{44}\end{vmatrix}-K_{11}\begin{vmatrix}-B_2 & K_{22} & K_{24}\\ -B_3 & K_{32} & K_{34}\\ -B_4 & K_{42} & K_{44}\end{vmatrix}+K_{12}\begin{vmatrix}-B_2 & K_{21} & K_{24}\\ -B_3 & K_{31} & K_{34}\\ -B_4 & K_{41} & K_{44}\end{vmatrix}\right.$$

$$\left.-K_{14}\begin{vmatrix}-B_2 & K_{21} & K_{22}\\ -B_3 & K_{31} & K_{32}\\ -B_4 & K_{41} & K_{42}\end{vmatrix}\right\}+\frac{\lambda}{|J|}\left\{B_1\begin{vmatrix}-B_2 & K_{22} & K_{24}\\ -B_3 & K_{32} & K_{34}\\ -B_4 & K_{42} & K_{44}\end{vmatrix}-B_2\begin{vmatrix}-B_2 & K_{21} & K_{24}\\ -B_3 & K_{31} & K_{34}\\ -B_4 & K_{41} & K_{44}\end{vmatrix}\right.$$

$$\left.+B_4\begin{vmatrix}-B_2 & K_{21} & K_{22}\\ -B_3 & K_{31} & K_{32}\\ -B_4 & K_{41} & K_{42}\end{vmatrix}\right\}$$

$$=-\frac{\varepsilon_1}{|J|}\left[-B_1\{K_{21}(K_{32}K_{44}-K_{42}K_{34})+K_{22}(K_{41}K_{34}-K_{31}K_{44})+K_{24}(K_{31}K_{42}-K_{41}K_{32})\}\right.$$

$$-K_{11}\{-B_2(K_{32}K_{44}-K_{42}K_{34})+K_{22}(-B_4K_{34}+B_3K_{44})+K_{24}(-B_3K_{42}+B_4K_{32})\}$$

$$+K_{12}\{-B_2(K_{31}K_{44}-K_{41}K_{34})+K_{21}(-B_4K_{34}+B_3K_{44})+K_{24}(-B_3K_{41}+B_4K_{31})\}$$

$$\left.-K_{14}\{-B_2(K_{31}K_{42}-K_{41}K_{32})+K_{21}(-B_4K_{32}+B_3K_{42})+K_{22}(-B_3K_{41}+B_4K_{31})\}\right]$$

$$+\frac{\lambda}{|J|}\left[B_1\left\{-B_2\left(K_{32}K_{44}-K_{42}K_{34}\right)+K_{22}\left(-B_4K_{34}+B_3K_{44}\right)+K_{24}\left(-B_3K_{42}+B_4K_{32}\right)\right\}\right.$$

$$-B_2\left\{-B_2\left(K_{31}K_{44}-K_{41}K_{34}\right)+K_{21}\left(-B_4K_{34}+B_3K_{44}\right)+K_{24}\left(-B_3K_{41}+B_4K_{31}\right)\right\}$$

$$\left.+B_4\left\{-B_2\left(K_{31}K_{42}-K_{41}K_{32}\right)+K_{21}\left(-B_4K_{32}+B_3K_{42}\right)+K_{22}\left(-B_3K_{41}+B_4K_{31}\right)\right\}\right]$$

$$=-\frac{\varepsilon_1}{|J|}\left\{-B_1K_{21}K_{32}K_{44}+B_1K_{21}K_{42}K_{34}-B_1K_{22}K_{41}K_{34}+B_1K_{22}K_{31}K_{44}-B_1K_{24}K_{31}K_{42}+B_1K_{24}K_{41}K_{32}\right.$$

$$+B_2K_{11}K_{32}K_{44}-B_2K_{11}K_{42}K_{34}+B_4K_{11}K_{22}K_{34}-B_3K_{11}K_{22}K_{44}+B_3K_{11}K_{24}K_{42}-B_4K_{11}K_{24}K_{32}$$

$$-B_2K_{12}K_{31}K_{44}+B_2K_{12}K_{41}K_{34}-B_4K_{12}K_{21}K_{34}+B_3K_{12}K_{21}K_{44}-B_3K_{12}K_{24}K_{41}+B_4K_{12}K_{24}K_{31}$$

$$\left.+B_2K_{14}K_{31}K_{42}-B_2K_{14}K_{41}K_{32}+B_4K_{14}K_{21}K_{32}-B_3K_{14}K_{21}K_{42}+B_3K_{14}K_{22}K_{41}-B_4K_{14}K_{22}K_{31}\right\}$$

$$+\frac{\lambda}{|J|}\left\{-B_1B_2K_{32}K_{44}+B_1B_2K_{42}K_{34}-B_1B_4K_{22}K_{34}+B_1B_3K_{22}K_{44}-B_1B_3K_{24}K_{42}+B_1B_4K_{24}K_{32}\right.$$

$$+B_2^2K_{31}K_{44}-B_2^2K_{41}K_{34}+B_2B_4K_{21}K_{34}-B_2B_3K_{21}K_{44}+B_2B_3K_{24}K_{41}-B_2B_4K_{24}K_{31}$$

$$\left.-B_2B_4K_{31}K_{42}+B_2B_4K_{41}K_{32}-B_4^2K_{21}K_{32}+B_3B_4K_{21}K_{42}-B_3B_4K_{22}K_{41}+B_4^2K_{22}K_{31}\right\}$$

$$=-\frac{1}{|J|}\frac{A^3\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\left\{-k\varepsilon_1\varepsilon_3pq^2rs(s-1)+k\varepsilon_1\varepsilon_3pq^2rs^2-k\varepsilon_1\varepsilon_3pq(q-1)rs^2-k\varepsilon_1\varepsilon_3pq^2rs^2\right.$$

$$+k\varepsilon_1\varepsilon_3pq(q-1)rs(s-1)+k\varepsilon_1\varepsilon_3pq^2rs^2+l\varepsilon_2\varepsilon_3p(p-1)qrs(s-1)-l\varepsilon_2\varepsilon_3p(p-1)qrs^2$$

$$+n\varepsilon_3\varepsilon_4p(p-1)q(q-1)rs-m\varepsilon_3^2p(p-1)q(q-1)s(s-1)+m\varepsilon_3^2p(p-1)q^2s^2-n\varepsilon_3\varepsilon_4p(p-1)q^2rs$$

$$-l\varepsilon_2\varepsilon_3x^2qrs(s-1)+l\varepsilon_2\varepsilon_3p^2qrs^2-n\varepsilon_3\varepsilon_4p^2q^2rs+m\varepsilon_3^2p^2q^2s(s-1)-m\varepsilon_3^2p^2q^2s^2$$

$$+n\varepsilon_3\varepsilon_4p^2q^2rs+l\varepsilon_2\varepsilon_3p^2qrs^2-l\varepsilon_2\varepsilon_3p^2qrs^2+n\varepsilon_3\varepsilon_4p^2q^2rs-m\varepsilon_3^2p^2q^2s^2+m\varepsilon_3^2p^2q(q-1)s^2$$

$$\left.-n\varepsilon_3\varepsilon_4p^2q(q-1)rs\right\}+\frac{1}{|J|}\frac{A^3\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}\nabla}{\varepsilon_1^2\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2B}\left\{-kl\varepsilon_1^2\varepsilon_2\varepsilon_3qrs(s-1)+kl\varepsilon_1^2\varepsilon_2\varepsilon_3qrs^2\right.$$

$$-kn\varepsilon_1^2\varepsilon_3\varepsilon_4q(q-1)rs-km\varepsilon_1^2\varepsilon_3^2q^2s^2+km\varepsilon_1^2\varepsilon_3^2q(q-1)s(s-1)+kn\varepsilon_1^2\varepsilon_3\varepsilon_4q^2rs$$

$$+l^2\varepsilon_1\varepsilon_2^2\varepsilon_3prs(s-1)-l^2\varepsilon_1\varepsilon_2^2\varepsilon_3prs^2+nl\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs-lm\varepsilon_1\varepsilon_2\varepsilon_3^2pqs(s-1)+lm\varepsilon_1\varepsilon_2\varepsilon_3^2pqs^2$$

$$-nl\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs-nl\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs+nl\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs-n^2\varepsilon_1\varepsilon_3\varepsilon_4^2pq^2r+mn\varepsilon_1\varepsilon_3^2\varepsilon_4pq^2s$$

$$\left.-mn\varepsilon_1\varepsilon_3^2\varepsilon_4pq(q-1)s+n^2\varepsilon_1\varepsilon_3\varepsilon_4^2pq(q-1)r\right\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3\varepsilon_4^2\nabla}\ \{-pq(s-1)\ -p(q-1)s\ +p(q-1)(s-1)\ +pqs\ +(p-1)q(s-1)$$

$$-pq(s-1)\ +pqs\ -(p-1)qs\ -2pqs\ +p(q-1)s\ -(p-1)(q-1)(s-1)\ +pq(s-1)\ +(p-1)qs$$

$$+(2\varepsilon_4-1)(p-1)(q-1)s\quad -(2\varepsilon_4-1)(p-1)qs\quad +(2\varepsilon_4-1)pqs\quad -(2\varepsilon_4-1)p(q-1)s\}$$

$$+\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3\varepsilon_4^2\nabla}\ \{-p(s-1)+qs\ +(q-1)(s-1)-qs\ +qs\ -(2\varepsilon_4-1)(q-1)s\ +q(s-1)$$

$$-qs\ -q(s-1)+qs\ +(2\varepsilon_4-1)qs\ -(q-1)s-(2\varepsilon_4-1)^2qs\ +(2\varepsilon_4-1)^2(q-1)s\}$$

$$=-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3\varepsilon_4^2\nabla}\ \{-pq(s-1)\ +p(q-1)(s-1)\ +(p-1)q(s-1)\ -(p-1)(q-1)(s-1)$$

$$+(2\varepsilon_4-1)(p-1)(q-1)s\quad -(2\varepsilon_4-1)(p-1)qs\quad +(2\varepsilon_4-1)pqs\quad -(2\varepsilon_4-1)p(q-1)s\}$$

$$+\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3\varepsilon_4^2\nabla}\quad \{-q(s-1)\quad +qs\quad -(q-1)s\quad +(q-1)(s-1)\quad -(2\varepsilon_4-1)(q-1)s$$

$$+(2\varepsilon_4-1)qs\ -(2\varepsilon_4-1)^2qs\ +(2\varepsilon_4-1)^2(q-1)s\}$$

$$\frac{\partial\varepsilon_3}{\partial k}=\frac{1}{|J|}\frac{A^3 4\varepsilon_4 pqrs^2\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3\varepsilon_4^2\nabla}(1-\varepsilon_4). \tag{22}$$

If $\varepsilon_4<1$ in equation (22) we have,

$$\frac{\partial\varepsilon_3}{\partial k}>0. \tag{23}$$

From inequality (23) we see that if the interest rate of the capital ε_1 increases, the purchasing of principal raw material ε_3 also increases. Hence, the production rate of the firm of course increases, and in parallel profit of the firm increases that tends to maximization. It seems that principal raw material ε_3 is essential for the firm; it has no substitutes (Moolio et al., 2009; Roy et al., 2021; Mohajan, 2022).

If $\varepsilon_4>1$ in equation (21) we have,

$$\frac{\partial\varepsilon_3}{\partial k}<0. \tag{24}$$

From inequality (24) we see that if the interest rate of the capital ε_1 increases; the purchasing level of principal raw material ε_3 decreases. The firm invests more fortune for increased interest of the capital and less to buy principal raw material. Consequently, the production rate of the firm may decrease. In this situation it seems that the firm may face difficulties for its sustainability (Moolio et al., 2009; Mohajan, 2021a).

Now we check the effect on irregular input ε_4 when interest rate of capital increases. Taking T_{51} (i.e., term of 5th row and 1st column) from both sides of (17) we get (Ferdous & Mohajan, 2022; Mohajan, 2021c),

$$\frac{\partial\varepsilon_4}{\partial k}=\frac{\varepsilon_1}{|J|}[C_{15}]+\frac{\lambda}{|J|}[C_{25}]$$

$$=\frac{\varepsilon_1}{|J|}\text{Cofactor of } C_{15}+\frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{25}$$

$$=\frac{\varepsilon_1}{|J|}\begin{vmatrix}-B_1 & K_{11} & K_{12} & K_{13}\\ -B_2 & K_{21} & K_{22} & K_{23}\\ -B_3 & K_{31} & K_{32} & K_{33}\\ -B_4 & K_{41} & K_{42} & K_{43}\end{vmatrix}-\frac{\lambda}{|J|}\begin{vmatrix}0 & -B_1 & -B_2 & -B_3\\ -B_2 & K_{21} & K_{22} & K_{23}\\ -B_3 & K_{31} & K_{32} & K_{33}\\ -B_4 & K_{41} & K_{42} & K_{43}\end{vmatrix}$$

$$=\frac{\varepsilon_1}{|J|}\left\{-B_1\begin{vmatrix}K_{21} & K_{22} & K_{23}\\ K_{31} & K_{32} & K_{33}\\ K_{41} & K_{42} & K_{43}\end{vmatrix}-K_{11}\begin{vmatrix}-B_2 & K_{22} & K_{23}\\ -B_3 & K_{32} & K_{33}\\ -B_4 & K_{42} & K_{43}\end{vmatrix}+K_{12}\begin{vmatrix}-B_2 & K_{21} & K_{23}\\ -B_3 & K_{31} & K_{33}\\ -B_4 & K_{41} & K_{43}\end{vmatrix}-K_{13}\begin{vmatrix}-B_2 & K_{21} & K_{22}\\ -B_3 & K_{31} & K_{32}\\ -B_4 & K_{41} & K_{42}\end{vmatrix}\right\}$$

$$-\frac{\lambda}{|J|}\left\{B_1\begin{vmatrix}-B_2 & K_{22} & K_{23}\\ -B_3 & K_{32} & K_{33}\\ -B_4 & K_{42} & K_{43}\end{vmatrix}-B_2\begin{vmatrix}-B_2 & K_{21} & K_{23}\\ -B_3 & K_{31} & K_{33}\\ -B_4 & K_{41} & K_{43}\end{vmatrix}+B_3\begin{vmatrix}-B_2 & K_{21} & K_{22}\\ -B_3 & K_{31} & K_{32}\\ -B_4 & K_{41} & K_{42}\end{vmatrix}\right\}$$

$$=\frac{\varepsilon_1}{|J|}\left[-B_1\{K_{21}(K_{32}K_{43}-K_{42}K_{33})+K_{22}(K_{41}K_{33}-K_{31}K_{43})+K_{23}(K_{31}K_{42}-K_{41}K_{32})\}\right.$$

$$-K_{11}\{-B_2(K_{32}K_{43}-K_{42}K_{33})+K_{22}(-B_4K_{33}+B_3K_{43})+K_{23}(-B_3K_{42}+B_4K_{32})\}$$

$$+K_{12}\{-B_2(K_{31}K_{43}-K_{41}K_{33})+K_{21}(-B_4K_{33}+B_3K_{43})+K_{23}(-B_3K_{41}+B_4K_{31})\}$$

$$\left.-K_{13}\{-B_2(K_{31}K_{42}-K_{41}K_{32})+K_{21}(-B_4K_{32}+B_3K_{42})+K_{22}(-B_3K_{41}+B_4K_{31})\}\right]$$

$$-\frac{\lambda}{|J|}\left[B_1\{-B_2(K_{32}K_{43}-K_{42}K_{33})+K_{22}(-B_4K_{33}+B_3K_{43})+K_{23}(-B_3K_{42}+B_4K_{32})\}\right.$$

$$-B_2\{-B_2(K_{31}K_{43}-K_{41}K_{33})+K_{21}(-B_4K_{33}+B_3K_{43})+K_{23}(-B_3K_{41}+B_4K_{31})\}$$

$$\left.+B_3\{-B_2(K_{31}K_{42}-K_{41}K_{32})+K_{21}(-B_4K_{32}+B_3K_{42})+K_{22}(-B_3K_{41}+B_4K_{31})\}\right]$$

$$=\frac{\varepsilon_1}{|J|}\{-B_1K_{21}K_{32}K_{43}+B_1K_{21}K_{42}K_{33}-B_1K_{22}K_{41}K_{33}+B_1K_{22}K_{31}K_{43}-B_1K_{23}K_{31}K_{42}+B_1K_{23}K_{41}K_{32}$$

$$+B_2K_{11}K_{32}K_{43}-B_2K_{11}K_{42}K_{33}+B_4K_{11}K_{22}K_{33}-B_3K_{11}K_{22}K_{43}+B_3K_{11}K_{23}K_{42}-B_4K_{11}K_{23}K_{32}$$

$$-B_2K_{12}K_{31}K_{43}+B_2K_{12}K_{41}K_{33}-B_4K_{12}K_{21}K_{33}+B_3K_{12}K_{21}K_{43}-B_3K_{12}K_{23}K_{41}+B_4K_{12}K_{23}K_{31}$$

$$+B_2K_{13}K_{31}K_{42}-B_2K_{13}K_{41}K_{32}+B_4K_{13}K_{21}K_{32}-B_3K_{13}K_{21}K_{42}+B_3K_{13}K_{22}K_{41}-B_4K_{13}K_{22}K_{31}\}$$

$$-\frac{\lambda}{|J|}\{-B_1B_2K_{32}K_{43}+B_1B_2K_{42}K_{33}-B_1B_4K_{22}K_{33}+B_1B_3K_{22}K_{43}-B_1B_3K_{23}K_{42}+B_1B_4K_{23}K_{32}$$

$$+B_2^2K_{31}K_{43}-B_2^2K_{41}K_{33}+B_2B_4K_{21}K_{33}-B_2B_3K_{21}K_{43}+B_2B_3K_{23}K_{41}-B_2B_4K_{23}K_{31}$$

$$-B_2B_3K_{31}K_{42}+B_2B_3K_{41}K_{32}-B_3B_4K_{21}K_{32}+B_3^2K_{21}K_{42}-B_3^2K_{22}K_{41}+B_3B_4K_{22}K_{31}\}$$

$$=\frac{1}{|J|}\frac{A^3\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\{-k\varepsilon_1\varepsilon_4pq^2r^2s+k\varepsilon_1\varepsilon_4pq^2r(r-1)s-k\varepsilon_1\varepsilon_4pq(q-1)r(r-1)s-k\varepsilon_1\varepsilon_4pq^2r^2s$$

$$+k\varepsilon_1\varepsilon_4pq(q-1)r(r-1)s+k\varepsilon_1\varepsilon_4pq^2r^2s+l\varepsilon_2\varepsilon_4p(p-1)qr^2s-l\varepsilon_2\varepsilon_4p(p-1)qr(r-1)s$$

$$+n\varepsilon_4^2p(p-1)q(q-1)r(r-1)-m\varepsilon_3\varepsilon_4p(p-1)q(q-1)rs+m\varepsilon_3\varepsilon_4p(p-1)q^2rs-n\varepsilon_4^2p(p-1)q^2r^2$$

$$-l\varepsilon_2\varepsilon_4p^2qr^2s+l\varepsilon_2\varepsilon_4p^2yqr(r-1)s-n\varepsilon_4^2p^2q^2r(r-1)+m\varepsilon_3\varepsilon_4p^2q^2rs-m\varepsilon_3\varepsilon_4p^2q^2rs$$

$$+n\varepsilon_4^2p^2q^2r^2+l\varepsilon_2\varepsilon_4p^2qr^2s-l\varepsilon_2\varepsilon_4p^2qr^2s+n\varepsilon_4^2p^2q^2r^2-m\varepsilon_3\varepsilon_4p^2q^2rs+m\varepsilon_3\varepsilon_4p^2q(q-1)rs$$

$$-n\varepsilon_4^2p^2q(q-1)r^2\}-\frac{\lambda}{|J|}\frac{A^2\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_1^2\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\{-kl\varepsilon_1^2\varepsilon_2\varepsilon_4qr^2s+kl\varepsilon_1^2\varepsilon_2\varepsilon_4qr(r-1)s$$

$$-kn\varepsilon_1^2\varepsilon_4^2q(q-1)r(r-1)+km\varepsilon_1^2\varepsilon_3\varepsilon_4q(q-1)rs-km\varepsilon_1^2\varepsilon_3\varepsilon_4q^2rs+kn\varepsilon_1^2\varepsilon_4^2q^2r^2+l^2\varepsilon_1\varepsilon_2^2\varepsilon_4pr^2s$$

$$-l^2\varepsilon_1\varepsilon_2^2\varepsilon_4pr(r-1)s+nl\varepsilon_1\varepsilon_3\varepsilon_4^2pqr(r-1)-lm\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs+lm\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs-nl\varepsilon_1\varepsilon_2\varepsilon_4^2pqr^2$$

$$-lm\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs+lm\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4pqrs-mn\varepsilon_1\varepsilon_3\varepsilon_4^2pq^2r+m^2\varepsilon_1\varepsilon_3^2\varepsilon_4pq(q-1)s-m^2\varepsilon_1\varepsilon_3^2\varepsilon_4pq^2s$$

$$+mn\varepsilon_1\varepsilon_3\varepsilon_4^2pq(q-1)r\}$$

$$=\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2}\{-k\varepsilon_1qr+k\varepsilon_1q(r-1)+l\varepsilon_2(p-1)r-l\varepsilon_2(p-1)(r-1)+l\varepsilon_2p(r-1)$$

$$-l\varepsilon_2pr-m\varepsilon_3(p-1)(q-1)+m\varepsilon_3(p-1)q-m\varepsilon_3pq+m\varepsilon_3p(q-1)+n\varepsilon_4(p-1)(q-1)(r-1)s^{-1}$$

$$-n\varepsilon_4(p-1)qrs^{-1}-n\varepsilon_4pq(r-1)s^{-1}+2n\varepsilon_4pqrs^{-1}-n\varepsilon_4p(q-1)rs^{-1}\}$$

$$-\frac{1}{|J|}\frac{A\varepsilon_1^p\varepsilon_2^q\varepsilon_3^r\varepsilon_4^s\nabla}{B}\frac{A^2\varepsilon_1^{2p}\varepsilon_2^{2q}\varepsilon_3^{2r}\varepsilon_4^{2s}}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4}\{-kl\varepsilon_1\varepsilon_2qr^2s+kl\varepsilon_1\varepsilon_2qr(r-1)s-kn\varepsilon_1\varepsilon_4q(q-1)r(r-1)$$

$$+kn\varepsilon_1\varepsilon_4q^2r^2+km\varepsilon_1\varepsilon_3q(q-1)rs-km\varepsilon_1\varepsilon_3q^2rs+l^2\varepsilon_2^2pr^2s-l^2\varepsilon_2^2pr(r-1)s+nl\varepsilon_2\varepsilon_4pqr(r-1)$$

$$-nl\varepsilon_1\varepsilon_4pqr^2+m^2\varepsilon_3^2pq(q-1)s-m^2\varepsilon_3^2pq^2s-mn\varepsilon_3\varepsilon_4pq^2r+mn\varepsilon_3\varepsilon_4pq(q-1)r\}$$

$$=\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4^2\nabla}\{-3pqr+2pq(r-1)-(p-1)q(r-1)+2(p-1)qr+p(q-1)r$$

$$-(p-1)(q-1)r+(2\varepsilon_4-1)(p-1)(q-1)(r-1)-(2\varepsilon_4-1)(p-1)qr-(2\varepsilon_4-1)pq(r-1)$$

$$+2(2\varepsilon_4-1)pqr-(2\varepsilon_4-1)p(q-1)r\}-\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4\nabla}\{-qr+q(r-1)$$

$$-(2\varepsilon_4-1)(q-1)(r-1)\ +(2\varepsilon_4-1)qr\ +(q-1)r\ -qr\ +l^2\varepsilon_2^2pr^2s\ -q(r-1)\ +(2\varepsilon_4-1)q(r-1)$$

$$-(2\varepsilon_4-1)qr\ +(q-1)r-qr\ -(2\varepsilon_4-1)qr\ +(2\varepsilon_4-1)(q-1)r\}$$

$$=\frac{1}{|J|}\frac{A^3pqrs\varepsilon_1^{3p}\varepsilon_2^{3q}\varepsilon_3^{3r}\varepsilon_4^{3s}B}{\varepsilon_1\varepsilon_2^2\varepsilon_3^2\varepsilon_4\nabla}\ \{2\varepsilon_4(2pqr+p+2r)+(-2pqr-p+q-pq)\}. \quad (25)$$

We consider, $p=\frac{1}{4}$, $q=\frac{1}{4}$, $r=\frac{1}{4}$, and $s=\frac{1}{4}$ in equation (25), then for constant returns to scale, $\nabla=1$ we get;

$$\frac{\partial\varepsilon_4}{\partial k}=\frac{1}{|J|}\frac{A^3B}{2^{13}\varepsilon_1^{1/4}\varepsilon_2^{5/4}\varepsilon_3^{5/4}\varepsilon_4^{1/4}\nabla}(52\varepsilon_4-3). \quad (26)$$

If $\varepsilon_4>3/52$ in (26) we get,

$$\frac{\partial\varepsilon_4}{\partial k}>0. \quad (27)$$

From (27) we see that if the interest rate of the capital ε_1 increases, the purchasing of irregular input ε_4 also increases. It seems that irregular input ε_4 is essential for the firm; it has no substitutes (Moolio et al., 2009; Mohajan, 2022).

If $\varepsilon_4<3/52$ in (26) we get,

$$\frac{\partial\varepsilon_4}{\partial k}<0. \quad (28)$$

From (28) we see that if the interest rate of the capital ε_1 increases; the purchasing level of irregular input ε_4 decreases. Consequently, the production rate of the firm may decrease for constant returns to scale. In this situation it seems that the firm may face difficulties for its sustainability (Moolio et al., 2009; Mohajan, 2022).

We consider, $p=\frac{1}{8}$, $q=\frac{1}{4}$, $r=\frac{1}{4}$, and $s=\frac{1}{4}$ in equation (24), then for decreasing returns to scale, $\nabla=\frac{7}{8}<1$ we get;

$$\frac{\partial\varepsilon_4}{\partial k}=\frac{1}{|J|}\frac{A^3B}{2^{13}\nabla\varepsilon_1^{5/8}\varepsilon_2^{5/4}\varepsilon_3^{5/4}\varepsilon_4^{1/4}}(82\varepsilon_4+5)>0,\forall\varepsilon_4>0. \quad (29)$$

From the relation (29) we see that if the interest rate of the capital ε_1 increases, the purchasing of irregular input ε_4 also increases. Hence, the production rate of the firm of course increases, but the firm faces the decreasing returns to scale. It seems that irregular input ε_4 is essential for the firm; it has no substitutes (Moolio et al., 2009; Mohajan, 2022).

11. Conclusions

In this study we have dealt with profit maximization activities where we have considered nonlinear budget constraint. We have not tried to verify whether the profit of the firm is maximized or not. Rather, we have considered that profit is maximized, and we have studied the effects of various inputs of the firm if interest rate of capital is increased. For the proper measurement we have considered Cobb-Douglas productions function as our profit function. Side by side we have also used 5×5 bordered Hessian matrix and 5×5 Jacobian. Throughout the study we have displayed the mathematical calculations elaborately.

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