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Transportation Problem via Median Approximation Technique for IBFS and Revised MODI's Technique

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Abstract

The work sought to improve the modified distribution method (MODI) and propose an approach for estimating the initial basic feasible solution (IBFS) for a transportation problem. The Median Approximation Technique (MAT), an explicit and effective approach for locating an IBFS to a transportation problem, was created and successfully tested on 10 problem sets. The created methodology, MAT, was compared to the five existing approaches for obtaining the IBFS: Vogel Approximation Method{VAM}; North West Corner Method {NWCM}; Least Cost Method{LCM}; Column Minimum Method{CMM}; and Row Minimum Method{RMM} in terms of iterations' number required to reach the optimum stage. Conclusively, the MAT under development provided a better IBFS across the board, as it produced optimum solutions for around 60% of the solved problem cases, as opposed to the IBFS created by the five existing approaches employed in this study, which gave optimum solutions for roughly 0%, 20%, 20%, 10%, and 10% of the examples, respectively.

Keywords: MAT, Linear Transportation Problem, Amended MODI Method, IBFS, Optimum Solution

1. Introduction

Transportation problem (TP), first proposed by Hitchcock in 1941, is a known and acceptable optimization network problem (Babu *et al*, 2014). The fundamental goal of TP is to discover the most economical way to move commodities from different supply sites to different demand points while minimizing overall transportation expenses. The amount readily available at a stage of conveyance, aggregate needed at a stage of request, and cost (unit price) of conveying a unit from a specific stage of supply to a certain stage of order are the parameters of a TP, however. A product or material flow between two businesses seeks to find feasible ways to ship comparable goods from various sources to various destinations in order to cut expenses overall (Haruna, 2010).

Transportation strategies can be used to solve the special class of linear programming issues known as linear TP (Chen *et al*, 2020). In order to satisfy the linear constraints, the objective function of the Linear TP is optimized using a linear programming approach because that is the major objective (Charkhgard *et al*, 2018). For a balanced TP, the IBFS can be obtained in a variety of ways (Chhibber *et al*, 2021). After the initial basic feasible solution (IBFS) has been determined, the primary objective of TP is to achieve the lowest transportation costs (Zhu *et al*, 2021).

The goal of this study is to create an algorithm that is effective in obtaining the IBFS, which would be extremely close to the optimal solution before increasing its IBFS or optimal in some circumstances. Reducing the total cost of a TP is one of its top priorities.

2. Materials and Methods

2.1 A TP in Linear Form

If the transportation unit cost of is denoted as R_{ij} the distance from the origin (m_i), to the landing point (n_j), and the distribution cost (z) is assumed to equal the total, then the real formulation of the Linear TP is as follows:

$$\left.\begin{array}{c} \min\ Z = \sum_{ij} r_{ij} w_{ij} \\ s.t.: \sum_i w_{ij} = n_j\,(j = 1, \cdots, p) \\ \sum_j w_{ij} = m_i\,(i = 1, \cdots, q) \\ w_{ij} \geq 0 \end{array}\right\} \quad (1)$$

Eq. (1) is balanced if and only if $\sum_i m_i = \sum_j n_j$

2.2 Linear case of a Transportation Tableau (TT)

A linear case of a TT is shown in Table 1.

Table 1. TT in Linear Case

	Landing point						
Origin	1	2	....	j	....	p	S_i
1	$\boxed{R_{11}}$ W_{11}	$\boxed{R_{12}}$ W_{12}	....	$\boxed{R_{1j}}$ W_{1j}	....	$\boxed{R_{1p}}$ W_{1p}	m_1
2	$\boxed{R_{21}}$ W_{21}	$\boxed{R_{22}}$ W_{22}	....	$\boxed{R_{2j}}$ W_{2j}	....	$\boxed{R_{2p}}$ W_{2p}	m_2
⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞
i	$\boxed{R_{i1}}$ W_{i1}	$\boxed{R_{i2}}$ W_{i2}	....	$\boxed{R_{ij}}$ W_{ij}	....	$\boxed{R_{ip}}$ W_{ip}	m_i
⁞	⁞	⁞	⁞	⁞	⁞	⁞	⁞
q	$\boxed{R_{q1}}$ W_{q1}	$\boxed{R_{q2}}$ W_{q2}	....	$\boxed{R_{qj}}$ W_{qj}	....	$\boxed{R_{qp}}$ W_{qp}	m_q
D_j	n_1	n_2	....	n_j	....	n_p	$\sum_i m_i = \sum_j n_j$

3. Algorithms for Median Approximation Technique (Researchers-Developed)

The new approach for calculating IBFS of a TP is known as the median approximation technique (MAT), and the procedures are listed below.

Stage 1: The TP should be in the form $\sum m_i = \sum n_j$.

Stage 2: Locate the smallest unit cost (R_i) in the TT {Choose the Allocation Cell Value (ACV) with the lowest aggregate cost by breaking ties arbitrarily, and but if a tie occurs in the minimum cell aggregate cost, select the cell with largest average subscripts (i, j)}.

Stage 3: Find the median price for each column and row.

Stage 4: Choose the ACV with the lowest total cost to arbitrarily break ties and find the row or column with the highest median for the rows and columns. Choose the column or row with the lowest cost and allocate as many units as you can to that cell.

Stage 5: Reduce the column demand and row supply by the number of units allotted to the cell, expunging column demand and row supply which are met before a new table is created.

Stage 6: When a row and a column are both filled at once, only one of them crosses out, while the other is given zero demand or supply; future median calculations shouldn't include rows or columns with no supply or demand.

Stage 7: Continue to stage 4 after recalculating the column/row medians of the decreased TT (as indicated in stage 3).

Stage 8: The total price of the TP is determined via Equation (2) after this process is completed and all supply and demand have been satisfied.

$$Z = \sum_{ij} r_{ij} w_{ij} \tag{2}$$

The outcome of the self-developed methodology will be contrasted with the outcomes of other existing methodologies like RMM, NWCM, CMM, VAM, and LCM.

4. A TP Solution

Procedures for finding a TP:

Stage a: Utilize the MAT to determine the IBFS as previously described.

Stage b: Calculate the TT's shadow costs A_i & B_j. Starting by putting A_1 as the number of basic cell in the first row; the multipliers A_i & B_j fulfill Eq. (3) for each basic variable in the present solution.

$$A_i + B_j = R_{ij} \tag{3}$$

Stage c: The potential benefits (V_{lt}) is calculated for each route, which is not currently in use using Eq. (4).

$$V_{lt} = R_{lt} - A_l - B_t \tag{4}$$

Stage d: The cell with the greatest minus V_{lt} value is chosen. The current allocation is optimum if $V_{lt} \geq 0$, otherwise, go to stage e.

Stage e: Assign the proportion "ɸ" to selected cells to get the class of cells now in use whose values n_j and m_i remain constant when the size "ɸ" is changed. This entails creating a closed loop for the input variable being used right now (the loop begins and finishes with the input variable). A loop is made up of several vertical and horizontally connected lines, all of whose endpoints — aside from those connected to input variables — must be simple variables. For a specific basic solution, loops can be traced in either a clockwise or counterclockwise direction.

Per non-basic variable, only one distinct loop is permitted. Then use to modify the loop corner basis variable with "ɸ".

Stage f: Make "ɸ" as large as you can. Stage b will follow when you redraft the Tableau.

5. Providing Examples

Ten examples were used to illustrate the points being made. Example 1 was taken from Shweta and Vishal (2022), examples 2 and 4 from Mollah et al (2016), example 3 from Inyama (2007), example 5 from Okenwe (2018), example 6 from Abdul-Salam (2014), example 7 from Eghbal (2017), example 8 from a study by Opara et al (2019), and example 9 from Opara et al (2016) and example 10 from Shweta and Vishal (2022).

Ex. 1

Table 2. Data for Ex. 1

	Mkt.				SUP
W/H	1	2	3	4	
1	4	6	8	8	40
2	6	8	6	7	60
3	5	7	6	8	50
DEM	20	30	50	50	

Employing the new approach MAT, the IBFS is calculated as follows:

The TC with lowest unit is 4 at cell (1, 1), which is;

$$w_{11} = MIN(m_1, n_1) = MIN(40, 20) = 20$$

This implies that Mkt. 1 is met fully and stroke out whereas W/H 1 remained 40 – 20 = 20 units more; and the adjusted table is:

Table 3. First table solution from Ex. 1

	Mkt.				
W/H	1	2	3	4	Sup
1	$\boxed{4}$ (20)	$\boxed{6}$	$\boxed{8}$	$\boxed{8}$	20
2	$\boxed{6}$ 0	$\boxed{8}$	$\boxed{6}$	$\boxed{7}$	60
3	$\boxed{5}$ 0	$\boxed{7}$	$\boxed{6}$	$\boxed{8}$	50
Dem	0	30	50	50	

Go to stage 3 after crossing out the smallest unit of TC that satisfied the requirements.

Table 4. Second table solution from Ex. 1

	Mkt.					
W/H	1	2	3	4	Sup	MDN
1	$\boxed{4}$ (20)	$\boxed{6}$	$\boxed{8}$	$\boxed{8}$	20	8
2	$\boxed{6}$ 0	$\boxed{8}$	$\boxed{6}$	$\boxed{7}$	60	7
3	$\boxed{5}$ 0	$\boxed{7}$	$\boxed{6}$	$\boxed{8}$	50	7
Dem	0	30	50	50		
MDN	–	7	6	8		

The row 1 and column 4 have the highest median (8, tied), and the cells with the lowest unit costs are (1, 2) and (2, 4), respectively. Thus;

$$w_{12} = MIN(m_1, n_2) = MIN(20, 30) = 20 \Rightarrow TCC = 20(6) = 120$$

$$w_{24} = MIN(m_2, n_4) = MIN(60, 50) = 50 \Rightarrow TCC = 50(7) = 350$$

The lowest Total Cell Cost (TCC) is chosen, which is w_{12}. This implies that W/H. 1 is met fully and stroke out whereas Mkt. 2 remained 30 – 20 = 10 units more; and the adjusted table is:

Table 5. Third table solution from Ex. 1

	Mkt.					
W/H	1	2	3	4	Sup	MDN
1	$\boxed{4}$ (20)	$\boxed{6}$ (20)	$\boxed{8}$ 0	$\boxed{8}$ 0	0	–
2	$\boxed{6}$	$\boxed{8}$	$\boxed{6}$	$\boxed{7}$	60	7

	0					
3	$\boxed{5}$ 0	$\boxed{7}$	$\boxed{6}$	$\boxed{8}$	50	7
Dem	0	10	50	50		
MDN	–	7.5	6	7.5		

The columns 2 and 4 have the highest median (7.5, tied), and the cells with the lowest unit costs are (3, 2) and (2, 4), respectively. Thus;

$$w_{32} = MIN(m_3, n_2) = MIN(50, 10) = 10 \Rightarrow TCC = 10(7) = 70$$

$$w_{24} = MIN(m_2, n_4) = MIN(60, 50) = 50 \Rightarrow TCC = 50(7) = 350$$

The lowest TCC is chosen, which is w_{32}. This implies that Mkt. 2 is met fully and stroke out whereas W/H 3 remained 50 – 10 = 40 units more; and the adjusted table is:

Table 6. Fourth table solution from Ex. 1

	Mkt.					
W/H	1	2	3	4	Sup	MDN
1	$\boxed{4}$ (20)	$\boxed{6}$ (20)	$\boxed{8}$ 0	$\boxed{8}$ 0	0	–
2	$\boxed{6}$ 0	$\boxed{8}$ 0	$\boxed{6}$	$\boxed{7}$	60	6.5
3	$\boxed{5}$ 0	$\boxed{7}$ (10)	$\boxed{6}$	$\boxed{8}$	40	7
Dem	0	0	50	50		
MDN	–	–	6	7.5		

The median largest value is 7.5, which corresponds to column 4 with cell (2, 4) being the lowest unit price. Thus;

$$w_{24} = MIN(m_2, n_4) = MIN(60, 50) = 50$$

This implies that Mkt. 4 is met fully and stroke out whereas W/H 2 remained 60 – 50 = 10 units more; and the adjusted table is:

Table 7. Fifth table solution from Ex. 1

	Mkt.					
W/H	1	2	3	4	Sup	MDN
1	$\boxed{4}$ (20)	$\boxed{6}$ (20)	$\boxed{8}$ 0	$\boxed{8}$ 0	0	–
2	$\boxed{6}$ 0	$\boxed{8}$ 0	$\boxed{6}$	$\boxed{7}$ (50)	10	–
3	$\boxed{5}$ 0	$\boxed{7}$ (10)	$\boxed{6}$	$\boxed{8}$ 0	40	–
Dem	0	0	50	0		
MDN	–	–	6	–		

The median is 6, being the only one left at column 3 with its smallest costs (tied 6) at cells (2, 3) and (3, 3). Thus, their respective costs allocations become:

$$w_{23} = MIN(m_2, n_3) = MIN(10, 50) = 10$$

$$w_{33} = MIN(m_3, n_3) = MIN(40, 50) = 40$$

At this point, all the markets and warehouses are met fully and consequently, the finalized updated table is shown below;

Table 8. Sixth table solution from Ex. 1

	Mkt.				
W/H	1	2	3	4	Sup
1	$\boxed{4}$ 20	$\boxed{6}$ 20	$\boxed{8}$	$\boxed{8}$	40
2	$\boxed{6}$	$\boxed{8}$	$\boxed{6}$ 10	$\boxed{7}$ 50	60
3	$\boxed{5}$	$\boxed{7}$ 10	$\boxed{6}$ 40	$\boxed{8}$	50
Dem	20	30	50	50	

The IBFS is W_{11} = 20, W_{12} = 20, W_{23} = 10, W_{24} = 50, W_{32} = 10, W_{33} = 40; and which is equivalent to: = (20, 20, 0, 0, 0, 0, 10, 50, 0, 10, 40, 0).

$$\overline{W} = (W_{B11}, W_{B21}, W_{13}, W_{14}, W_{21}, W_{22}, W_{B23}, W_{B24}, W_{31}, W_{B32}, W_{B33}, W_{34})$$

The total TC is 20(4) + 20(6) + 10(6) + 50(7) + 10(7) + 40(6) = ₦ 920 is the total TC.

We now go on to **stage 2** by enhancing the IBFS calculated by first computing the shadow costs $(A_i \,\&\, B_j)$ in order to acquire the best TP solution.

Employing Eq. (3) to achieve:

Stage 2:

W_{11}: $A_1 + B_1 = 4$

W_{12}: $A_1 + B_2 = 6$

W_{23}: $A_2 + B_3 = 6$

W_{24}: $A_2 + B_4 = 7$

W_{32}: $A_3 + B_2 = 7$

W_{33}: $A_3 + B_3 = 6$

$A_1 = 2$ and since the occurrence of basic cell in row one is two and solving the equation sets produce:

$$A_1 = 2; A_2 = 2; A_3 = 3.$$

$$B_1 = 2; B_2 = 4; B_3 = 3; B_4 = 5.$$

Stage 3:

Using Eq. (4), the following results are obtained for each of the six non-basic variables (V_{lt}) :

W_{13}: $V_{13} = R_{13} - A_1 - B_3 = 8 - 2 - 3 = 3$

W_{14}: $V_{14} = R_{14} - A_1 - B_4 = 8 - 2 - 5 = 1$

W_{21}: $V_{21} = R_{21} - A_2 - B_1 = 6 - 2 - 2 = 2$

$$W_{22}: V_{22} = R_{22} - A_2 - B_2 = 8 - 2 - 4 = 2$$

$$W_{31}: V_{31} = R_{31} - A_3 - B_1 = 5 - 3 - 2 = 0$$

$$W_{34}: V_{34} = R_{34} - A_3 - B_4 = 8 - 3 - 5 = 0$$

Stage 4: The present IBFS is optimal because none of the V_{lt} are negative, thus we stop.

Ex. 2

Table 9. Data for Ex. 2

	Mkt.			SUP
W/H	1	2	3	
1	$\boxed{4}$	$\boxed{3}$	$\boxed{5}$	90
2	$\boxed{6}$	$\boxed{5}$	$\boxed{4}$	80
3	$\boxed{8}$	$\boxed{10}$	$\boxed{7}$	100
DEM	70	120	80	

The IBFS of Ex. 2 is calculated using MAT and the results are displayed below:
W_{11} = 50, W_{12} = 40, W_{22} = 80, W_{31} = 20, W_{33} = 80; which is equivalent to; = (50, 40, 0, 0, 80, 0, 20, 0, 80).

$$\overline{W} = (W_{B11}, W_{B12}, W_{13}, W_{21}, W_{B22}, W_{23}, W_{B31}, W_{22}, W_{B33})$$

50(4) + 40(3) + 80(5) + 20(8) + 80(7) = ₦ 1440 is the total TC.

We now go on to **stage 2** by enhancing the IBFS calculated by first computing the shadow costs $(A_i \ \& \ B_j)$ in order to acquire the best TP solution.
Employing Eq. (3) to achieve:

Stage 2:

$$W_{11}: A_1 + B_1 = 4$$

$$W_{12}: A_1 + B_2 = 3$$

$$W_{22}: A_2 + B_2 = 5$$

$$W_{31}: A_3 + B_1 = 8$$

$$W_{33}: A_3 + B_3 = 7$$

$A_1 = 2$ and since the occurrence of basic cell in row one is two and solving the equation sets produce:

$$A_1 = 2; A_2 = 4; A_3 = 6.$$

$$B_1 = 2; B_2 = 1; B_3 = 1.$$

Stage 3: Using Eq. (4), the following results are obtained for each of the six non-basic variables (V_{lt}) :

$$W_{13}: V_{13} = R_{13} - A_1 - B_3 = 5 - 2 - 1 = 2$$

$$W_{21}: V_{21} = R_{21} - A_2 - B_1 = 6 - 4 - 2 = 0$$

$$W_{23}: V_{23} = R_{23} - A_2 - B_3 = 4 - 4 - 1 = -1$$

$$W_{32}: V_{32} = R_{32} - A_3 - B_2 = 10 - 6 - 1 = 3$$

The summarized result is displayed in the table as shown below:

Table 10. Initial summarized data for Ex. 2

	Mkt.				
W/H	1	2	3	A_i	SUP
1	$\boxed{4}$ 50	$\boxed{3}$ 40	$\boxed{5}$ (2)	2	90
2	$\boxed{6}$ (0)	$\boxed{5}$ 80	$\boxed{4}$ (-1)	4	80
3	$\boxed{8}$ 20	$\boxed{10}$ (3)	$\boxed{7}$ 80	6	100
B_j	2	1	1		
DEM	70	120	80		

Stage 4:

The greatest negative V_{lt} number, though the only one, is in cell (2, 3). W_{23} is the entering variable as a result.

Stage 5:

We now create loops for the variable W_{23} that is entering.

Table 11. Summarized loop data for Ex. 2

	Mkt.				
W/H	1	2	3	A_i	SUP
1	− $\boxed{4}$ (50)	+ $\boxed{3}$ (40)	$\boxed{5}$	2	90
2	$\boxed{6}$	$\boxed{5}$ (80) −	$\boxed{4}$ +	4	80
3	$\boxed{8}$ + (20)	$\boxed{10}$	$\boxed{7}$ (80) +	6	100
B_j	2	1	1		
DEM	70	120	80		

In the formed loop, the corners with minus signs with the least number among the basic is the leaving variable. The leaving variable has a minimum value of 50 and falls in W_{11}.

Stage 6:

The tableau was changed as shown below by adding 50 to the corners with plus signs and subtracting 50 from the corners with minus signs:

Table 12. Final summarized loop data for Ex. 2

	Mkt.			
W/H	1	2	3	SUP
1	$\boxed{4}$	$\boxed{3}$ 90	$\boxed{5}$	90
2	$\boxed{6}$	$\boxed{5}$	$\boxed{4}$	80

		30	50	
3	8 70	10	7 30	100
DEM	70	120	80	

The basic solution becomes:

$W_{12} = 90$, $W_{22} = 30$, $W_{23} = 50$, $W_{31} = 70$, $W_{33} = 30$ and we move to stage 2:

Stage 2:

W_{12}: $A_1 + B_2 = 3$

W_{22}: $A_2 + B_2 = 5$

W_{23}: $A_2 + B_3 = 4$

W_{31}: $A_3 + B_1 = 8$

W_{33}: $A_3 + B_3 = 7$

$A_1 = 1$ and since the occurrence of basic cell in row one is 1 and solving the equation sets produce:

$$A_1 = 1; A_2 = 3; A_3 = 6.$$

$$B_1 = 2; B_2 = 2; B_3 = 1.$$

Stage 3

Using Eq. (4), the following results are obtained for each of the four non-basic variables (V_{lt}) :

W_{11}: $V_{11} = R_{11} - A_1 - B_1 = 4 - 1 - 2 = 1$

W_{13}: $V_{13} = R_{13} - A_1 - B_3 = 5 - 1 - 1 = 3$

W_{21}: $V_{21} = R_{21} - A_2 - B_1 = 6 - 3 - 2 = 1$

W_{32}: $V_{32} = R_{32} - A_3 - B_2 = 10 - 6 - 2 = 2$

Stage 4:

The present basic solution is optimal because none of the V_{lt} are negative, thus we stop, and the lowest price of TP is R = 90(3) + 30(5) + 50(4) + 70(8) + 30(7) = ₦ 1390 with optimal solution as $W_{12} = 90$, $W_{22} = 30$, $W_{23} = 50$, $W_{31} = 70$, $W_{33} = 30$.

Table 13. Information concerning examples 3 to 10

Examples	**Size**	**SUP**	**DEM**	**Cost**
3	3 × 4	[12 14 10]	[10 6 8 12]	$\begin{bmatrix} 8 & 6 & 3 & 9 \\ 2 & 6 & 3 & 4 \\ 7 & 8 & 6 & 3 \end{bmatrix}$
4	4 × 4	[30 25 20 15]	[30 30 20 10]	$\begin{bmatrix} 7 & 5 & 9 & 11 \\ 4 & 3 & 8 & 6 \\ 3 & 8 & 10 & 5 \\ 2 & 6 & 7 & 3 \end{bmatrix}$

5	3 × 3	[13000 13000 15000]	[11000 18000 12000]	$\begin{bmatrix} 5 & 4 & 6 \\ 7 & 6 & 5 \\ 9 & 11 & 8 \end{bmatrix}$
6	3 × 4	[15000 25000 10000]	[20000 10000 8000 12000]	$\begin{bmatrix} 15 & 10 & 4 & 20 \\ 7 & 6 & 8 & 3 \\ 1 & 9 & 5 & 3 \end{bmatrix}$
7	3 × 4	[50 75 25]	[20 20 50 60]	$\begin{bmatrix} 3 & 5 & 7 & 6 \\ 2 & 5 & 8 & 2 \\ 3 & 6 & 9 & 2 \end{bmatrix}$
8	4 × 4	[6 9 7 12]	[10 4 6 14]	$\begin{bmatrix} 2 & 5 & 6 & 3 \\ 9 & 6 & 2 & 1 \\ 5 & 2 & 3 & 6 \\ 7 & 7 & 2 & 4 \end{bmatrix}$
9	4 × 3	[11 12 10 7]	[16 10 14]	$\begin{bmatrix} 4 & 3 & 4 \\ 10 & 7 & 5 \\ 8 & 8 & 3 \\ 5 & 6 & 6 \end{bmatrix}$
10	5 × 5	[6 2 4 10 9]	[2 8 7 10 4]	$\begin{bmatrix} 12 & 8 & 11 & 18 & 11 \\ 14 & 22 & 8 & 12 & 14 \\ 14 & 14 & 16 & 14 & 15 \\ 19 & 11 & 14 & 17 & 15 \\ 13 & 9 & 17 & 20 & 11 \end{bmatrix}$

6. Summary

The study compared the findings produced with the new MAT methodology and the improved MODI method with those obtained with the existing methods after computing the IBFS using the method established using a set of 10 problems.

Table 14. Results using existing approaches and MAT compared

		NWCM	**LCM**	**VAM**	**RMM**	**CMM**	**MAT (Self-Developed)**
Ex. 1	IBS	980	960	960	960	920	920
	NOITO	4	2	2	2	1	1
	OV	**920**					
Ex. 2	IBS	1500	1450	1500	1450	1500	1440
	NOITO	3	2	3	2	3	2
	OV	**1390**					
Ex. 3	IBS	178	118	118	124	128	118
	NOITO	3	1	1	2	2	1
	OV	**118**					

Ex. 4	IBS	540	435	470	470	435	440
	NOITO	6	3	4	4	3	3
	OV	**410**					
Ex. 5	IBS	270	261	270	261	270	261
	NOITO	3	2	3	2	3	2
	OV	**253,000**					
Ex. 6	IBS	420	264	236	236	327	236
	NOITO	4	2	1	1	2	1
	OV	**236**					
Ex. 7	IBS	670	650	650	745	630	610
	NOITO	3	3	2	5	2	1
	OV	**610**					
Ex. 8	IBS	149	83	92	92	88	83
	NOITO	6	1	2	2	2	1
	OV	**83**					
Ex. 9	IBS	230	199	199	204	187	183
	NOITO	4	2	2	3	2	1
	OV	**183**					
Ex. 10	IBS	459	383	383	428	383	381
	NOITO	7	2	2	6	2	2
	OV	**378**					

Key: NOITO = No. of Iteration to Optimality, OV = Optimal Value, IBS = Initial Basic Solution.

The TC for each of the 10 exemplary situations is shown in Table 14. The LCM and VAM only yielded two optimum solutions in its IBS, which constitutes 20%, RMM and CMM yielded one optimum results in its IBS, which constitutes 10%; but the MAT yielded six optimum solutions in its IBS, which constitutes 60%. NWCM provided no single optimum solution in its IBS. The three additional problems that have been solved for which IBS performs less well than the MAT technique for which it was designed are likewise extremely near to ideal. Based on the utilized instances, the MAT delivers an IBFS that is more acceptable than the traditional approaches.

7. Conclusion

The MAT, being optimal, is shown to be closer to the ideal solution and, in some situations, results in the best IBS. Therefore, rather than initially obtaining the IBS, future research should think about creating algorithms that directly find the TP's optimal solution. The work invites forthcoming study to write a programme to handle the developed algorithm.

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