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A Mathematical Modeling of Newtonian Blood Flow Through Arterial Mild Stenosis

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Abstract

Mathematical models in health science are usually used to study the blood flow behaviors into the artery. The blood flow through the artery is modeled as an incompressible and homogenous Newtonian fluid, and the flow is assumed to be steady and laminar. Stenosis is an abnormal narrowing in arteries for excess fat deposition that restricts the normal pattern of blood flow through arteries and may cause a heart attack. It is evident that the stenosis in artery may develop due to the accumulation of fat, cholesterol, and abnormal intravascular growth of tissue. The stenosis is to be symmetric about the axis of artery and the flow of blood is axisymmetric. The shear stress has direct proportional relation with both length and height of stenosis. This study uses Navier-Stokes equations in a cylindrical coordinate system and assumes axial symmetry under laminar flow to discuss the blood flow through the mild stenosis artery.

Keywords: artery, blood flow, Newtonian fluid, shear stress, stenosis

1. Introduction

Blood is an essential life-maintaining fluid, which circulates within our whole body through the arteries, veins, and capillaries. Main ingredients of blood are red blood cells, white blood cells, platelets, and plasma (Maton et al., 1993; Guyton & Hall, 2000). The plasma is an aqueous solution composed of about 90% water and 7% protein. In a milliliter of healthy human blood there are about 5×10^9 cells; of which about 94% are red blood cells (erythrocytes), about 1% are white blood cells (leucocytes), and about 5% are platelets (thrombocytes) (Phaijoo, 2013; Boron & Boulpaep, 2017). Main function of erythrocytes is to transport oxygen from the lungs to all the cells to the body and the removal of carbon-dioxide formed by metabolic processes in the body through the transportation to the lungs, leucocytes play a role in the resistance of the body to infection, and platelets perform a function related to blood clotting (Kapur, 1985).

During the flowing of blood through the larger diameter arteries at high shear rates, behaves like a Newtonian fluid. Blood behaves as an incompressible Newtonian fluid when it flows through arteries with a larger diameter at a high shear rate within the range of Reynolds numbers 5,000-10,000 (Pralhad & Schultz, 2004). When stenosis is developed in the artery, the blood flow behaves as a non-Newtonian fluid. Stenosis in an artery is the accumulation of fat, cholesterol, etc. in the walls of artery that narrows the passage of blood, and the abnormal growth of tissue, which losses the elasticity that leads to cardiovascular diseases, such as stroke and heart attack (Phaijoo, 2013). A common cause of this restriction is a chronic disease process named atherosclerosis. Stroke is the second most common cause of death that follows heart disease (Moayeri & Zendehbudi, 2003).

It is believed that deposits of cholesterol, fatty substances, cellular waste products, calcium, and fibrin may be responsible for the development of the cardiovascular disease (Mishra et al., 2011). The development of atherosclerotic plaque creates the most effective changes in the pressure, velocity, wall shear stress, and flow resistance of blood flow (Kamangar et al., 2017). After development of stenosis, various complications are seen due to turbulence flow, such as flow resistance, high shear stress on the blood vessel wall, tensile stress in endothelial cell membrane, change in blood rheology due to deformability of red blood cells, the surface cell loss as well as internal cell motion due to pressure and shear stress (Varghese & Frankel, 2003). In this review article mathematical model of Newtonian blood flow through mild stenosis artery is studied to address the mechanics of cardiovascular system (Pokhrel et al., 2020).

2. Literature Review

Literature review is an introductory section of a research, where works of previous researchers are presented in briefly that helps the novice researchers to understand the subject area of the research (Creswell, 2007; Polit & Hungler, 2013). A rich and furnished literature review can ensure a research about the proper research methodology (Baglione, 2012; Torraco, 2016). Puskar R. Pokhrel and his coworkers have analyzed the blood flow dynamics in cylindrical form by evaluating pressure, pressure drop against the wall, and shear stress on the wall. They have also discussed the dynamics by evaluating the ratio of pressure drop with stenosis to the pressure drop without stenosis against the wall, and the ratio of maximum to minimum shear stresses with the ratios of various thicknesses of stenosis to radius of the artery (Pokhrel et al., 2020).

Somchai Sriyab has developed a mathematical model of non-Newtonian blood flow through different stenosis, such as bell shape and cosine shape. He has provided some potential aspects to further study the causes and development of cardiovascular diseases (Sriyab, 2020). Shailesh Mishra and his coworkers have run the fluid mechanical study on the effects of the permeability of the wall through an artery with a composite stenosis (Mishra et al., 2011). Sarfaraz Kamangar and his coauthors have investigated the blood flow behavior, and the severity of blockage caused in the arterial passage due to the different geometries, such as elliptical, trapezium and triangular shapes of stenosis (Kamangar et al., 2017).

Hamed M. Sayed and Emad H. Aly have examined the flow of the blood through asymmetric artery with mild stenosis with the effect of asymmetrical shape and non-Newtonian behavior of blood. They have observed that the resistance impedance increases as the stenosis length, stenosis size and flow rate increase, and it decreases as the stenosis shape parameter and yield value increases (Sayed & Aly, 2015). Rupesh K. Srivastavr has worked on the flow of blood through a narrow catheterized artery with axially non-symmetrical stenosis. He has derived the impedance, the wall shear stress, and the shear stress at stenosis throat (Srivastavr, 2015). Devajit Mohajan and Haradhan Kumar Mohajan have studied obesity, BMI, diabetes mellitus, eating disorders, and various anthropometric indices (Mohajan & Mohajan, 2023a-m). They have also studied on insulin and various oral medications for the treatment of T2D. They have stressed that overweight and obesity are the roots of many non-communicable diseases (Mohajan & Mohajan, 2023n-v).

3. Research Methodology of the Study

Research is a procedure of systematic investigations that requires collection, interpretation and refinement of collected data that finally prepares a research work (Pandey & Pandey, 2015). It is considered as a hard-working search, scholarly inquiry, and investigation for a researcher, which targets for the discovery of new concepts (Adams et al., 2007). Most of the researchers often try to write a methodology section that explains how a research work is carried out (Kothari, 2008). It provides the research design and analysis procedures to perform a good research (Hallberg, 2006). Therefore, research methodology is a set of principles to the researchers for organizing, planning, designing, and conducting a good research (Legesse, 2014). It helps to identify research areas and projects within these areas (Blessing et al., 1998). In this mini review article, we have tried to discuss blood flow related complexities. Ethical approval is essential to maintain the possible benefits, and to minimize harm to participants, science, and society (NASW, 2021). Throughout this review article we have tried to maintain the reliability and validity as far as possible (Mohajan, 2017, 2018, 2020). To prepare this article we have dependent on the secondary data sources related to mild stenosis. We have used books handbooks, theses, and various research reports of renowned authors. We have also collected some valuable information from websites and internets to enrich the paper (Mohajan & Mohajan, 2024a-j).

4. Objective of the Study

Main objective of this article is to discuss the basic concept of blood flow through the artery and its related diseases when a mild stenosis is formed in the artery. When a stenosis is developed in the artery, decreases the blood flow to the heart, causes chest pain, sometimes happens shortness of breath, and also other coronary artery diseases are visible. Other minor objectives of the study are as follows:

- to focus on the blood disorders that develop various cardiovascular diseases,

- to highlight mathematical structure of biomechanics, and
- to show the pressure drop and optimum stress when a mild stenosis is developed.

5. Some Definitions Related to Blood Flow

Flux: Flux can be defined as the total volume of the fluid crossing any section per unit time. In mathematical notation, the flux Q of the blood flow is the total volume of the blood crossing any section of the artery of radius $r = a$ units per unit length, and is given by (Phaijoo, 2013),

$$Q = \int_0^a v.2\pi r dr \tag{1}$$

where v is the velocity of the blood towards the direction of the flow (Figure 1).

Pressure Gradient: A blood flow through the artery is driven by a pressure gradient that is normally represented by the difference between the arterial and venous pressures across the organ. The pressure gradient is the difference in pressure across a length of a tube in z -direction and is given by (Phaijoo, 2013),

$$\frac{dp}{dz} = -\frac{P_1 - P_2}{L} \tag{2}$$

where L is the length of the tube, P_1 and P_2 are the values of pressure P at the initial and terminal extremities of the tube respectively, and z is the direction of the flow (Figure 1).

Pressure Drop: It is the difference in pressure between two points of a fluid carrying network, and is caused by the resistance to flow, is denoted by ΔP , and is given by $\left(P_L - P_0\right)$ (Phaijoo, 2013). High flow velocities or high fluid viscosities result in a larger pressure drop and low velocity will result in lower, or no pressure drop across a section of pipe (Figure 1).

6. Mathematical Structure of Fluid Dynamics

The blood flow through the artery is modeled flow through cylinder. Let r be the radius of the artery, p be the pressure, three velocity components are v^r along the radius vector, v^θ perpendicular to the radius vector, and v^z parallel to the z-axis respectively. The blood flow through artery can be represented by the equation of continuity and Navier-Stokes equations (NSEs). Now equation of continuity in cylindrical coordinates is (Appendix I4),

$$\frac{1}{r}\frac{\partial r v^r}{\partial r} + \frac{\partial v^z}{\partial z} = 0 . \tag{3}$$

The NSEs for axially symmetric flow are (Appendix II6),

for r -momentum; $$\rho\left(\frac{\partial v^r}{\partial t} + v^r\frac{\partial v^r}{\partial r} + v^z\frac{\partial v^r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v^r}{\partial r^2} + \frac{\partial^2 v^r}{\partial z^2} + \frac{1}{r}\frac{\partial v^r}{\partial r} - \frac{v^r}{r^2}\right) \tag{4}$$

for z -momentum; $$\rho\left(\frac{\partial v^z}{\partial t} + v^r\frac{\partial v^z}{\partial r} + v^z\frac{\partial v^z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v^z}{\partial r^2} + \frac{\partial^2 v^z}{\partial z^2} + \frac{1}{r}\frac{\partial v^z}{\partial r}\right). \tag{5}$$

In axisymmetric case we consider $v^\theta = 0$, and v^r , v^z , and p are independent of θ. For the steady flow we consider constant viscosity μ , and constant density ρ . It is assumed that the fluid is Newtonian, and the flow is laminar and steady, the artery is of constant diameter. It is assumed that the stenosis develops in an axially symmetric manner due to some abnormal growth over a length $2z_0$ (Figure 1). We consider that the rate of growth of stenosis is expected to be a function of time t. We have $v^r = 0$, $v^\theta = 0$, and $v^z = v$, then equation (3) becomes (Pokhrel et al., 2020),

$$\frac{\partial v^z}{\partial z} = 0 . \tag{6}$$

Equation (4) becomes,

$$\frac{\partial p}{\partial r} = 0 . \tag{7}$$

Equation (5) becomes,

$$0=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right). \tag{8}$$

Let $P(z)=-\frac{\partial p}{\partial z}$ then (8) becomes,

$$\frac{\partial}{\partial r^2}\left(r\frac{\partial v}{\partial r}\right)=-P(z)\frac{r}{\mu}. \tag{9}$$

The boundary conditions are, $v=0$ at $r=R(z)$ for $-z_0\le z\le z_0$ and $v=0$ at $r=R_0$ for $|z|\ge z_0$ (Figure 1).

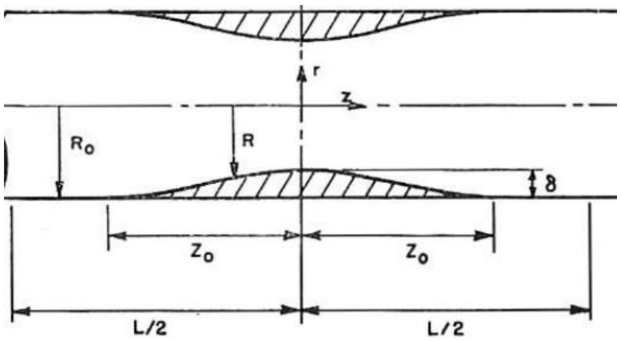

Figure 1. Section of an artery for mild stenosis

Source: Young (1968).

7. Mathematical Study of the Blood Flow

We also assume that the time rate of change of the radius $R(t)$ of the cylindrical pipe and cosine shape stenosis model is given as (Kapur, 1985),

$$\frac{\partial R}{\partial t}=-\alpha_0\left(1+\cos\frac{\pi z}{z_0}\right)e^{-\frac{t}{\tau}} \tag{10}$$

for $-z_0\le z\le z_0$, where $\alpha_0=\text{constant}$, and τ is the time constant for the stenotic growth, R_0 is the radius of a normal artery, $R(z)$ is the radius of the mild stenosed artery. For all other z we have,

$$\frac{\partial R(z)}{\partial t}=0. \tag{11}$$

Integrating (10) we get (Pokhrel et al., 2020),

$$R(z)=\alpha_0\left(1+\cos\frac{\pi z}{z_0}\right)e^{-\frac{t}{\tau}}\tau+A(z) \tag{12}$$

where $A(z)$ is integration constant. Using boundary conditions $R(z)=R_0$ for $t=0$ in (12) we get,

$A(z)=R_0-\alpha_0\tau\left(1+\cos\frac{\pi z}{z_0}\right)$ then (12) becomes,

$$R(z)=R_0-\alpha_0\tau\left(1-e^{-\frac{t}{\tau}}\right)\left(1+\cos\frac{\pi z}{z_0}\right). \tag{13}$$

For $t\to\infty$, $(R-R_0)_{z=0}\to\delta_m=2\tau\alpha_0$, where δ_m is the maximum depth on of the stenosis into the lumen. Hence, equation (13) becomes (Pokhrel et al., 2020),

$$\frac{R}{R_0} = 1 - \frac{\delta_m}{2R_0}\left(1 - e^{-\frac{t}{\tau}}\right)\left(1 + \cos\frac{\pi z}{z_0}\right). \tag{14}$$

Integrating (9) we get,

$$r\frac{\partial v}{\partial r} = -P(z)\frac{r^2}{2\mu} + B(z) \tag{15}$$

where $B(z)$ is integration constant. Applying boundary condition $\frac{\partial v}{\partial r} = 0$ for $r = 0$ in (15) gives $B(z) = 0$, then (15) becomes,

$$\frac{\partial v}{\partial r} = -P(z)\frac{r}{2\mu}. \tag{16}$$

Integrating (14) we get,

$$v(r) = -P(z)\frac{r^2}{4\mu} + C(z) \tag{17}$$

where $C(z)$ is integration constant. Applying boundary condition $v = 0$ for $r = R$ in (17) gives $C(z) = P(z)\frac{R^2}{4\mu}$, then (17) provides the velocity of the fluid as (Phaijoo, 2013),

$$v(r) = \frac{P(z)}{4\mu}\left(R^2 - r^2\right). \tag{18}$$

Equation (18) indicates that maximum velocity is attained along the axis and varies on the surface parallel flow of the artery.

8. Pressure Drop for Mild Stenosis

Now the flux through the cylindrical tube becomes (Maton et al., 1993),

$$Q = 2\pi\int_0^{R(z)} rv\,dr = \int_0^{R(z)} \frac{P(z)\pi}{2\mu}\left(R^2 r - r^3\right)dr = \frac{P(z)\pi}{8\mu}R^4(z). \tag{19}$$

Since $Q(z)$ is independent of z, and $P(z)$ is a function of z, then from (19), $P(z)$ is proportional to $\frac{1}{R^4(z)}$.

Therefore, pressure gradient varies inversely as the fourth power of the surface distance of the stenosis from the axis of artery. Hence, the pressure gradient is minimum at the middle of the stenosis and maximum at the ends (Kapur, 1985).

Force due to pressure is

$$F = P(z)A = 2\pi P(z)r\,dr. \tag{20}$$

Let $\tau(r)$ be the shear stress at a distance $R(z)$ from the axis and force due to inner cylinder is

$$F = 2\pi\frac{d}{dr}(\tau r)dr. \tag{21}$$

Comparing (20) and (21) we get,

$$P(z)r = \frac{d}{dr}(\tau r). \tag{22}$$

Integrating (22) we get,

$$\tau r = P(z)\frac{r^2}{2} + D(z). \tag{23}$$

where $D(z)$ is integrating constant. At $r = 0$ the shear stress is finite, (23) gives $D(z) = 0$ then (23) becomes,

$$\tau = \frac{1}{2}P(z)r . \tag{24}$$

From the fluid power law relating shear stress and shear rate we have,

$$\frac{dv(r)}{dr} = \left(\frac{\tau}{\mu}\right)^{\frac{1}{n}} = \left(\frac{P(z)r}{2\mu}\right)^{\frac{1}{n}}. \tag{25}$$

Integrating (25) from $r = R$ to r we get,

$$v(r) = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1+n}{n}} - r^{\frac{1+n}{n}}\right). \tag{26}$$

The total flux is given by (Pokhrel et al., 2020),

$$Q = \int_0^R 2\pi r v(r)dr = \frac{n\pi}{3n+1}\left(\frac{P(z)}{2\mu}\right)^{\frac{1}{n}} R^{\frac{1+3n}{n}}$$

$$P(z) = \left(\frac{3n+1}{n\pi}\right)^n \frac{2\mu Q^n}{R^{3n+1}} . \tag{27}$$

The pressure drop is developed due to the presence of mild stenosis, and across the stenosis area it has significant effect on the blood flow characteristics and is given by (Pralhad & Schultz, 2004),

$$\Delta P = \int_{-z_0}^{z_0} P(z)dz$$

$$= \int_{-z_0}^{z_0} \left(\frac{3n+1}{n\pi}\right)^n \frac{2\mu Q^n}{R^{3n+1}} dz . \tag{28}$$

Let $a = 1 - \frac{\delta}{2R_0}$, $b = \frac{\delta}{2R_0}$ and $\frac{\pi z}{z_0} = u$ then (14) becomes

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0} - \frac{\delta}{2R_0}\cos\left(\frac{\pi z}{z_0}\right) = a - b\cos u . \tag{29}$$

Hence (27) becomes (Pokhrel et al., 2020),

$$\Delta P = \frac{4\mu z_0 Q^n}{\pi R^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n \int_0^\pi \frac{du}{(a - b\cos u)^{3n+1}} . \tag{30}$$

When there is no stenosis, $\delta = 0$, and $f\left(\frac{\delta}{R_0}\right) = 1$, then pressure drop across the stenosis length is given by (Pokhrel et al., 2020),

$$(\Delta P)_P = \frac{4\mu z_0 Q^n}{\pi R^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n . \tag{31}$$

Dividing (30) by (31) we get the ratio of pressure drop across the stenosis as,

$$\frac{\Delta P}{(\Delta P)_P}=\frac{1}{\pi}\int_0^{\pi}\frac{du}{(a-b\cos u)^{3n+1}}. \tag{32}$$

For Newtonian fluid, we put $n=1$ and applying Libnitz's rule we get from (32) (Pokhrel et al., 2020),

$$\frac{\Delta P}{(\Delta P)_P}=\left(1-\frac{\delta}{2R_0}\right)\left(1-\frac{\delta}{R_0}-\frac{5\delta^2}{8R_0^2}\right)\left(1-\frac{\delta}{R_0}\right)^{-\frac{1}{2}}. \tag{33}$$

9. Optimum Stress during Mild Stenosis

Shear stress at wall $\tau = P(z)\frac{R(z)}{2}$ gives (Guyton & Hall, 2000),

$$\tau=\frac{R(z)}{2}\frac{2\mu Q^n}{R(z)^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n=\mu Q^n\left(\frac{3n+1}{n\pi}\right)^n\frac{1}{R(z)^{3n}}. \tag{34}$$

When there is no stenosis then, $\delta=0$, and $f\left(\frac{\delta}{R_0}\right)=1$, so the pressure drop across the stenosis length is given by,

$$\tau_P=Q^n\left(\frac{3n+1}{n\pi}\right)^n\frac{1}{R_0(z)^{3n}}. \tag{35}$$

The ratio of shear stress at the wall (Pokhrel et al., 2020),

$$\frac{\tau}{\tau_P}=\frac{1}{\left(1-\frac{\delta}{2R_0}-\frac{\delta}{2R_0}\cos\frac{\pi z}{z_0}\right)^{3n}}. \tag{36}$$

The ratio of the maximum stress to the minimum stress neglecting higher terms for $\cos\frac{\pi z}{z_0}=1$,

$$\frac{\tau_{\max}}{\tau_{\min}}=\left(1-\frac{\delta}{R_0}\right)^{-3n}=1+\frac{3\delta n}{R_0}. \tag{37}$$

10. Conclusions

In this study, we have tried to present the mathematical model for blood flow in a mildly stenosis artery that is assumed to be symmetric in size. We have assumed that the blood flow through the artery behaves like a Newtonian fluid. At present stenosis is one of the main causes of the death in the worldwide. We have observed that the effect of stenosis reduces and the flow rate high blood viscosity is very dangerous for the cardiovascular disorders. We have used Navier-Stokes equation for the determination of pressure drop in the artery for various radii of stenosis. Stenosis creates the abnormal blood flow through the arteries because of narrowing of an artery due to the accumulation of cholesterol, fats, and the abnormal growth of tissue. It develops cardiovascular diseases, such as heart attack and stroke. It is very important to understand the severity of stenosis that may lead to fatal situation if not diagnosed and measured accurately. Therefore, the cause and development of many arterial diseases leading to the malfunction of the cardiovascular system related to flow characteristics of the blood are necessary.

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Appendix I

Equation of Continuity

For continuous fluid motion conservation of mass and energy is necessary that obeys the Newton's second law of motion. Let us consider the velocities of fluid along x_1, x_2, x_3 directions are u_1, u_2, u_3, respectively; density is $\rho(t, x_1, x_2, x_3)$ and internal energy per unit mass is $e(t, x_1, x_2, x_3)$. The rate at which the fluid leaves the elementary volume is

$$\delta v = \left[\frac{\partial}{\partial x_1}(\rho u_1) + \frac{\partial}{\partial x_2}(\rho u_2) + \frac{\partial}{\partial x_3}(\rho u_3)\right]\delta x_1 \delta x_2 \delta x_3 . \tag{AI1}$$

The rate of decrease in mass inside the elementary volume is

$$-\frac{\partial \rho}{\partial t}\delta x_1 \delta x_2 \delta x_3 . \tag{AI2}$$

According to the conservation of mass,

$$-\frac{\partial \rho}{\partial t}\delta x_1 \delta x_2 \delta x_3 = \left[\frac{\partial}{\partial x_1}(\rho u_1) + \frac{\partial}{\partial x_2}(\rho u_2) + \frac{\partial}{\partial x_3}(\rho u_3)\right]\delta x_1 \delta x_2 \delta x_3$$

$$\left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i}\right)\rho + \rho \frac{\partial u_i}{\partial x_i} = 0 . \tag{AI3}$$

In three-dimensional case the equation of continuity is

$$\frac{D\rho}{Dt} + \rho \nabla . \mathbf{q} = 0 . \tag{AI4}$$

Appendix II

Navier-Stokes Equation (NSE)

The total linear momentum of the fluid is

$$M = \int_V \rho \mathbf{q} dV \tag{AII1}$$

and the total force acting on it is

$$\int_V \mathbf{F} \rho dV + \int_S \boldsymbol{\sigma} \mathbf{n} dS . \tag{AII2}$$

Newton's second law of motion indicates,

$$\frac{d}{dt}\int_V \rho\mathbf{q}dV = \int_V \mathbf{F}\rho dV + \int_S \boldsymbol{\sigma}\mathbf{n}dS \tag{AII3}$$

$$\frac{d\mathbf{q}}{dt} = \mathbf{F} + \frac{1}{\rho}\nabla.\boldsymbol{\sigma} \tag{AII4}$$

$$\frac{d\mathbf{q}}{dt} = \mathbf{F} + \frac{\mu}{\rho}\nabla^2\mathbf{q} + \frac{\mu}{3\rho}\nabla(\nabla.\mathbf{q}) - \frac{1}{\rho}\nabla p \tag{AII5}$$

$$\frac{\partial\mathbf{q}}{\partial t} + \nabla\times\left(\frac{1}{2}\nabla\times\mathbf{q}\right) - \mathbf{q}\times(\nabla\times\mathbf{q}) = \mathbf{F} + \frac{\mu}{\rho}\nabla^2\mathbf{q} + \frac{\mu}{3\rho}\nabla(\nabla.\mathbf{q}) - \frac{1}{\rho}\nabla p\ .$$

For incompressible viscous fluid NSE without body force for unsteady motion becomes,

$$\frac{\partial\mathbf{q}}{\partial t} - \mathbf{q}\times(\nabla\times\mathbf{q}) = -\nabla\left(\frac{p}{\rho} + \frac{1}{2}\mathbf{q}^2\right) + \frac{\mu}{\rho}\nabla^2\mathbf{q}. \tag{AII6}$$