

# The Riemann Hypothesis Applies Not Only to Prime Numbers

Peter Matveevich Mazurkin<sup>1</sup>

<sup>1</sup> Volga State University of Technology, Yoshkar-Ola, the Republic of Mari El, Russia

Correspondence: Peter Mazurkin, Volga State University of Technology, Yoshkar-Ola, the Republic of Mari El, Russia.

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## Abstract

The analysis of a series of natural numbers 0, 1, 2, 3, 4, ..., 1024 is given after expansion in the binary number system. Particular cases of this series are composite, even and odd numbers, and the latter are divided into prime numbers and composite odd numbers. Comparison of these series is carried out according to the general formula for the distribution of binary numbers for all 9 digits of the binary number system. It is shown that the shift of the oscillation of a binary number is different for varieties of natural numbers. It is proved that the real root of the Riemann zeta function  $1/2$  exists for any series of numbers obtained from the series of natural numbers. In the limit, with an increase in the power of the series, a sinusoid is subtracted from the real root (for series of odd, prime and odd composite numbers) and a cosine function (for series of natural, composite and integer numbers), the amplitude of which is also equal to  $1/2$ , and the half-period of the trigonometric function are two numbers: 1—for series of natural and composite natural numbers; 2—for series of odd natural, prime, composite odd and even numbers. Moreover, under the sine and cosine functions, the varieties of series of natural numbers are located on the Riemannian critical lines in the following way: a) on the zero vertical of the binary expansion of series of natural and composite numbers, the value  $\pi$ ; b) on the first vertical of the binary expansion of series of odd, prime, composite odd and integer natural numbers  $\pi/2$ .

**Keywords:** natural numbers, prime numbers, other varieties, binary number system, expansion of series, patterns, comparison, critical line, root  $1/2$

## 1. Introduction

In the nascent mathematical analysis of the 17th-18th centuries, there were two main approaches: a visual, non-rigorous *mechanical-geometric approach*; formal *algebraic*. From these two points of view, the concept of a function was also perceived (Wikipedia—the free encyclopedia). In the scientific discussion *about the string* that unfolded in the 18th century between the leading scientists of that time, the concept of a function was considered in detail.

Daniel Bernoulli entered into a dispute between L. Euler and d'Alembert, criticizing their decisions from the point of view of physics as extremely abstract. In his publications, he noted that these are wonderful mathematical results, but asked: "what does the sounding strings have to do with it?" Based on ideas about the nature of oscillations, he develops the idea of the important role of pure oscillations of a sinusoidal form, which appeared in Taylor. His guess was that an arbitrary vibration could be represented as a superposition or the sum of several pure vibrations (the principle of superposition), which corresponded to the observation of a string: the sound emitted by it consists of a fundamental tone and many overtones. Bernoulli found a solution to the oscillation equation as the sum of a trigonometric series and argued (again, based on physical considerations) that such a series can represent an arbitrary function. He could not confirm this assumption mathematically - in particular, he did not know the formula for calculating the coefficients of such a series. Nevertheless, he believed that his solution not only has a greater physical meaning than the solutions of d'Alembert and Euler but is also

more general (Wikipedia—the free encyclopedia).

Fourier's results answered one of the key questions in the dispute about the string - the representation of a wide class of functions by a trigonometric series. In the published article (Mazurkin P.M., 2014), we proved that any piece of a series of prime numbers can be represented as a sum of asymmetric wavelets.

The key point in the analysis of series of primes was the transition from the decimal number system to the binary number system (Mazurkin P.M., 2014), which allowed us to prove the Riemann hypothesis about the real root  $1/2$  (Mazurkin P.M., 2014).

## 2. Exploring a Series of Prime Numbers

Prior to the studies of L. Euler, since the time of Euclid, a series of prime numbers was studied only as a set of some illogically arranged numbers. From the beginning of the 17th century, mathematicians began to pay more attention to the distribution of prime numbers in a series of natural numbers in decimal places. Starting with Gauss, in the end, until now, the series of primes itself has not been studied. An analytical method for analyzing the distribution of prime numbers over decimal places, including the consideration of their distribution in the complex number system, has not been created.

For almost 400 years, the study of the distribution of primes among a series of natural numbers has come to the strongest pessimism, which is expressed by the judgments of famous mathematicians (cited by Constantine Adraktas *has proven on the b.pdf*):

- "Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that this is a mystery into which the mind will never penetrate" (cited in *The Prime Numbers have a definitive Orde.pdf*), L. Euler;

- In 1975, Don Zagier noted (Zagier D, n.d.): "There are two facts about the distribution of prime numbers that I hope to convince you overwhelmingly that they will be finally engraved in your hearts. First, despite their simple definition and role as building blocks of natural numbers, prime numbers grow like weeds among natural numbers, apparently obey no other law than chance, and no one can predict where the next one will grow. The second fact is even more striking because it says the opposite: the numbers show a startling pattern that there are laws governing their behavior and that they obey those laws with almost military precision" (quoted in *The Prime Numbers have a definitive Orde.pdf*);

- "At least a million years before we understand Primes" Paul Erdos;

- Terence Tao - "Our belief in the random nature of prime numbers and cryptography".

Prime numbers are not random, which is confirmed by the fundamental theorem of arithmetic. At the same time, the fundamental theorem of arithmetic states that every positive integer (except the number 1) can be represented in exactly one way, except by rearrangement, as a product of one or more prime numbers (Hardy & Wright, 1979, pp. 2-3).

## 3. Number Theory Before Gauss

Important theorems about integers were proved already in antiquity. So, for example, in the school of Pythagoras (5th century BC) such coprime squares of integers were found in an infinite number, the sum of which is also a square  $x^2+y^2=z^2$ . In the Elements of Euclid (III century BC) we find a proof of the infinity of the series of prime numbers. In the work of Diophantus (3rd century AD), methods for solving various indefinite equations in rational numbers are considered (Delaunay BN, n.d.).

The first who, in the modern times of the 17th century, began a deep study of integers was Pierre de Fermat (1601-1665) from Toulouse. The problems solved by Fermat, and especially the theorems stated by him, but the proofs of which he concealed in accordance with the custom of that time, made a great impression on subsequent mathematicians. Fermat had a special ability to penetrate the deepest mysteries of numbers.

Most of the problems Fermat left to succeeding generations of mathematicians were solved by Euler (1707-1783) and Lagrange (1736-1813), who also posed and solved many further problems of their own. Legendre (1752-1833) published the first major treatise on number theory, in which he collected everything that had been done before him and added many new things.

Such was the position of number theory before Gauss (Delaunay BN, n.d.).

The whole difficulty of number theory lies in the fact that the properties of integers with respect to multiplication (multiplicative properties) with their properties with respect to addition (with their additive properties) are connected very intricately.

According to A.N. Kolmogorov (Kolmogorov A.N., 1991, pp. 28), the time before the 6th-5th centuries was the *period of the birth of mathematics*, and from that moment on, the beginning of the *period of elementary*

*mathematics* is observed. “With the use of variables in the analytical geometry of the French scientist R. Descartes and the creation of differential and integral calculus, the *period of mathematics of variables begins*, which can also be conditionally called the period of higher mathematics. It is natural, however, that neither this nor the next period stopped the further development of elementary mathematics” (Kolmogorov A.N., 1991, pp. 29).

The development of elementary mathematics slows down (in Western Europe at the beginning of the 17th century) when the center of gravity of mathematical interests is transferred to the field of mathematics of variables (Kolmogorov A.N., 1991, pp. 33). For Vieta, the concept of trigonometric functions with an argument from 0 to  $+\infty$  appears only in the 16th century. But until now, a number of prime numbers, thanks to Gauss, begin with the number 2, although at the age of 15 he was presented with a table of prime numbers starting with the number 1. Thus, the binary number system 0 and 1 was completely cut off from a number of prime numbers. We restore the functional representation of the full series of prime numbers 0, 1, 2, 3, 5, 7, ... and thus it becomes possible to obtain trigonometric functions after decomposing prime numbers from the decimal number system in the binary number system. Then it turns out that the series of natural numbers 0, 1, 2, 3, 4, 5, ... also contains a binary number system at the beginning.

#### 4. A Number of Prime Numbers as a Distribution in Decimal Notation

L. Euler was the first to use (1737, 1748, 1749) the zeta function to study prime numbers, thereby laying the foundation for analytic number theory (Kolmogorov A.N., 1991, pp. 57).

L. Euler was the first mathematician who began to create general methods and apply other branches of mathematics, in particular mathematical analysis, to solving problems in the *analytic theory of prime numbers*. The zeta function was introduced in 1737 by L. Euler as a function of a real variable. Then this function was considered by Dirichlet and, especially successfully, by Chebyshev when studying the *law of distribution of prime numbers*. However, the most profound properties of the zeta function were discovered later, after the work of Riemann (1859), where the zeta function was considered as a function of a complex variable.

*Analytic number theory* has its origins in Euler. Then Dirichlet and Chebyshev continued to develop these ideas within the class of real numbers. New number theory was poured by Riemann, who entered the complex plane and began to use a new analytic continuation. The methods of analytic number theory were used in the theory of algebraic numbers by Dirichlet, Dedekind, Kronecker, Frobenius, Chebotarev, Hecke, Siegel, Heilbronn and others. An important methodical device for additive problems was invented by Voronoi. The same idea, although apparently independently of Voronoi, received a brilliant development in the works of Ramanujan, Hardy, and Littlewood. An important chapter in analytic number theory is the evaluation of trigonometric sums, the first far-reaching result in which was obtained by G. Weil in 1914. The most profound new ideas in this area were developed by Vinogradov, who, among other things, obtained estimates for trigonometric sums over prime numbers (Kolmogorov A.N., 1991, pp. 894-895).

The period that began in 1914 by the works of G. Weyl and I.M. Vinogradov and continued until 1934, can be called the second stage in the development of the theory of trigonometric sums in number theory.

According to I. M. Vinogradov, the characteristic features of this stage were two:

- 1) refusal to find the exact values of the sums and the transition to their estimates, but instead;
- 2) exemption from the requirement that the statements obtained with their help refer only to the full system of deductions. However, the condition was preserved that the summation was carried out over a segment of the numbers of the natural series, taken in a row (Kolmogorov A.N., 1991, pp. 904).

#### 5. Theory of Functions in the Field of Real Numbers

During the period of enthusiasm for the theory of functions of a complex variable, the largest representative of interest in specific issues of the theory of functions in the real region of numbers is P.L. Chebyshev (Chebyshev P.L., n.d.). The most striking expression of this trend was the theory of best approximations created (starting from 1854) by Chebyshev based on the requirements of the theory of mechanisms (Kolmogorov A.N., 1991, pp. 72). Then P.L. Chebyshev (1848, 1850) obtained the main results on the density of arrangement in the natural series of primes. Prior to this, Dirichlet proves (1837) a theorem on the existence of an infinite number of primes in arithmetic progressions (Kolmogorov A.N., 1991, pp. 73).

Of the many discoveries of Chebyshev, it is necessary to mention, first of all, the works on number theory. They began in the appendices to Chebyshev's doctoral dissertation *Theory of Comparisons*, published in 1849 (Chebyshev P.L., n.d.).

The number of primes not exceeding a given natural number  $x$  is denoted by the symbol  $\pi(x)$ . Of course, some values of this function  $\pi(x)$  can be determined exactly from the table of primes. So, for example, without taking into account the binary number system 0, 1 on the segment  $[1; 10]$   $\pi(10)=4$  (2; 3; 5; 7); on the interval  $[1; 100]$

density value  $\pi(100)=25$  primes, etc.

After Euclid (3rd century BC), who proved by graceful rigorous reasoning that there is no largest in a sequence of primes, it became clear that  $\pi(x)$  increases indefinitely as  $x$  increases; but by what law?

Century followed century, and only Chebyshev was the first to cut a window into the mysterious and seemingly impregnable area of the theory of the distribution of prime numbers. With great wit and depth of analysis, he proved that for sufficiently large values of  $x$ , the true value of  $\pi(x)$  can be found from the Chebyshev inequality (<http://math4school.ru/chebyshev.html>).

$$0.92x/\ln x < \pi(x) < 1.06x/\ln x$$

Studying the table of primes, you can see that with increasing  $x$ , the so-called “average density” of primes on the segment of natural numbers from 1 to  $x$  decreases. The first significant contribution to the solution of problems related to the distribution of prime numbers was made by P.L. Chebyshev. From his two famous memoirs of 1848 and 1852, the elementary methods of the theory of the distribution of prime numbers originate, i.e. methods that do not use the theory of functions of a complex variable. Note that in 1949 A. Selberg, supplementing Chebyshev’s ideas with his own fundamental considerations, found a proof of the asymptotic law of distribution of prime numbers, without using, like Hadamard and Vallée-Poussin, the theory of functions of a complex variable. This was a worldwide sensation in mathematics, as many famous mathematicians thought it was impossible! (Laptev V.N., Sergeev A.E. & Sergeev E.A., 2015).

Let us return to the ancient Greeks, who laid the foundations of the modern science of number. Eratosthenes was a contemporary and friend of Archimedes. Among his inventions is the so-called “*sieve of Eratosthenes*” - sifting numbers and allowing you to select prime ones from them. In fact, it was the world’s first algorithm - a set of rules, strictly following which you will certainly get the right result: placing a series of numbers in their natural (natural) sequence.

In modern literature on number theory, or higher arithmetic as it is sometimes called by professionals, the algorithm for finding prime numbers using the sieve of Eratosthenes is usually given at the beginning of the presentation of the material.

The development of the foundations of number theory was carried out by such luminaries of mathematics as Euler, Gauss, Legendre, Chebyshev and his student Zolotarev, as well as the well-known Fermat. Thus, Gauss once wrote about number theory: “Higher arithmetic offers us an inexhaustible supply of interesting truths - truths that do not stand in isolation but are connected by deep internal connections and between which, as our knowledge increases, we constantly discover new ones and sometimes completely unexpected connections” (Bayandin A.V., 1999).

Comparison of P.L. Chebyshev and Eratosthenes shows that since the time of Euclid, who studied the series of prime numbers itself and proved the infinity of the continuation of this series, there has been a separation of understanding of the object of study. Euclid and Eratosthenes considered directly the series of natural and prime numbers, and then the scientists could not continue direct research and switched to another path - this is the study of the density of prime numbers in some given series of natural numbers. As a result, the problem of the distribution of prime numbers, as you know, has again come to a standstill. Then it turns out that in higher arithmetic it is time to return to Euclid and Eratosthenes again, then carefully study the structure and functions of a series of prime numbers from within the series itself. It is necessary to refuse to study a series of prime numbers from the outside, and even more so to refuse the zeta function in complex numbers.

Next, consider the achievements of mathematics in the study of the distribution of prime numbers (different from the placement of prime numbers among natural numbers) in the complex domain.

## 6. Transition to the Distribution in the Complex Domain

In our opinion, the enthusiasm of mathematicians for the study of a number of prime numbers is indirect, through the distribution in the form of a zeta function from L. Euler, P.L. Chebyshev and, especially, later to B. Riemann in complex variables, greatly hampered the development of the elementary theory of prime numbers, and even more so of higher arithmetic.

In the 17th century, complex numbers were firmly established in mathematics, the contribution to the study of which was made by Abraham de Moivre (1667-1754) and Leonard Euler (1707-1783). Then, at the beginning of the 19th century, the French mathematician O. Cauchy (1821) created the foundations of the theory of functions of a complex variable. According to academician A.N. Kolmogorov (Kolmogorov A.N., 1991, pp. 60), great new mathematical theories also arise from the internal needs of mathematics itself. This was basically the development of the theory of functions of a complex variable, which at the beginning and in the middle of the 19th century occupied a central position in all mathematical analysis. The main line here was that the transition to the complex domain made the properties of the functions under study clearer and more visible.

Based on a clear understanding of the nature of complex numbers, the theory of functions of a complex variable arises. Gauss knew a lot in this area but published almost nothing. Only much later (complex numbers were first studied by Argan, 1806), in 1831, Gauss explicitly expounded the theory of complex numbers. The nature of the theory of functions of a complex variable was further enhanced in the middle of the 19th century by the German mathematician B. Riemann. The natural geometric carrier of an analytic function in the case of its multivaluedness is not the plane of a complex variable, but the corresponding Riemann surface. B. Riemann’s geometric ideas turn out to be more and more defining the whole style of thinking in the field of the theory of functions of a complex variable (Kolmogorov A.N., 1991, pp. 71-72).

As a consequence, the so-called special series of primes, as well as many additional hypotheses (open problems of number theory) about primes, have arisen in special cases of distribution density.

**7. Riemann Zeta Function**

Thus, B. Riemann in 1859 published a zeta function with respect to a complex variable, and his approach to revealing the law of distribution of prime numbers in the decimal number system is now considered a classical approach (Chubarikov V.N., 2011). As a result, all special cases cannot cover the ideas of the Riemann zeta function, as she herself did with the elementary theory of prime numbers by P.L. Chebyshev. From the entire theory of B. Riemann, we single out only the postulate of a real root 1/2.

The Riemann hypothesis of the distribution of zeros of the zeta function states: “All non-trivial zeros of the zeta function have a real part equal to 1/2.” Riemann discovered that the number of primes not greater than x, the distribution function of primes, denoted π(x), is expressed in terms of the distribution of the so-called non-trivial zeros of the zeta function. Little is known about the values of the zeta function in odd integers. The numbers -2, -4, -6, ... form the trivial zeros of the Riemann zeta function, and this function has no other real zeros. All other zeros, called *nontrivial*, are located in the strip 0<Re(s)<1 symmetrically with respect to the so-called “critical line” 1/2+it, t∈R. B. Riemann’s discovery in 1859 that the complex zeros of the zeta function determine the law of distribution of prime numbers made an era in the theory of prime numbers. Riemann found an explicit formula relating the function π(x) to the sum over zeros of the zeta function (Sharafeev I.G., 2015).

The proof of the Riemann hypothesis on the rational root 1/2 was given by us in (Mazurkin P.M., 2014; Mazurkin P.M., 2012; Mazurkin P.M., 2015).

**8. Numerical Systems**

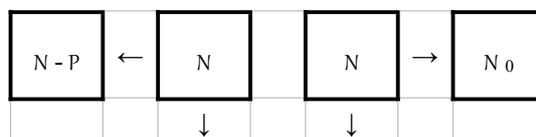
Among prime numbers, there are pairs of such, the difference between which is equal to two (simple twins), but the finiteness or infinity of such pairs has not been proven. In other articles, we will show that the structure of a series of prime numbers is much more complicated.

Euclid considered it obvious that by multiplying only prime numbers, one can obtain all natural numbers, and each natural number can be represented as a product of prime numbers in a unique way (up to the order of factors). Thus, the prime numbers form a multiplicative basis of the natural series. The first problems about prime numbers were: how often they are located in natural numbers and how far apart they are from each other. The study of the placement of primes led to the creation of an algorithm (rule) that allows one to obtain tables of primes. Such an algorithm is the sieve of Eratosthenes (3rd century BC). Euclid in the Elements indicated a method for finding the greatest common divisor of two numbers (the Euclidean algorithm), the consequence of which is the theorem on the unique decomposition of natural numbers into prime factors (Karatsuba A.A., n.d.).

In elementary number theory, for known sets of numbers (without complex numbers), the expression is true in which the following numerical systems are given.

$$P \subset N \subset Z \subset Q \subset R \tag{1}$$

Prime numbers P (Figure. 1) are natural numbers that are divisible only (dividing the number itself into itself is excluded) by one. In this case, the complete series of primes has the form P = {0, 1, 2, 3, 5, 7, 11, 13, 17, ...}. When divided by 1, we get the series P/1 = {0/1, 1/1, 2/1, 3/1, 5/1, 7/1, 11/1, 13/1, 17/1, ...}, equal to itself, and when dividing by itself, we get a series of units P/P = {0/0, 1, 1, 1, 1, 1, 1, 1, 1, ...}. The ratio 0/0 becomes an indeterminacy, and a series of units reduces prime numbers to the number 1. Then the series of prime numbers 2, 3, 5, 7, 11, 13, 17, ..., proposed by the great Gauss, when divided by itself, reduces to the number 1.



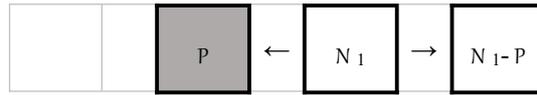


Figure 1. Graph of connections between numerical systems

The first ten of the decimal number system contains six prime and four composite numbers. This group **0, 1, 2, 3, 4, 5, 6, 7, 8, 9** of ten natural numbers is asymmetric. At the beginning there are prime numbers  $4 + 1 + 1$ , and from the end to the beginning - composite numbers according to the arrangement scheme  $2 + 1 + 1$ . The ratio of prime numbers to the first ten natural numbers is 60%, which is very close to the golden ratio of 0.618.

Then the binary number system will be the beginning of the complete series of primes in the form of the set  $P_2 = \{0;1\}$ , and the Gaussian series of primes  $P_G = \{2, 3, 5, 7, 11, 13, 17, \dots\}$  will be its continuation. As a result, we get  $P = P_2 + P_G$ . It is quite possible to display the continuation of  $P_G$  through the binary number system  $P_2$ . moreover, any series according to (1) can also be written in binary codes.

Why 0 and 1 are not accepted in number theory by Gauss, we do not understand (it seems that the series of prime numbers is adjusted to the desired result of the density of prime numbers in the decimal number system). In practice, the OEIS library knows special series of prime numbers, for example, the Mersenne numbers  $2^N - 1$  (URL: <http://oeis.org/A000225>), which start with the digit 0, and some start with the digit 1. Therefore, 0 and 1 - These are also prime numbers.

The natural numbers obtained by natural counting are denoted by N. We accept the set of natural numbers with zero, that is,  $N = \{0, 1, 2, 3, \dots\}$ .

In Figure 1, for a series of primes, the following additions are formed (primes are in bold):

$N-P$  - composite natural numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, ...

$N_0$  - are even natural numbers **0, 2, 4, 6, 8, 10, 12, 14, 16, ...**

$N_1$  - odd natural numbers **1, 3, 5, 7, 9, 11, 13, 15, 17, ...**

$N_{1-P}$  - are composite odd natural numbers 9, 15, 21, 25, ....

The complete series of prime numbers will contain the first two numbers 0 and 2 from the series of even natural numbers, and the remaining prime numbers are placed in the series of odd numbers.

Integers are obtained by combining natural numbers with a set of negative numbers and zero, denoted by  $Z = \{\dots-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, \dots\}$ . With negative numbers, we obtain a series of integer primes  $P_z = \{\dots -7, -5, -3, -2, -1, 0, 1, 2, 3, 5, 7, \dots\}$ , the results of which were published in (Mazurkin P.M., 2014; Mazurkin P.M., 2015).

Rational numbers are numbers represented as a fraction  $Q = Z / N$  ( $N \neq 0$ ). For rational numbers, all four classical arithmetic operations are defined: addition, subtraction, multiplication, and division (except division by zero).

For example, from the existing definition of a prime number (divided by 1 and by itself), we obtain *rational prime numbers*  $P_Q = \{\dots 1/11, 1/7, 1/5, 1/3, 1/2, 1/1, 0/1, 1/1, 2/1, 3/1, 5/1, 7/1, 11/1, \dots\}$ .

Real numbers (real numbers) are an extension of the set of rational numbers, closed with respect to some (important for mathematical analysis) operations of passage to the limit. The set of real numbers is denoted by R. We will calculate the binary number z of the decomposition of prime and other types of numbers in the decimal notation as a real number. The error of this decomposition depends on the representation of the number  $\pi = 3.14159\dots$ , which is calculated in our computer with 18 decimal places.

After binary decomposition of any series of numbers by codes 0 or 1 in each vertical, the resulting series of binary numbers is taken as a separate statistical sample, depending on the type of distribution of numbers (natural, simple, etc.). Next, we identify some initial statistical regularity using these statistical samples. In this case, the measurement error in these samples is zero. Then we select the vertical of the binary expansion along which the highest correlation coefficient is observed, equal to 1.0000.

### 9. Prime Number Placement Methods

As you know, many mathematical functions work only in the first positive trigonometric quadrant. The complex plane can consider functions in all four trigonometric quadrants. However, for a series of primes 0, 1, 2, 3, 5, 7, ... with positive numbers, it is sufficient to consider only the positive semi-axis of the abscissa. Therefore, the Riemann zeta function in complex variables becomes functionally redundant.

For a long time, it was not possible to obtain proofs of the law of distribution of prime numbers without applying the methods of the theory of functions of a complex variable, the first proofs (J. Hadamard, Ch. &

Vallee-Poussin) used these methods. Many mathematicians (A.E. Ingham, G. Hardy, & others) even believed that an elementary proof could not exist. The first completely elementary proof was obtained in 1946 by A. Selberg and P. Erdős (Zenkin V.I., 2008, pp. 78).

We have considered the so-called “increments of prime numbers” (Mazurkin P.M., 2014) and the relationship between physical laws and the regularity of distribution in a series of prime numbers (Mazurkin P.M., 2014).

**10. Prime Numbers in the Binary System**

We take the prime numbers according to (Prime numbers) as a complete set from a series of integers (Mazurkin P.M., 2012; Mazurkin P.M., 2015). In the article, we will consider only positive prime numbers, including 0 and 1, as well as positive composite numbers 4, 6, 8, 9, 10, ... up to the number 1024 (Table 1) and other sets according to the scheme in Figure 1.

Table 1. An example of decomposition in binary codes of series of natural and prime numbers

Binary system		Natural numbers		Prime numbers		Actual binary numbers $z_f$ of the binary system $i_2(2^i)$										
$i_2$	$2^i$	$i_{10}$	$N$	$j$	$P_j$	0	1	2	3	4	5	6	7	8	9	
						1	2	4	8	16	32	64	128	256	512	
$-\infty$	0	$-\infty$	<b>0</b>	<b>0</b>	<b>0</b>	0	0	0	0	0	0	0	0	0	0	
0	1	0	<b>1</b>	<b>1</b>	<b>1</b>	1	0	0	0	0	0	0	0	0	0	
1	2		<b>2</b>	<b>2</b>	<b>2</b>	0	1	0	0	<i>Trivial zeros</i>				0	0	
			<b>3</b>	<b>3</b>	<b>3</b>	1	1	0	0					0	0	
2	4		4			0	0	1	0	0	0	0	0	0	0	
			<b>5</b>	<b>4</b>	<b>5</b>	1	0	1	0	0	0	0	0	0	0	
			6			0	1	1	0	0	0	0	0	0	0	
			<b>7</b>	<b>5</b>	<b>7</b>	1	1	1	0	0	0	0	0	0	0	
3	8		8			0	0	0	1	0	0	0	0	0	0	
			9			1	0	0	1	0	0	0	0	0	0	
		1	10			0	1	0	1	0	0	0	0	0	0	
			<b>11</b>	<b>6</b>	<b>11</b>	1	1	0	1	0	0	0	0	0	0	
			12			0	0	1	1	0	0	0	0	0	0	
			<b>13</b>	<b>7</b>	<b>13</b>	1	0	1	1	0	0	0	0	0	0	
			14			0	1	1	1	0	0	0	0	0	0	
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
		2	100			0	0	1	0	0	1	1	0	0	0	
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
			1018			0	1	0	1	1	1	1	1	1	1	
			<b>1019</b>	<b>172</b>	<b>1019</b>	1	1	0	1	1	1	1	1	1	1	
			1020			0	0	1	1	1	1	1	1	1	1	
			<b>1021</b>	<b>173</b>	<b>1021</b>	1	0	1	1	1	1	1	1	1	1	
			1022			0	1	1	1	1	1	1	1	1	1	
			1023			1	1	1	1	1	1	1	1	1	1	
10	1024		1024			0	0	0	0	0	0	0	0	0	0	
								Non-trivial zeros								

For an example of statistical modeling (Mazurkin P.M., 2016; Mazurkin P.M., 2015), a set of 1024 natural

numbers are adopted, which has approximately  $10^3$  decimal numbers,  $2^{10}$  numbers in the binary system, and 36 groups of increments in a series of primes starting with an increment of 2 (Mazurkin P.M., 2014). It can be seen from the data in Table 1 that when the positions of the binary expansion are arranged from left to right (for clarity), trivial zeros are located behind the right ones. In this case, non-trivial zeros are located from column  $i=0$  to one on the right side of the matrix.

Thus, the binary decomposition of decimal numbers makes it possible to increase the visibility of the distribution of prime and other numbers.

In Table 1, the symbol  $z_f$  denotes the actual codes 0 and 1 in the binary number system, and then the symbol  $z$  – denotes the real values of the binary number obtained by identifying statistical patterns according to stable laws (Mazurkin P.M., 2016; Mazurkin P.M., 2015).

Then in the binary system, the series of numbers according to the scheme in Figure 1 are determined by the expression

$$P_j \vee N_N = \sum_{i=0}^m z_{fi} 2^i \tag{2}$$

The parameter  $z_f$  of formula (2) forms the so-called incidence table. In other words, codes 0 or 1 of the binary system turn into logical yes or no.

**11. Distributions of Binary Numbers over Binary Digits**

In table 1 we have  $0 \leq i \leq m=9$  and in the book (Mazurkin P.M., 2012) it was proved that any private series of numbers along the vertical  $i$ , composed of some numbers from the binary system, is characterized approximately by the equation

$$z = 1/2 - \mathcal{A} \cos(\pi x / p - a) \tag{3}$$

where  $z$  – is the calculated value of a binary real number according to the formula (3);

$1/2$  – is the real root of the Riemann zeta function;

$\mathcal{A}$  – is the amplitude (half) of the oscillation of a binary real number relative to the line of the real root  $1/2$  of the Riemann zeta function;

$x$  – the value of a number in a certain series of prime, natural or other types of numbers, distributed according to the scheme in Figure 1 to all types of partial series of natural numbers;

$p$  – is the half-period of oscillation of a binary number, which, like half the amplitude, can have a formula with a complex structure, for example, in the form of an asymmetric wavelet signal (Mazurkin P.M., 2014);

$a$  – the shift of a wave of an oscillatory perturbation of a binary real number, rad.

The cosine function is symmetric with respect to the positive semi-axis of the abscissa, so it is preferable compared to the sine for the process of identifying patterns from statistical data. At the same time, the full series of prime numbers  $P = 0, 1, 2, 3, 5, 7, \dots$ , as well as other series according to the scheme in Figure 1, as ideally reliable statistical samples, are mathematical objects that do not have measurement errors.

We tighten the requirement to prove the rational root  $1/2$  for any series of natural numbers, including primes, by assuming the conditions  $\mathcal{A} = 1/2$  and  $p = 2^i$ .

Then equation (3) is reduced to the form

$$z_i = 1/2 - 1/2 \cos(\pi x / 2^i - a_i) \tag{4}$$

Next, we identify equation (4) according to Table 1 for all series from the diagram in Figure 1 in the CurveExpert-1.40 software environment (URL: <http://www.curveexpert.net/>).

**12. Natural Numbers from 0 to 1024**

For the vertical  $i=0$ , the formula was obtained (Figure. 2)

$$z_0 = 1/2 - 1/2 \cos(\pi N + 0.0043088) \tag{5}$$

Here, the half-period of oscillation is  $2^{i=0} = 1$ . The wave shift is very small, and therefore it can be completely neglected in what follows. This means that natural numbers on the zero vertical are distributed with

half-period 1 (period 2). The remaining verticals of the binary decomposition receive less adequacy. Therefore, from the nine verticals of the binary expansion of 1024 natural numbers, the zero vertical becomes the most accurate according to the formula (5).

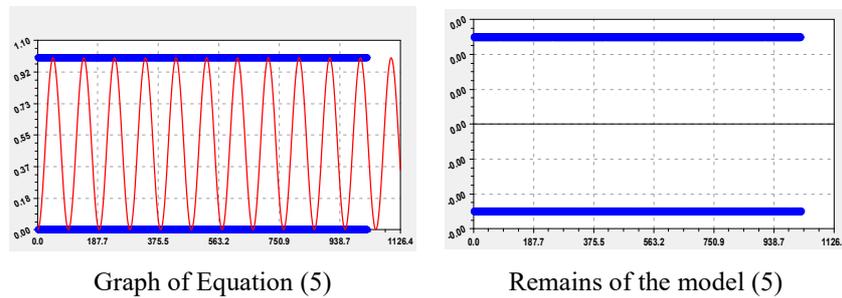


Figure 2. Graphs of the change of a binary number on the zero vertical of natural numbers in table 1.

The remainders after formula (5) are the same and equal to  $\pm 4.64148e-6$ . The residuals in the second graph of Figure 2 show that they form two horizontal lines.

On the first vertical expansion (Figure. 3), the formula ( $0.78514 \approx \pi / 4$ )

$$z_1 = 1/2 - 1/2\cos(\pi N/2 - 0.78514) \tag{6}$$

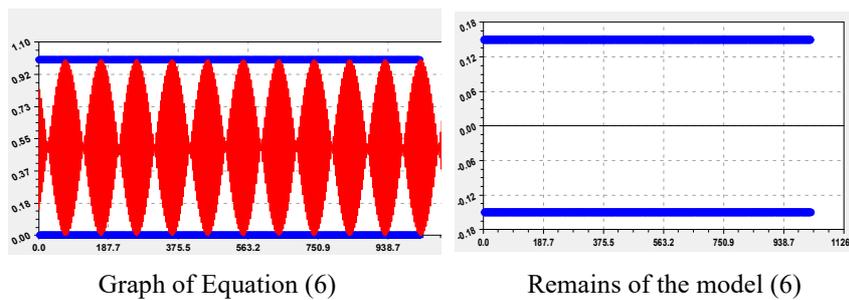


Figure 3. Graphs of a binary number on the first vertical of the decomposition of natural numbers.

The remainders of formula (6) are equal to  $\pm 0.146356$ . This is 14.64% of the number 1. Therefore, to improve the accuracy of modeling, it is possible to further identify the next component of the statistical model. Formula (5) is suitable for choosing by the minimum of residuals.

On the second vertical of the binary decomposition (Figure. 4), the formula was obtained

$$z_2 = 1/2 - 1/2 \cos(\pi N / 4 - 1.17725) \tag{7}$$

We notice that the shift of the wave increases: at  $i = 0$  from was close to zero, on the first vertical of the binary expansion  $i=1$  it became equal to 0.78514. This shift value is close to  $45^\circ$ , or to  $\pi/4 = 0.785397$ . On the second vertical, the shift of the wave of oscillatory perturbation of a binary real number became equal to 1.17725. This value can also be used to determine the angle by which the beginning of the perturbation wave rotates.

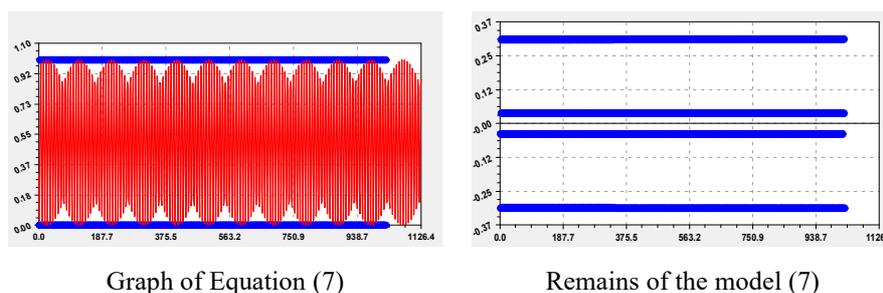


Figure 4. Graphs of a binary number on the second vertical of the expansion

As can be seen from the graphs in Figure 4, the residuals from formula (7) form two lines on the sides of the x-axis. In this case, the maximum deviation is  $\pm 0.3097$  or 30.97%. It can be concluded that with an increase in the bit of the binary number system, the remainders increase.

On the third vertical (Figure. 5), after identifying the model (4), the formula was obtained

$$z_3 = 1/2 - 1/2 \cos(\pi N / 8 - 1.37324) \tag{8}$$

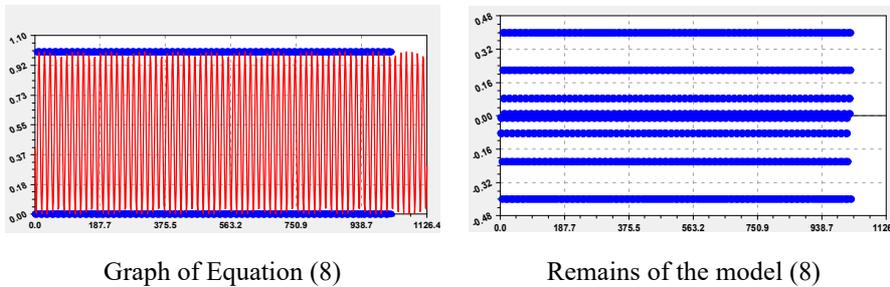


Figure 5. Graphs of a binary number on the third vertical of the decomposition

Of the four residual lines on both sides of the abscissa, the maximum value is  $\pm 0.4019$  or 40.19%.

On the fourth vertical (Figure 6) there is a further increase in the wave shear according to the expression

$$z_4 = 1/2 - 1/2 \cos(\pi N / 16 - 1.47124) \tag{9}$$

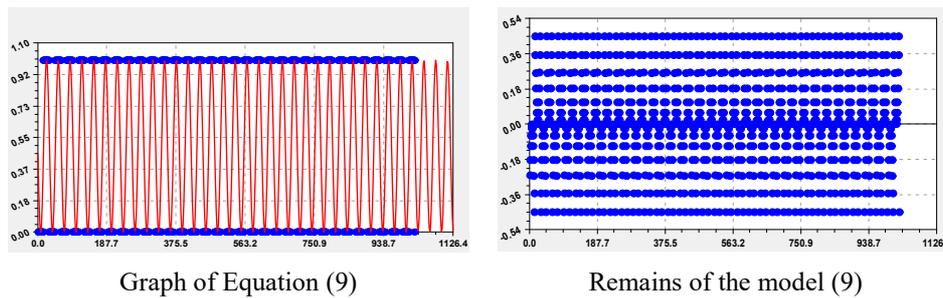


Figure 6. Graphs of a binary number on the fourth vertical of the decomposition of natural numbers

Of the eight lines on the sides of the abscissa axis, the maximum absolute error of formula (9) turned out to be  $\pm 0.4517$  or a relative error of 45.17%.

On the fifth vertical (Figure. 7) according to the formula

$$z_5 = 1/2 - 1/2 \cos(\pi N / 32 - 1.52028) \tag{10}$$

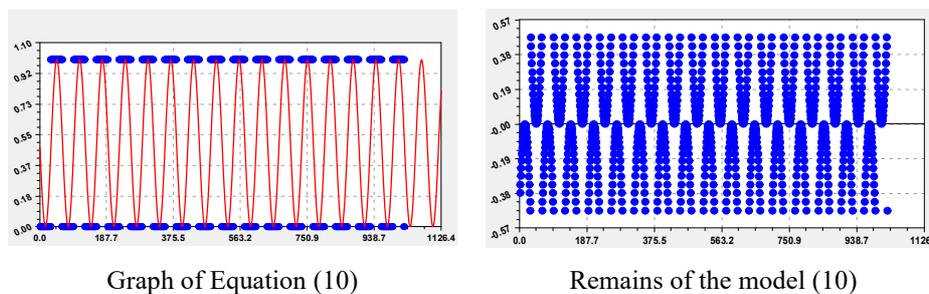


Figure 7. Graphs of a binary number on the fifth vertical of the decomposition of natural numbers

In this case, the remainders of formula (10) change from almost zero to 0.4831, which is 48.31% of the limiting value of the binary number 1.

On the sixth vertical (Figure. 8) of the binary expansion, the expression

$$z_6 = 1/2 - 1/2 \cos(\pi N / 64 - 1.54477) \tag{11}$$

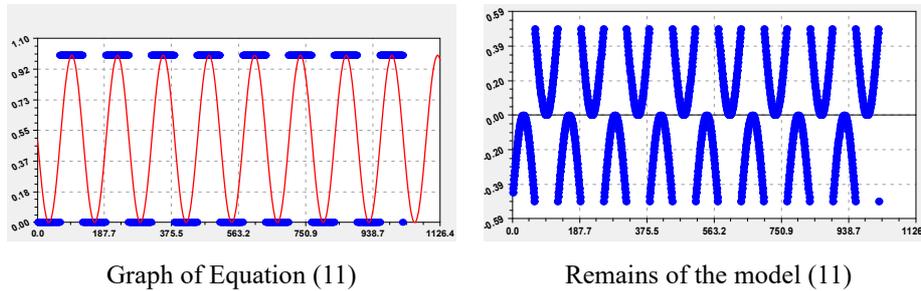


Figure 8. Graphs of a binary number on the sixth vertical of the expansion of natural numbers

Remains after (11) formed V-shaped figures with a maximum of 0.4885. It is noticeable that with an increase in the bit of the binary number system, the residuals from model (4) approach 0.5.

On the seventh vertical (Figure. 9), model (4) was identified and the formula

$$z_7 = 1/2 - 1/2 \cos(\pi N / 128 - 1.55699) \tag{12}$$

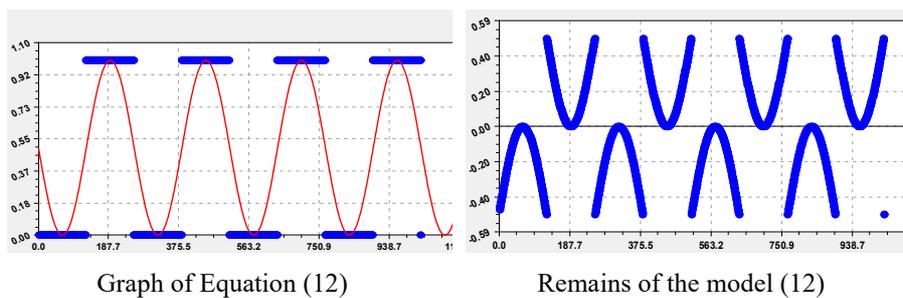


Figure 9. Graphs of a binary number on the seventh vertical of the decomposition of natural numbers

The maximum balances are 0.4946 (49.46%).

On the eighth vertical (Figure. 10) we get the expression

$$z_8 = 1/2 - 1/2 \cos(\pi N / 256 - 1.56313) \tag{13}$$

The residuals after formula (13) with a maximum of 0.4977 formed four pictures.

On the last ninth vertical (Figure. 11) we get the formula

$$z_9 = 1/2 - 1/2 \cos(\pi N / 512 - 1.56622) \tag{14}$$

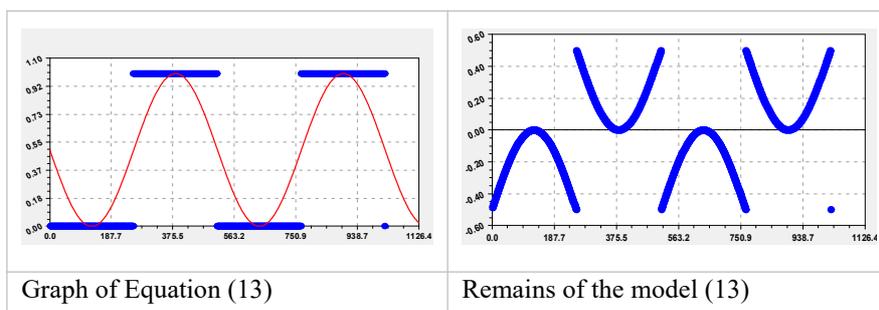


Figure 10. Graphs of a binary number on the eighth vertical of the expansion of natural numbers

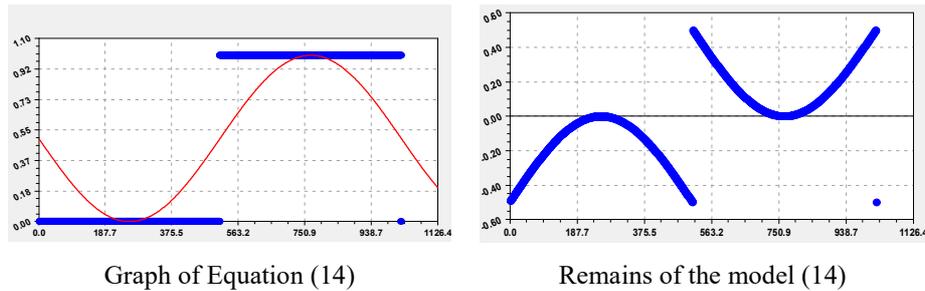


Figure 11. Graphs of a binary number on the ninth vertical of the decomposition of natural numbers

The residuals in the form of two V-shaped figures have a maximum value of 0.4992, which is already close to the limit of 0.5. At the same time, the wave shifts 1.56622 tends to  $\pi/2=1.570795$ . This fact means that with the growth of a natural number, when decomposing it in the binary number system, the cosine function (on the zero-decomposition vertical) changes to the sine function (on  $m \rightarrow \infty$  of the binary decomposition vertical).

**13. Odd Natural Numbers**

On the zero vertical row 1, 3, 5, 7, 9, ... we get  $z = 1$ .

On the first vertical (Figure. 12) we get the expression

$$z_1 = 1/2 - 1/2 \cos(\pi N_1 / 2 - 1.56790) \tag{15}$$

The remainders after formula (15) are the same and equal to  $\pm 2.10402e-6$ . This is almost two times less than the remainders on the first vertical of the binary decomposition of natural numbers. The wave shift approaches  $\pi/2 = 1.570795$ , and this allows us to replace the cosine with a sine in formula (15).

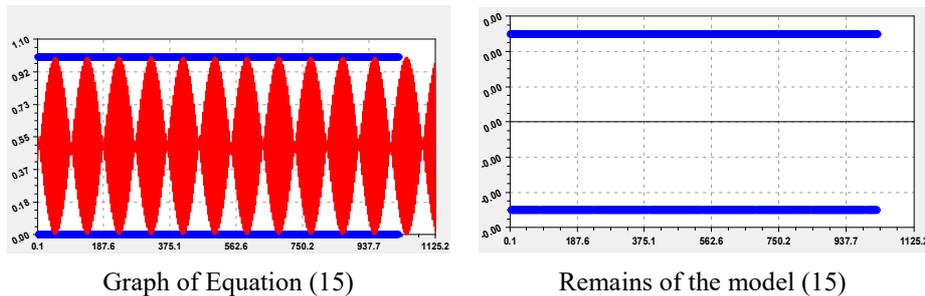


Figure 12. The first vertical of the decomposition of odd natural numbers

The second vertical (Figure 13) is characterized by the expression

$$z_2 = 1/2 - 1/2 \cos(\pi N_1 / 4 - 1.57082) \tag{16}$$

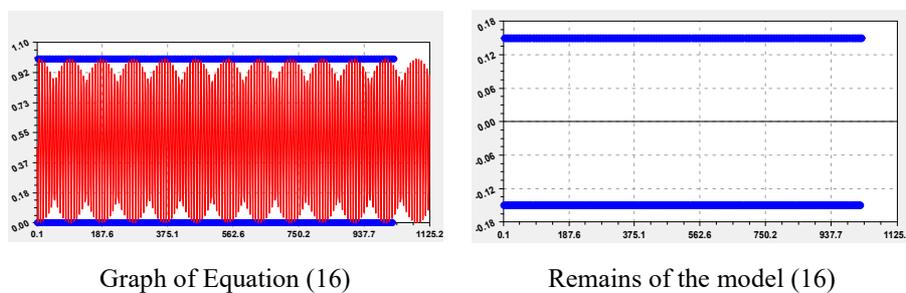
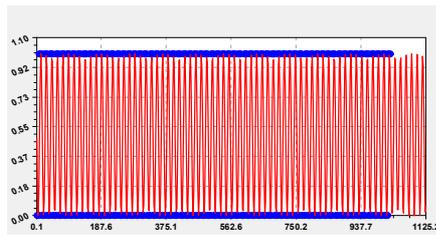


Figure 13. Second vertical expansion of odd natural numbers.

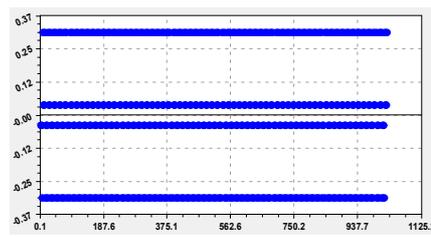
The residuals after formula (16) are the same and equal to  $\pm 0.1464$ .

On the third vertical (Figure 14) we get the formula

$$z_3 = 1/2 - 1/2 \cos(\pi N_1 / 8 - 1.57083) \tag{17}$$



Graph of Equation (17)



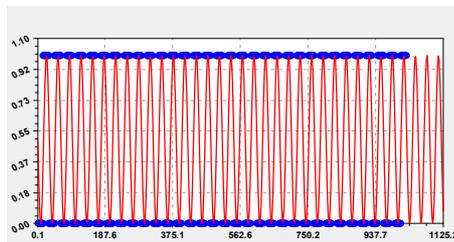
Remains of the model (17)

Figure 14. Third vertical expansion of odd natural numbers

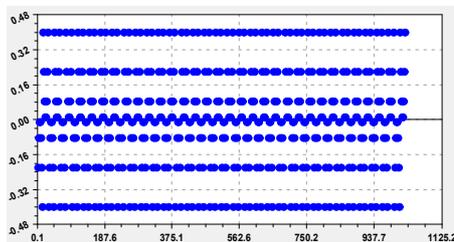
The maximum residuals after formula (18) are  $\pm 0.3087$ .

On the fourth vertical (Figure. 15) with an oscillation half-period of 16,

$$z_4 = 1/2 - 1/2 \cos(\pi N_1 / 16 - 1.57082) \tag{18}$$



Graph of Equation (18)



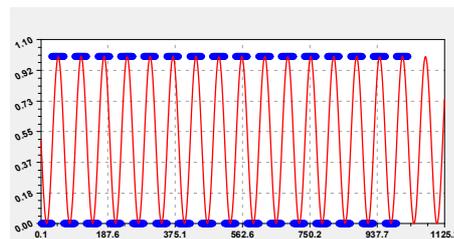
Remains of the model (18)

Figure 15. Fourth vertical expansion of odd natural numbers

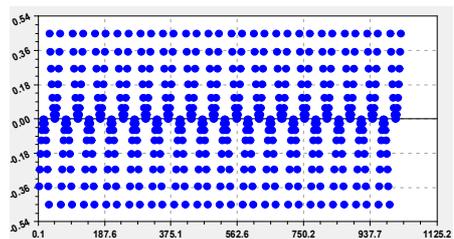
The maximum residuals after formula (18) are  $\pm 0.4025$ .

On the fifth vertical (Figure. 16) we get the expression

$$z_5 = 1/2 - 1/2 \cos(\pi N_1 / 32 - 1.57081) \tag{19}$$



Graph of Equation (19)



Remains of the model (19)

Figure 16. Fifth vertical expansion of odd natural numbers

The maximum residuals after formula (19) are  $\pm 0.4510$ .

The sixth vertical (Figure. 17) is determined by the equation

$$z_6 = 1/2 - 1/2 \cos(\pi N_1 / 64 - 1.57081) \tag{20}$$



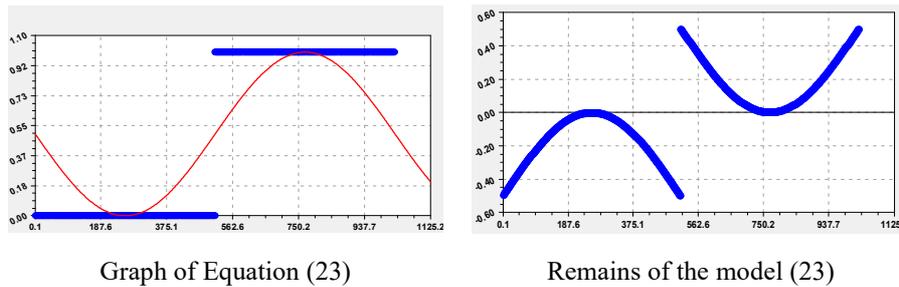


Figure 20. Ninth vertical expansion of odd natural numbers

The maximum residuals after formula (23) are  $\pm 0.4969$ . At the same time, it turned out that all expansions of odd numbers in the binary number system obey a sinusoid.

**14. Prime Numbers**

They refer to odd natural numbers, so on the zero vertical we get  $z = 1$ . On the first vertical, a regularity is revealed, which is determined by a formula

$$z_1 = 1/2 - 1/2 \cos(\pi P / 2 - 1.56539) \tag{24}$$

The remainders after formula (24) are the same and equal to  $\pm 7.29945e-6$ . Compared to a series of odd numbers, the residuals have almost quadrupled.

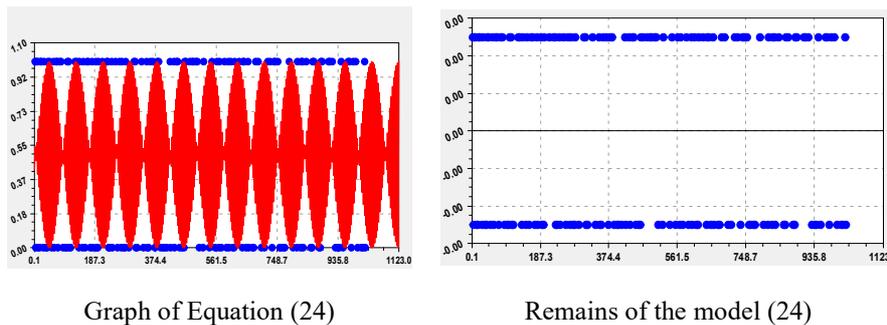


Figure 21. Graphs of a binary number on the first vertical of the decomposition of prime numbers

The wave shift is equal to 1.56539 and is approximately equal to  $\pi/2 = 1.570795$ , and this allows us to replace the cosine function with the sine function in formula (25) and subsequent regularities. On the first graph of Figure 21 (similarly to others) in the upper right corner are given: S - standard deviation of the residues of formula (24) 0.00000732; r is the correlation coefficient (instead of 1.00000000, it is sufficient to write the measure of the adequacy of formula (24) as 1.0000.

On the second vertical (Figure. 22) with almost the same shift, the formula

$$z_2 = 1/2 - 1/2 \cos(\pi P / 4 - 1.57765) \tag{25}$$

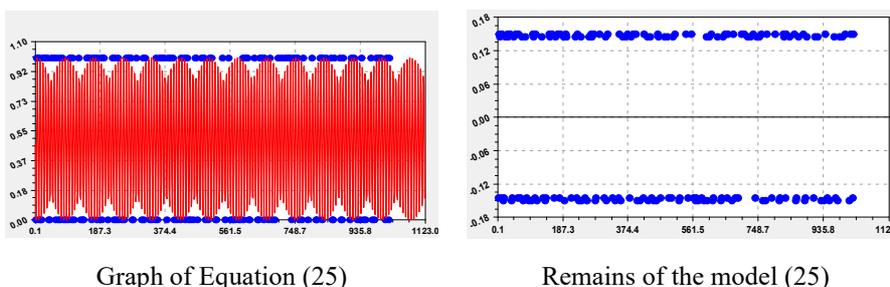


Figure 22. Graphs of a binary number on the second vertical of the decomposition of prime numbers

Here the residuals are  $\pm 0.144033$ . The standard deviation of the residuals is 0.14685 and the correlation

coefficient has decreased to 0.9561.

On the third vertical of the decomposition of prime numbers (Figure. 23), the formula

$$z_3 = 1/2 - 1/2 \cos(\pi P / 8 - 1.59173) \tag{26}$$

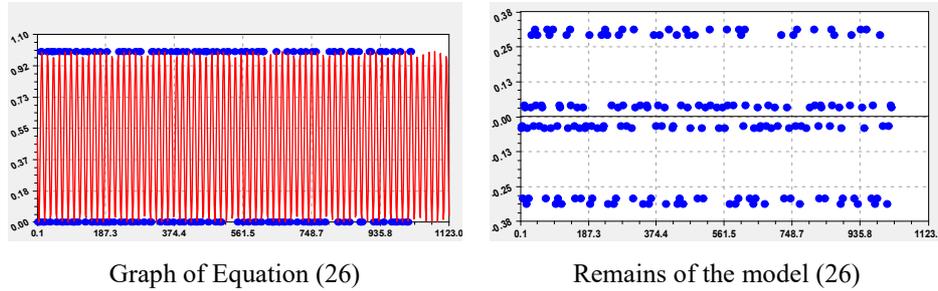


Figure 23. Graphs of a binary number on the third vertical of the expansion of prime numbers

Here, the remainders of formula (26) are equal to  $\pm 0.3484$ , which amounts to a relative error of 34.84% with respect to limit 1. The standard deviation of the residuals became equal to 0.21535, and the correlation coefficient decreased to 0.9029.

On the fourth vertical (Figure. 24), the binary expansion is determined by the equation

$$z_4 = 1/2 - 1/2 \cos(\pi P / 16 - 1.55614) \tag{27}$$

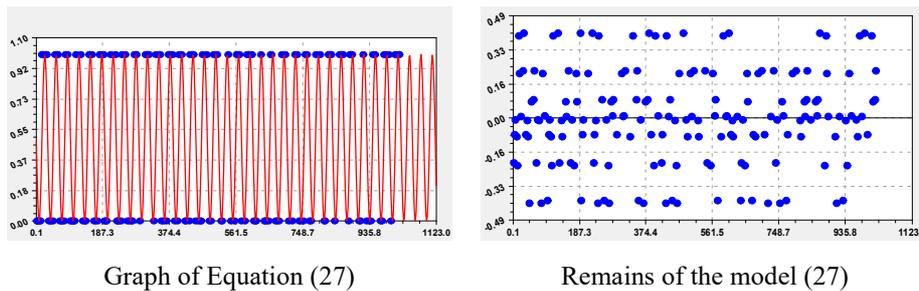


Figure 24. Graphs of a binary number on the fourth vertical of the decomposition of prime numbers

For formula (27), the maximum residuals are  $\pm 0.4097$ , which is up to 40.97% relative to unity. The standard deviation is 0.22911 and the correlation coefficient has become 0.8895. The plots of the residuals in Figure 24 have become more chaotic, and at the same time, chaos increases as the digit of the binary number system increases.

On the fifth vertical (Figure. 25), the binary expansion is determined by the expression

$$z_5 = 1/2 - 1/2 \cos(\pi P / 32 - 1.55210) \tag{28}$$

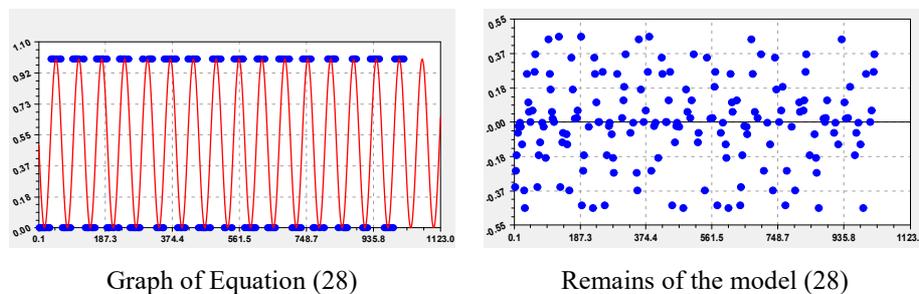


Figure 25. Graphs of a binary number on the fifth vertical of the expansion of prime numbers

For formula (28), the maximum residuals increased to  $\pm 0.4432$ , which is 44.32% relative to unity. The standard deviation of the residuals on the fifth vertical is 0.23415 and the correlation coefficient has become 0.8843. In

comparison with the previous category, the adequacy has decreased slightly.

On the sixth vertical (Figure. 26), after identifying the model (4), the equation was obtained

$$z_6 = 1/2 - 1/2 \cos(\pi P / 64 - 1.55977) \tag{29}$$

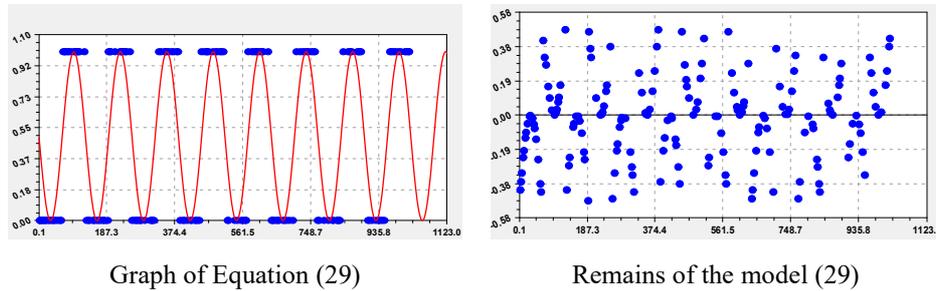


Figure 26. Graphs of a binary number on the sixth vertical of the expansion of prime numbers

For formula (29), the maximum residuals became equal to  $\pm 0.4700$ , which is a relative error of 47.00%. The standard deviation of the residuals is 0.23834 and the correlation coefficient has become 0.8797. Residuals plots have become more ordered in the form of V-shaped distributions.

The seventh vertical (Figure. 27) is characterized by a formula of the form

$$z_7 = 1/2 - 1/2 \cos(\pi P / 128 - 1.54046) \tag{30}$$

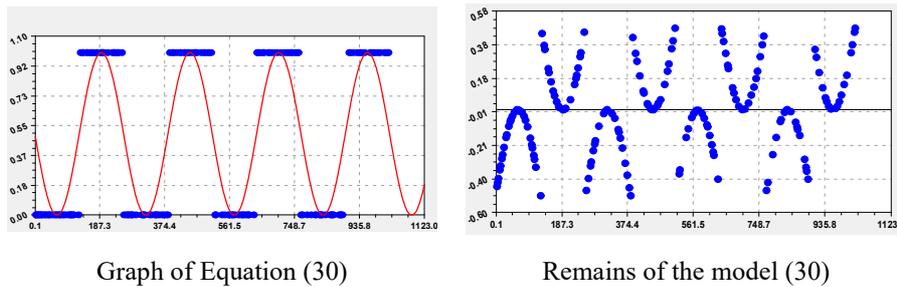


Figure 27. Graphs of a binary number on the seventh vertical of the expansion of prime numbers

For formula (30) with a half-period of 128, the maximum residuals became equal to  $\pm 0.4726$  with a relative error of 47.26%. The standard deviation of the residuals became 0.24091 and the correlation coefficient became 0.8768. Residual graphs have become more streamlined.

On the eighth vertical (Figure. 28), with a half-period of 256, a formula is obtained

$$z_8 = 1/2 - 1/2 \cos(\pi P / 256 - 1.53429) \tag{31}$$

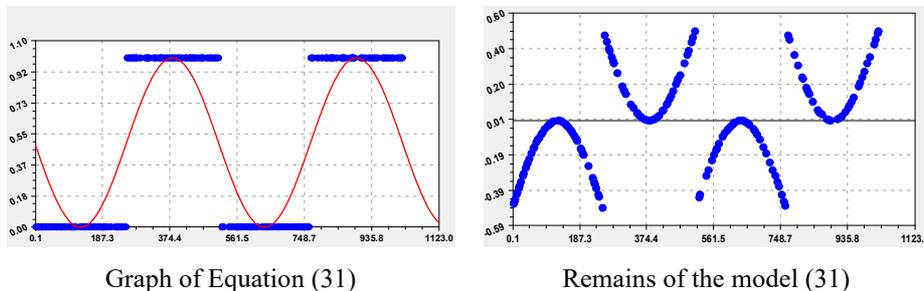


Figure 28. Graphs of a binary number on the eighth vertical of the expansion of prime numbers

The maximum balances are 0.4876 or 48.76%. From the graphs in Figure 28, it can be seen that the standard deviation of the residuals is 0.24363 with a correlation coefficient of 0.8734.

On the last ninth vertical (Figure. 29) a regularity of the form

$$z_9 = 1/2 - 1/2 \cos(\pi P / 512 - 1.51372) \tag{32}$$

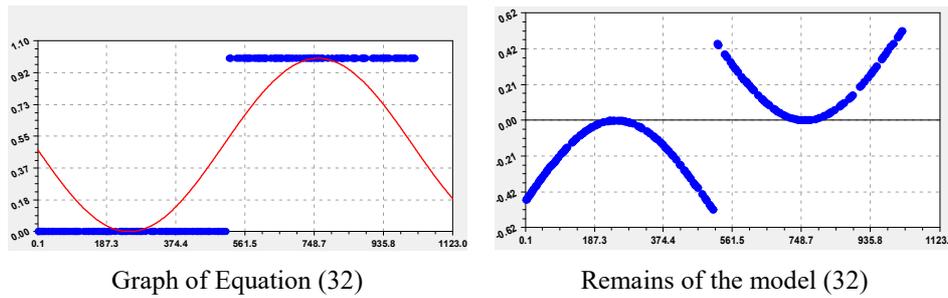


Figure 29. Graphs of a binary number on the ninth vertical of the expansion of prime numbers

The maximum balances became equal to 0.4887 or 48.87%. The graphs in Figure 29 show that the standard deviation is 0.24768 with a correlation coefficient of 0.8672.

**15. Composite Odd Numbers**

On the zero vertical we also get  $z = 1$ .

On the first vertical (Figure. 30) we have a pattern of the form

$$z_1 = 1/2 - 1/2 \cos(\pi(N_1 - P)/2 - 1.55539) \tag{33}$$

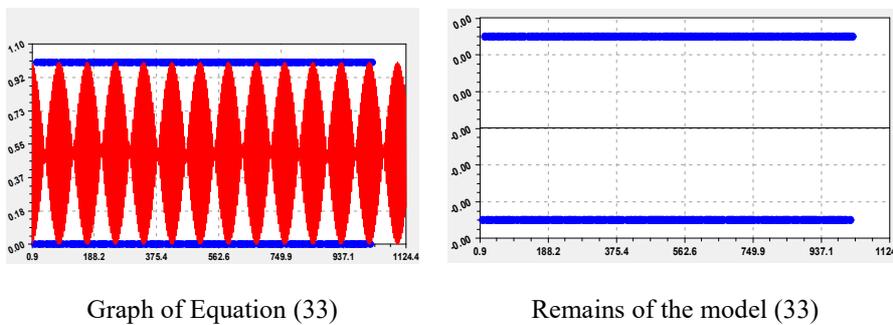


Figure 30. Graphs of a binary number on the first vertical decomposition of composite odd numbers

The remainders after formula (33) are the same and equal to  $\pm 7.29946e-6$ . The correlation coefficient of formula (34) is 1.0000.

On the second vertical (Figure. 31), model (4) takes the form of the equation

$$z_2 = 1/2 - 1/2 \cos(\pi(N_1 - P)/4 - 1.56731) \tag{34}$$

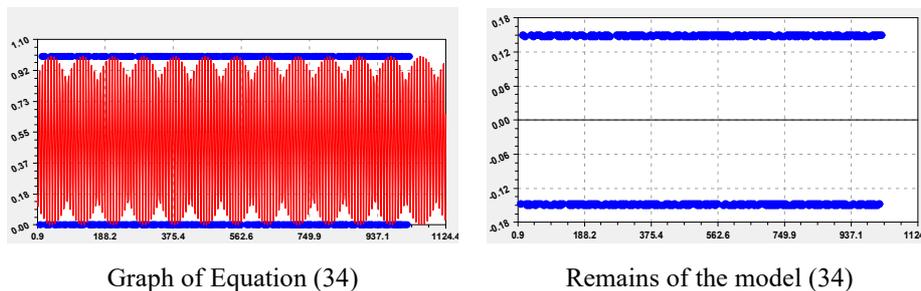


Figure 31. Graphs of a binary number on the second vertical decomposition of composite odd numbers

The remainders after formula (34) are approximately the same and equal to  $\pm 0.1477$  at the maximum. The correlation coefficient of formula (34) will be equal to 0.9561 with a standard deviation of residuals of 0.14666.

The third vertical according to the graphs in Figure 32 is determined by the formula

$$z_3 = 1/2 - 1/2 \cos(\pi(N_1 - P)/8 - 1.56070) \tag{35}$$

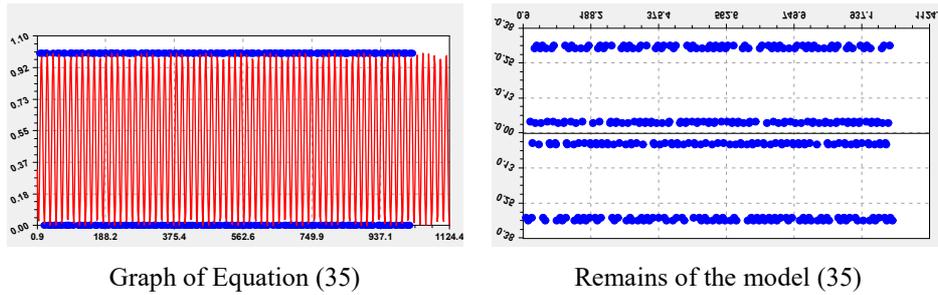


Figure 32. Graphs of a binary number on the third vertical decomposition of composite odd numbers

The residuals after formula (35) differ markedly and are equal to  $\pm 0.3133$  at the maximum. The correlation coefficient of formula (35) will be equal to 0.8956, with a standard deviation of residuals of 0.22269.

The fourth vertical (Figure. 33) is characterized by the mathematical expression

$$z_4 = 1/2 - 1/2 \cos(\pi(N_1 - P)/16 - 1.57814) \tag{36}$$

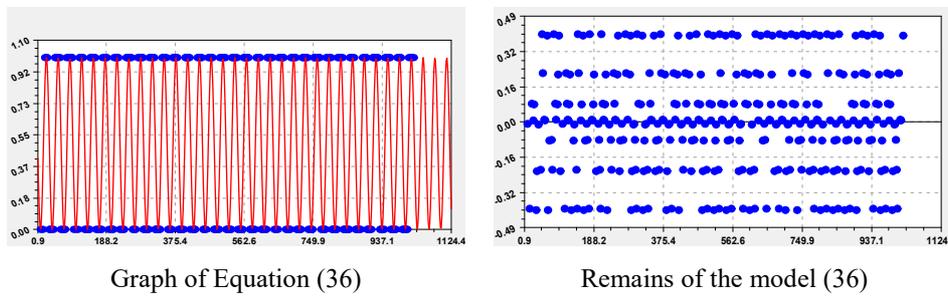


Figure 33. Binary numbers on the fourth vertical of the decomposition of composite odd numbers

The remainders of formula (36) are very different and equal to  $\pm 0.4061$  at the maximum. The correlation coefficient of formula (36) is 0.8812 with a standard deviation of the residuals of 0.23667.

The fifth vertical in Figure 34 is determined by an equation of the form

$$z_5 = 1/2 - 1/2 \cos(\pi(N_1 - P)/32 - 1.57865) \tag{37}$$

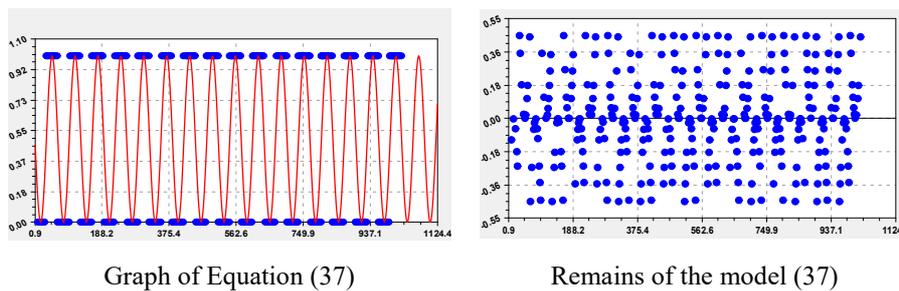


Figure 34. Graphs of a binary number on the fifth vertical of the expansion of composite odd numbers

The residuals from (37) are equal to  $\pm 0.4549$  to the maximum. The correlation coefficient is 0.8786 with a standard deviation of the residuals equal to 0.23908.

The sixth vertical (Figure. 35) is formed by a formula of the form

$$z_6 = 1/2 - 1/2 \cos(\pi(N_1 - P)/64 - 1.57651) \tag{38}$$

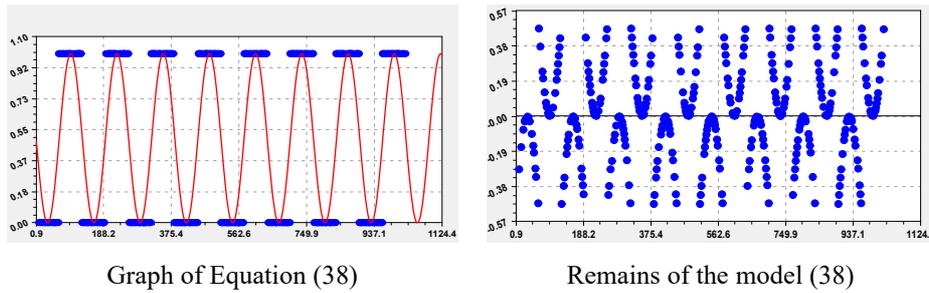


Figure 35. Graphs of a binary number on the sixth vertical of the expansion of composite odd numbers

The remainders of formula (38) are  $\pm 0.4726$ , the correlation coefficient is 0.8795, with a standard deviation of 0.23824. At the same time, the statistical indicators of identification even improved: the standard deviation became smaller, and the correlation coefficient became larger.

On the seventh vertical (Figure. 36), V-shaped eight plots of residuals were obtained after identifying the model (4) by a formula of the form

$$z_7 = 1/2 - 1/2 \cos(\pi(N_1 - P)/128 - 1.58634) \tag{39}$$

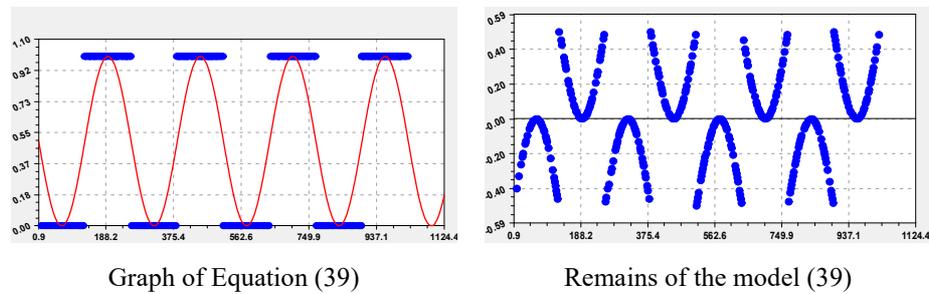


Figure 36. Graphs of a binary number on the seventh vertical of the decomposition of composite odd numbers

The residuals after regularity (39) are equal to  $\pm 0.4955$ , while the correlation coefficient became equal to 0.8808 with the standard deviation of the residuals equal to 0.23703. Compared to the previous vertical, there was also an improvement in the statistical indicators of identification.

The eighth vertical (Figure. 37) was determined by the expression

$$z_8 = 1/2 - 1/2 \cos(\pi(N_1 - P)/256 - 1.58942) \tag{40}$$

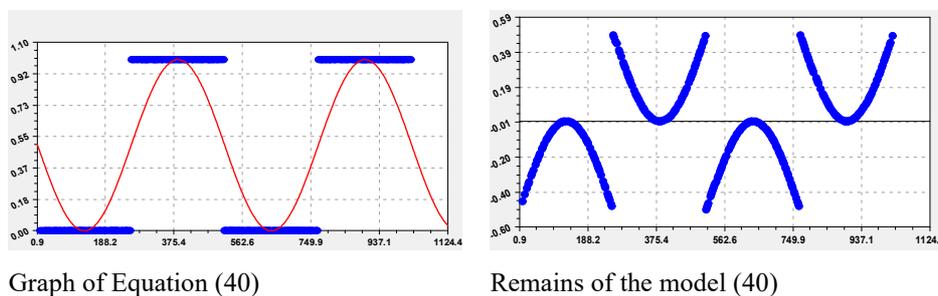


Figure 37. Graphs of a binary number on the eighth vertical of the decomposition of composite odd numbers

The residuals after regularity (40) are equal to  $\pm 0.4909$ , while the correlation coefficient became equal to 0.8822 with a standard deviation of the residuals equal to 0.23559.

On the last ninth vertical (Figure. 38) the formula was obtained

$$z_9 = 1/2 - 1/2 \cos(\pi(N_1 - P)/512 - 1.60117) \tag{41}$$

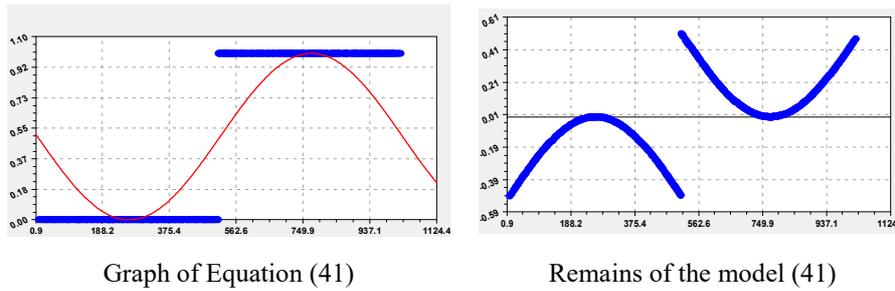


Figure 38. Graphs of a binary number on the ninth vertical of the expansion of composite odd numbers

The maximum residuals after regularity (41) are  $\pm 0.5121$ . The correlation coefficient became equal to 0.8847 with a standard deviation of the residuals of 0.23293.

**16. Composite Natural Numbers**

These numbers include both odd and fair numbers. For the vertical  $i = 0$ , the formula was obtained (Figure. 39)

$$z_0 = 1/2 - 1/2 \cos(\pi(N - P) - 0.0034809) \tag{42}$$

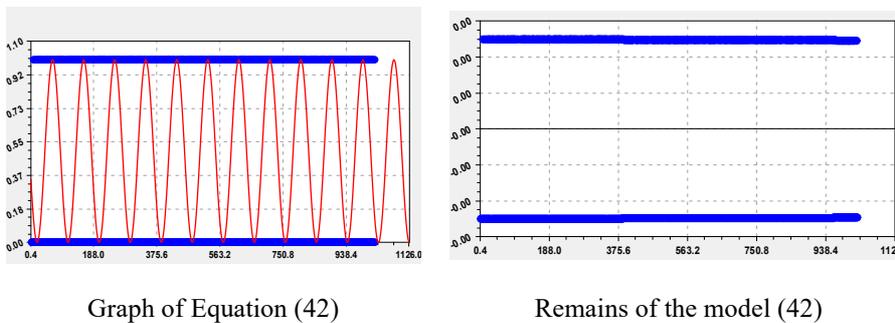


Figure 39. Graphs of a binary number on the zero vertical of composite natural numbers

Here, the half-period of oscillation is  $2^{i=0} = 1$ . The wave shift is very small. In many ways, the distribution of composite natural numbers is similar to the distribution of the natural numbers themselves.

The remainders after formula (42) are the same and equal to  $\pm 3.02848e-6$ . The standard deviation of the residuals from formula (42) will be equal to 0.00000301, and the measure of adequacy (43) by the correlation coefficient is equal to 1.0000. The binary expansion of composite natural numbers will be in second place after odd numbers with a variance of 0.00000211.

For the vertical  $i = 1$ , a formula was obtained (Figure. 40) of the form

$$z_1 = 1/2 - 1/2 \cos(\pi(N - P)/2 - 0.72583) \tag{43}$$

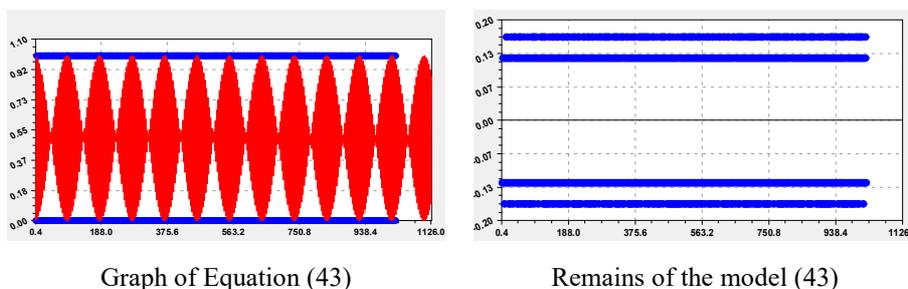


Figure 40. Graphs of a binary number on the first vertical decomposition of composite numbers

The remainders after formula (43) are different and equal to  $\pm 0.1681$  at the maximum. The standard deviation of the residuals from (43) is 0.14441, and the correlation coefficient of this pattern is 0.9574.

For the second vertical (Figure. 41), a regularity of the form

$$z_2 = 1/2 - 1/2 \cos(\pi(N - P)/4 - 1.09213) \tag{44}$$

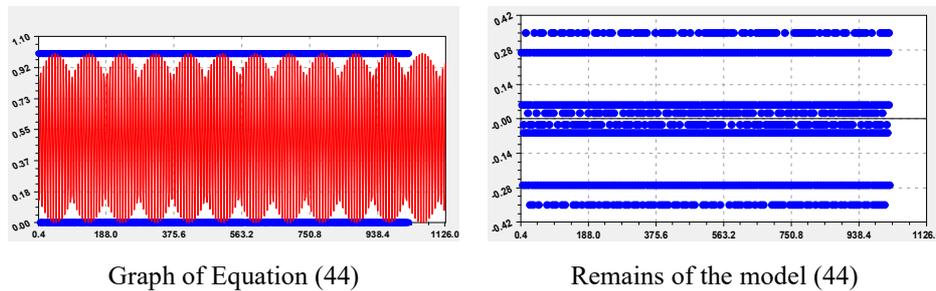


Figure 41. Graphs of a binary number on the second vertical of the expansion of composite numbers

The residuals after formula (44) are different and equal to  $\pm 0.3490$  at the maximum. The standard deviation of the residuals from formula (44) is 0.21689, and the correlation coefficient of this regularity is 0.9011.

For the third vertical (Figure. 42), a trigonometric formula was obtained

$$z_3 = 1/2 - 1/2 \cos(\pi(N - P)/8 - 1.33163) \tag{45}$$

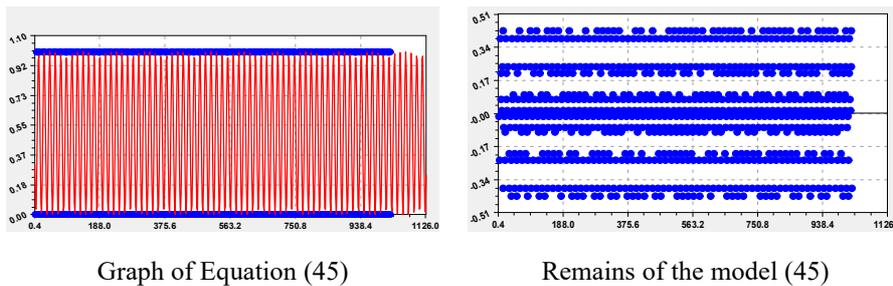


Figure 42. Graphs of a binary number on the third vertical of the expansion of composite numbers

The residuals after formula (45) are very different and equal to  $\pm 0.3816$  at the maximum. The standard deviation of the residuals is 0.23383, and the correlation coefficient of this pattern is 0.8849.

On the fourth vertical (Figure. 43) the formula was formed

$$z_4 = 1/2 - 1/2 \cos(\pi(N - P)/16 - 1.45677) \tag{46}$$

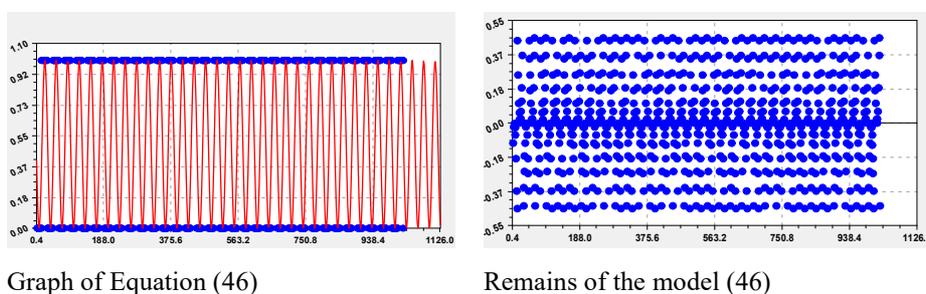


Figure 43. Graphs of a binary number on the fourth vertical of the expansion of composite numbers

The remainders after formula (46) also differ and are at a maximum equal to  $\pm 0.4431$ . The standard deviation of the residuals is 0.23827, and the correlation coefficient of this pattern is 0.8793.

According to the fifth vertical of the binary expansion (Figure. 44), a dependence of the form

$$z_5 = 1/2 - 1/2 \cos(\pi(N - P)/32 - 1.51636) \tag{47}$$

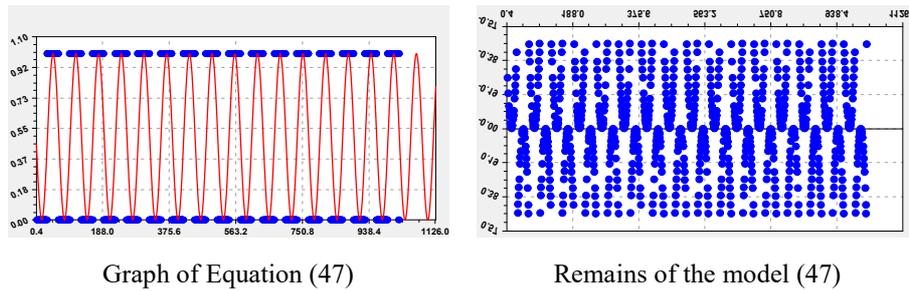


Figure 44. Graphs of a binary number on the fifth vertical of the expansion of composite numbers

The residuals from (47) become symmetrical and reach a maximum of  $\pm 0.4728$ . The standard deviation of the residuals is 0.23854, and the correlation coefficient of this pattern is 0.8790.

The sixth vertical (Figure. 45) receives a trigonometric formula of the form

$$z_6 = 1/2 - 1/2 \cos(\pi(N - P)/64 - 1.54507) \tag{48}$$

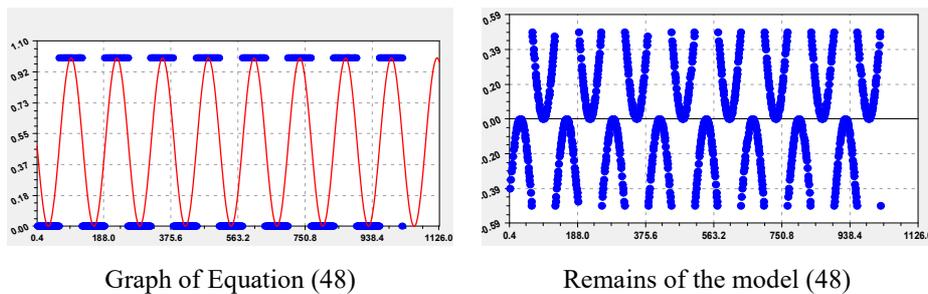


Figure 45. Graphs of a binary number on the sixth vertical of the expansion of composite numbers

The residuals from (48) acquire a clear symmetry and reach a maximum of  $\pm 0.4883$ . The standard deviation of the residuals is 0.23790, and the correlation coefficient of this pattern is 0.8797.

On the seventh vertical (Figure. 46), an expression of the form

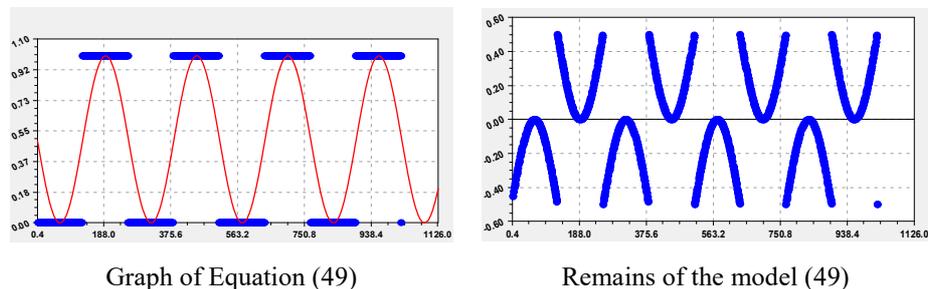


Figure 46. Graphs of a binary number on the seventh vertical of the expansion of composite numbers

The residuals from (49) also acquire a clear symmetry, the maximum reaches  $\pm 0.4966$ . The standard deviation of the residuals is 0.23735, and the correlation coefficient of this pattern is 0.8803. In this case, there is some increase in the adequacy of the model (4).

The eighth vertical (Figure. 47) of the binary expansion of composite numbers receives the formula

$$z_8 = 1/2 - 1/2 \cos(\pi(N - P)/256 - 1.57265) \tag{50}$$

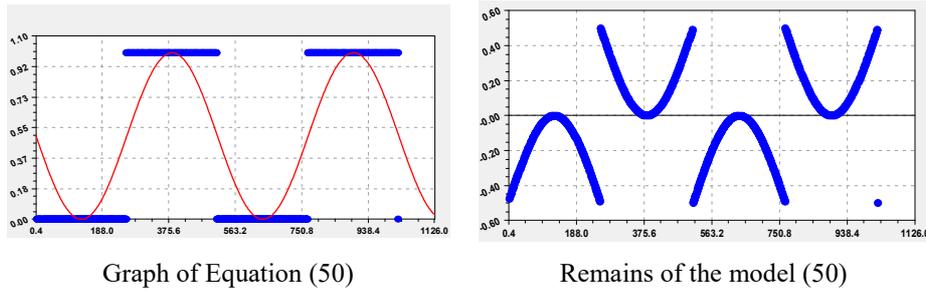


Figure 47. Graphs of a binary number on the eighth vertical of the expansion of composite numbers

The remainders from (50) also have a clear symmetry, the maximum reaches  $\pm 0.5009$ . The standard deviation of the residuals is 0.23672, and the correlation coefficient of this pattern is 0.8809. With an increase in the residuals, there is some increase in the adequacy of the model (4).

The ninth vertical (Figure. 48) is determined by a formula of the form

$$z_9 = 1/2 - 1/2 \cos(\pi(N - P)/512 - 1.58092) \tag{51}$$

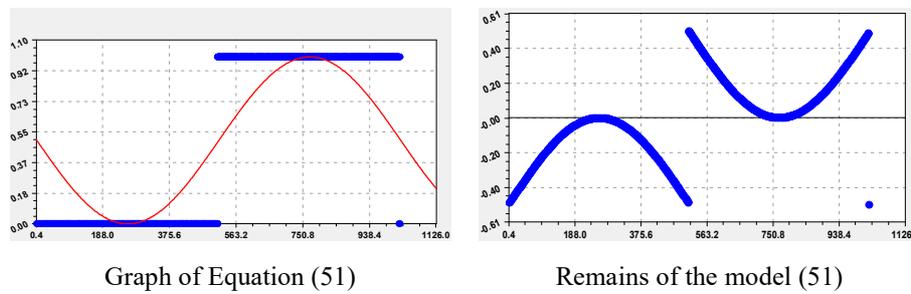


Figure 48. Graphs of a binary number on the ninth vertical of the decomposition of composite numbers

The remainders of formula (51) have two V-shaped figures and their maximum reaches  $\pm 0.5051$ . The standard deviation of the residuals is 0.23568, and the correlation coefficient of this pattern is 0.8820.

**17. Even Natural Numbers**

On the zero vertical, we obtain the condition  $z = 0$ .

On the first vertical (Figure. 49) the formula was obtained

$$z_1 = 1/2 - 1/2 \cos(\pi N_0 / 2 - 0.0043041) \tag{52}$$

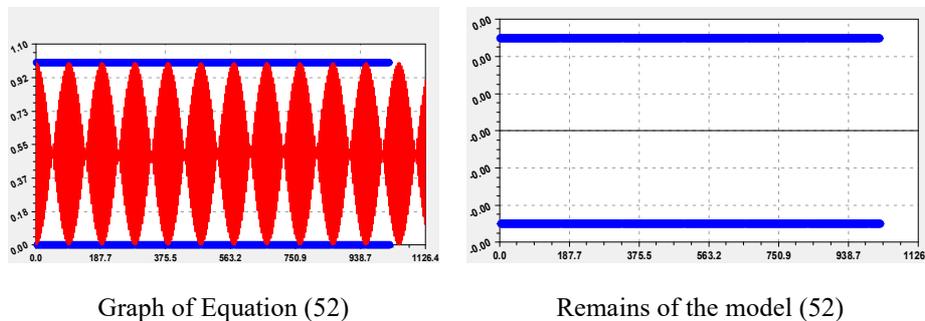


Figure 49. Graphs of a binary number on the first vertical of the decomposition of even natural numbers

The remainders after formula (52) are the same and equal to  $\pm 4.63126e-6$ . The shift of the perturbation wave is 0.0043041 radians, the correlation coefficient is 1.0000.

On the next second vertical (Figure. 50) the formula is obtained

$$z_2 = 1/2 - 1/2 \cos(\pi N_0 / 4 - 0.78480) \tag{53}$$

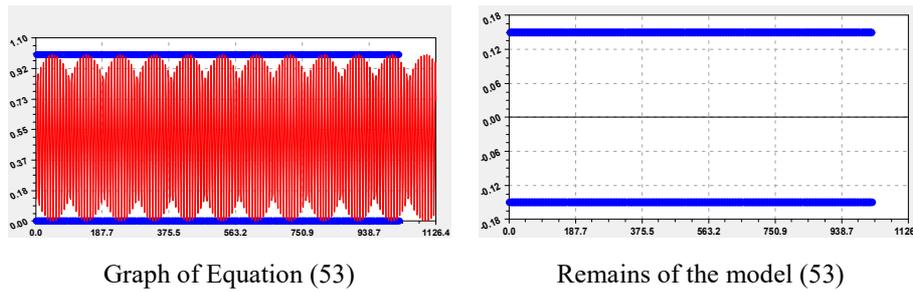


Figure 50. Graphs of a binary number on the second vertical of the decomposition of even natural numbers

The standard deviation of the residuals from formula (53) is 0.1466, and the correlation coefficient is 0.9561. In this case, the shift of the beginning of the oscillation is 0.78480 and this value is close to  $\pi/4$ .

On the third vertical of the binary expansion of even numbers (Figure. 51), we obtained the equation

$$z_3 = 1/2 - 1/2 \cos(\pi N_0 / 8 - 1.17638) \tag{54}$$

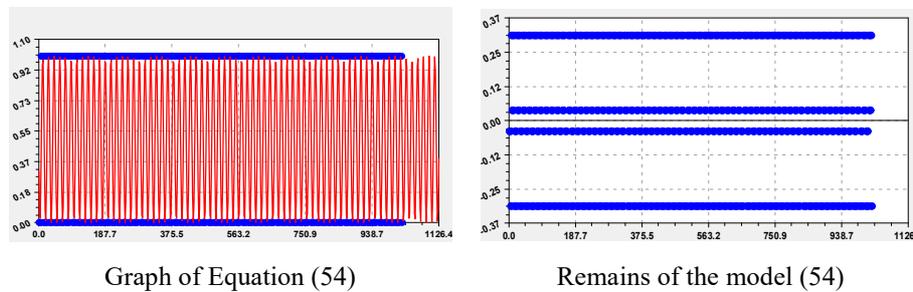


Figure 51. Graphs of a binary number on the third vertical of the decomposition of even natural numbers

Here, the wave shift has become larger than  $\pi/4$ , but it is less than  $\pi/2$ . Calculations showed that the shear angle is approximately 67.4 degrees. This value is close to the limit of  $3\pi/8=1.178096$ .

The standard deviation of equation (54) is 0.22033, its adequacy is 0.8979 in terms of the correlation coefficient. Thus, as in other distributions along the verticals of the binary expansion, with an increase in the digit of the binary number system, the error of model (4) increases.

On the fourth vertical (Figure. 52) we got the expression

$$z_4 = 1/2 - 1/2 \cos(\pi N_0 / 16 - 1.37204) \tag{55}$$

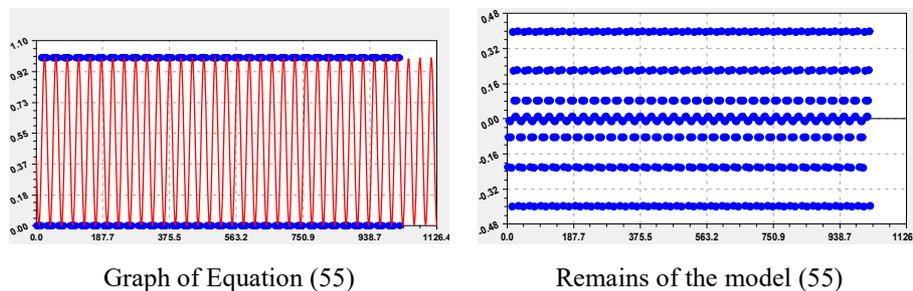


Figure 52. Graphs of a binary number on the fourth vertical of the expansion of even numbers

The maximum residual from the model is 0.4013. The standard deviation of equation (55) is 0.22442, its adequacy is 0.8835 in terms of the correlation coefficient.

On the next fifth (Figure. 53) vertical, a regularity of the form

$$z_5 = 1/2 - 1/2 \cos(\pi N_0 / 32 - 1.46987) \tag{56}$$

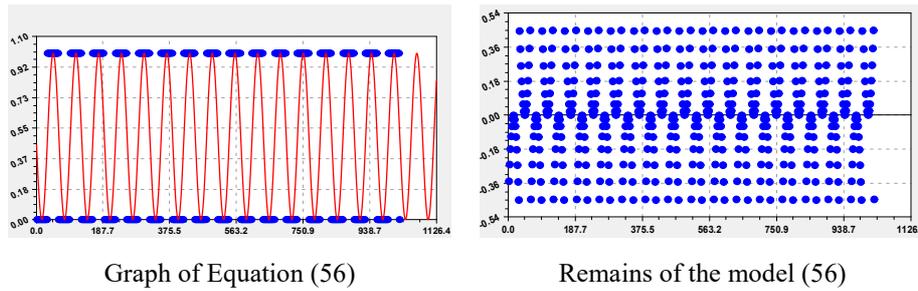


Figure 53. Graphs of a binary number on the fifth vertical of the decomposition of even natural numbers

The maximum balance became equal to 0.4496. The standard deviation of equation (56) became equal to 0.23785, its adequacy is equal to 0.8799 in terms of the correlation coefficient.

On the sixth vertical distribution of even numbers (Figure. 54), the expression

$$z_6 = 1/2 - 1/2 \cos(\pi N_0 / 64 - 1.51879) \tag{57}$$

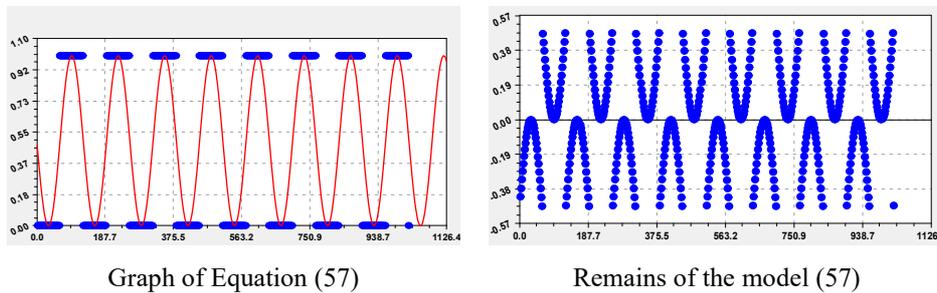


Figure 54. Graphs of a binary number on the sixth vertical of the decomposition of even natural numbers

The maximum balance became equal to 0.4740. The standard deviation of equation (57) reaches 0.23875, its adequacy is 0.8789 in terms of the correlation coefficient.

On the seventh vertical of the binary expansion of even numbers (Figure. 55), we obtained the formula

$$z_7 = 1/2 - 1/2 \cos(\pi N_0 / 128 - 1.54326) \tag{58}$$

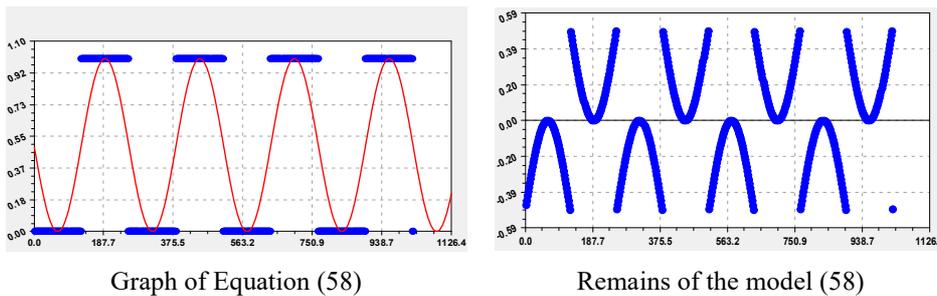


Figure 55. Graphs of a binary number on the seventh vertical of the decomposition of even natural numbers

The maximum remainder of equation (58) reached 0.4862. The standard deviation of equation (58) became equal to 0.23900, and its adequacy in terms of the correlation coefficient is equal to 0.8786. At the same time, the oscillation shift became equal to 1.54326, which is much closer to the  $\pi/2$  limit.

On the penultimate eighth vertical (Figure. 56), an expression of the form

$$z_8 = 1/2 - 1/2 \cos(\pi N_0 / 256 - 1.55549) \tag{59}$$

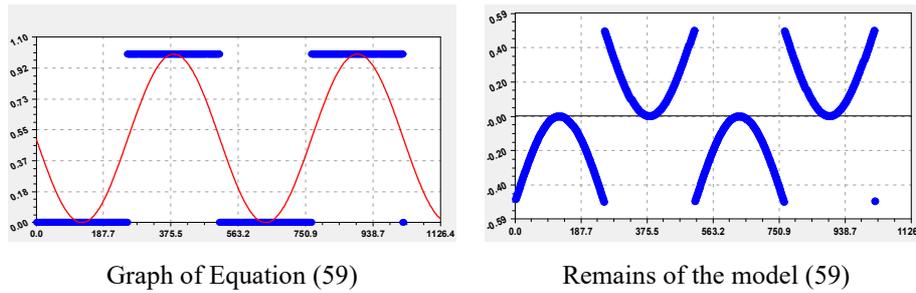


Figure 56. Graphs of a binary number on the eighth vertical of the decomposition of even natural numbers

The wave shift gradually approaches the limit  $\pi/2$ . At the same time, the maximum value of the residuals became equal to 0.4923, and the standard deviation - 0.23908, with a correlation coefficient of 0.8785.

On the last ninth vertical of the binary expansion of even numbers (Figure. 57), the formula appeared

$$z_9 = 1/2 - 1/2 \cos(\pi N_0 / 512 - 1.56161) \tag{60}$$

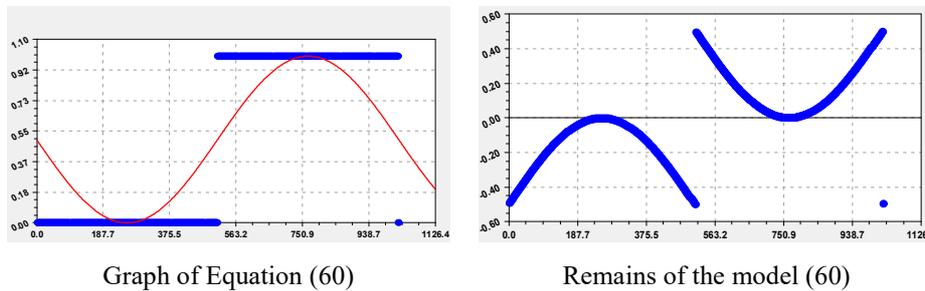


Figure 57. Graphs of a binary number on the ninth vertical of the decomposition of even natural numbers

The wave shift also approaches the  $\pi/2$  limit. The maximum value of the residuals became 0.4954, the standard deviation was 0.23910, and the correlation coefficient was 0.8785.

All formulas are the same in construction, except for the wave shift. Therefore, we will compare numerical systems by one statistical parameter - the shift of the cosine wave of an oscillatory perturbation of a binary number on different verticals of the binary expansion of numbers from the decimal number system.

**18. Comparison of Series of Numbers by the Shift of the Perturbation Wave of a Binary Number**

For comparison, the fluctuation shift parameter for all previous statistical models is given in Table 2. In it, the values of the parameter  $\alpha_i$  are rounded to the fifth significant figure after the dividing point.

Table 2. Comparison of varieties of natural numbers according to the shift of the oscillation in the model

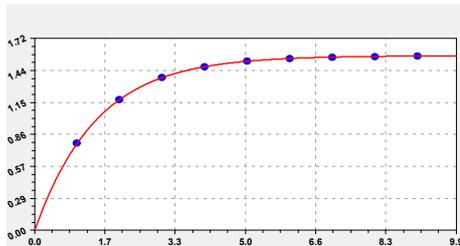
$$z_i = 1/2 - 1/2 \cos(\pi x / 2^i - \alpha_i)$$

$i_2$	Blocks $2^i$	At the end of the block	Parameter $\alpha$ shear wave perturbation of a binary number					
			natural numbers	odd numbers	prime numbers	composite odd numbers	composite natural numbers	even numbers
0	1	1	0.00043088	$z=1$	$z=1$	$z=1$	0.0034089	$z=0$
1	2	2	0.78514	1.56790	1.56539	1.56539	0.72583	0.0043041
2	4	3	1.17725	1.57082	1.57765	1.56731	1.09213	0.78480
3	8	7	1.37324	<b>1.57083</b>	<b>1.59173</b>	1.56070	1.33163	1.17638
4	16	13	1.47124	1.57082	1.55614	1.57814	1.45677	1.37204
5	32	31	1.52028	1.57081	1.55521	1.57865	1.51636	1.46987
6	64	61	1.54477	1.57081	1.55977	1.57651	1.54507	1.51879
7	128	127	1.55699	1.57081	1.54046	1.58634	1.56393	1.54326

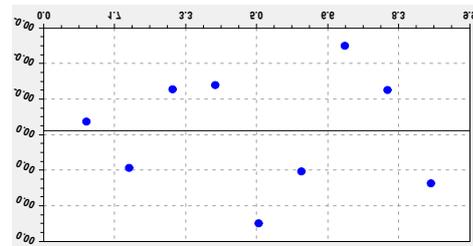
8	256	251	1.56313	1.57081	1.53429	1.58942	1.57265	1.55549
9	512	509	1.56622	1.57081	1.51372	1.60117	1.58092	1.56161
10	1024	1021						

Natural numbers receive (Figure. 58) an algebraic regularity in the form of the Weibull law of reaching a certain limit with increasing bit of the binary number system

$$a = 1.56927 - 1.56884 \exp(-0.69352i^{0.99970}) \tag{61}$$



Statistical Model Plot (61)



Remains of the model (61)

Figure 58. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of natural numbers

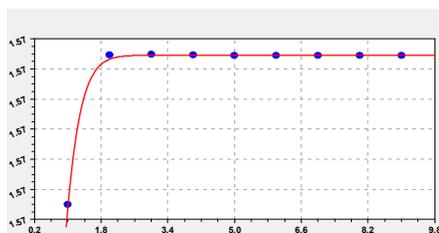
Formula (61) in the limit as  $i \rightarrow \infty$  tends to the value  $a \rightarrow \pi/2$ . Then from equation (4) we obtain the formula in the final form

$$z_0 = 1/2 - 1/2 \sin(\pi N) \tag{62}$$

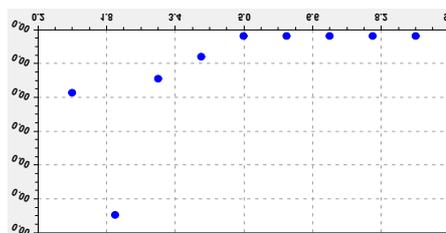
under the condition  $i \rightarrow \infty$ . Thus, at the beginning of a series of natural numbers up to about 1024, the oscillation of the binary number shifts from 0 to almost  $\pi/2$ . For the final proof, it is necessary to process series of natural numbers up to 10 and 100 thousand values. Then the meaning of oscillations to model (61) will be clear.

Odd natural numbers (Figure. 59), according to the Weibull law, showed an equation of the form

$$a = 1.57081 - 0.025702 \exp(-2.16897i^{1.42167}) \tag{63}$$



Statistical Model Plot (63)



Remains of the model (63)

Figure 59. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of odd numbers

Odd natural numbers reach the limit  $a \rightarrow \pi/2$  much faster, since already at the zero binary digit, in fact, according to Table 2, they get  $z=1$ . However, we will be careful in the conclusions, since the table has a maximum on the third vertical of a binary number. Additional research is needed on several series of odd numbers of different lengths.

Prime numbers (Figure. 60) from a series of natural numbers from 0 to 1024 gave the formula for a more complex statistical model

$$a = 1.46344 \exp(-0.0014426i^{1.14202}) - 0.12822i^{0.42844} \exp(-0.19633i^{0.89496}) \tag{64}$$

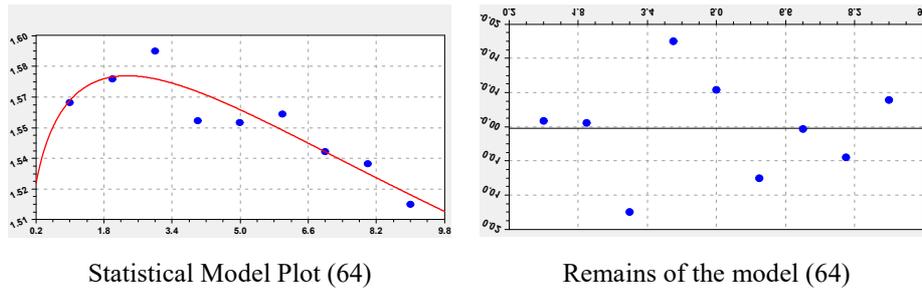


Figure 60. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of prime numbers

From the graphs in Figure 60, it can be seen that, according to the two-term trend, the shift in the fluctuation of the binary number changes in a complex way: first, up to the  $i=3$  digit, the shift increases, and then gradually decreases. In this case, the first term of the trend equation (64) is the law of exponential decay (the Laplace law in mathematics, Mandelbrot in physics, Zipf-Pearl in biology and Pareto in econometrics, modified by us), and the second component is the biotechnical law of prof. P.M. Mazurkin. It can be seen from the residuals that it is possible to identify the third component to model (64) in the form of an asymmetric finite-dimensional wavelet.

Under the condition  $i \rightarrow \infty$ , the second term of formula (64) receives uncertainty, therefore, additional statistical studies are needed for a number of prime numbers up to 10 million.

In the decimal number system, the proportion  $\alpha$  of prime numbers is arranged in relative descending order as follows:

$x = 10^i$	$10^0$	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$
$\pi(x)$	1	6	27	170	1231	7593	78499
$\alpha$	1	0.6	0.27	0.17	0.1231	0.09593	0.078499

The share of prime numbers in the decimal number system decreases according to the Mandelbrot law modified by us, starting from 1.

In addition, an asymmetric wavelet signal of the oscillatory perturbation of this fraction appears. Then it turns out that a series of prime numbers in the decimal number system is placed according to the hallmark of oscillatory adaptation. After an asymmetric universe, the laws of oscillatory perturbation of physical phenomena and processes appear.

The expansion of composite odd numbers (Figure. 61) in bits of the binary number system gave a statistical model of the form

$$a = 1.56602 \exp(-0.0020957i^{1.00567}) + 0.0023137i^{1.50148} \tag{65}$$

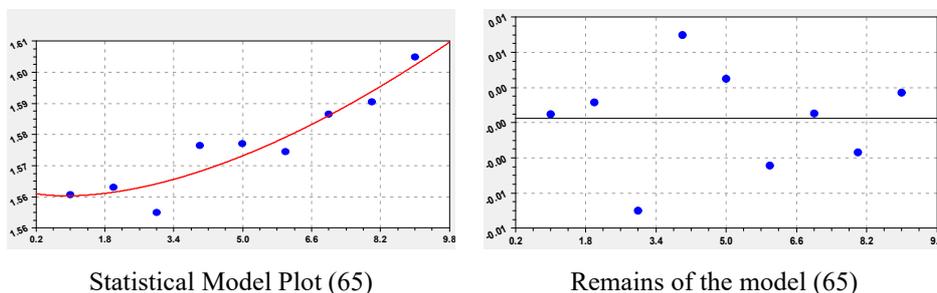


Figure 61. Influence of the expansion vertical on the shift of the fluctuation of a binary number in a series of composite odd numbers

Due to the exclusion of all prime numbers from the series of odd numbers, the distribution of the shift of the oscillation of a binary number became more complicated depending on the vertical of the binary number system. In formula (65), the sign in front of the second component has changed, which, by design, has been simplified from the biotechnical law to the law of a power function.

Composite natural numbers (Figure. 62) again gave the Weibull law

$$a = 1.58907 - 1.58443 \exp(-0.59640i^{1.00652}) \tag{66}$$

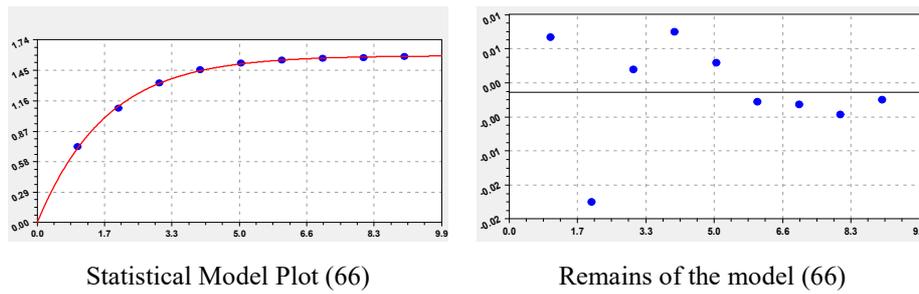


Figure 62. Influence of the discharge on the shift of the fluctuation of a binary number in a series of composite numbers

Again, in the limit as  $i \rightarrow \infty$  tends to the value  $a \rightarrow \pi/2$ .

Even natural numbers (Figure. 63) also showed a pattern of the form

$$a = 1.56758 - 3.11187 \exp(-0.68843i^{1.00354}) \tag{67}$$

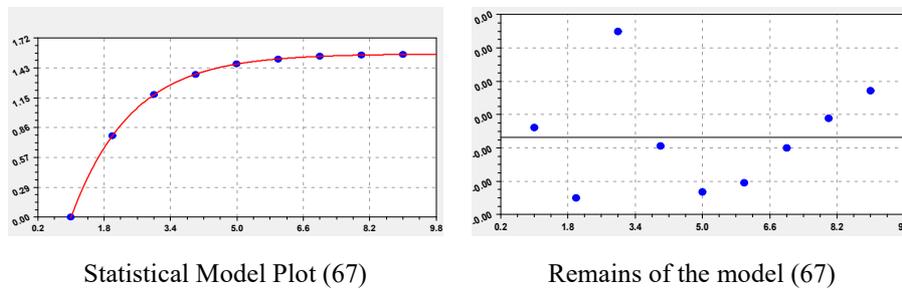


Figure 63. Influence of the discharge of the vertical on the shift of the oscillation binary number in a series of even numbers

An analysis of the statistical models of the influence of the rank of the binary number system along the verticals on the shift of the oscillation of a binary number showed that the most complex design of the trend was obtained by simple numbers, and then composite odd numbers were located. Natural numbers, composite natural numbers, composite odd numbers and even numbers have received a simple construction of the Weibull law of reaching the limit.

### 19. Comparison of Series of Numbers by the Correlation Coefficient

The data for analysis are given in Table 3. It is also marked in bold here that, with a zero digit of the binary number system, the distributions of natural numbers and composite natural numbers have the smallest error. This is the critical Riemann line.

And for the series of odd numbers, prime numbers and even numbers, the critical Riemann line is in the first digit of the binary number system under the condition  $i = 1$ .

Table 3. Comparison of varieties of natural numbers by model correlation coefficient

Binary digit $i$	Row of blocks $2^i$	Prime numbers at the block $P_i$	Correlation coefficient $r$ of the model					
			natural numbers	odd numbers	prime numbers	composite odd numbers	composite natural numbers	even numbers
0	1	1	<b>1.0000</b>	1	1	1	<b>1.0000</b>	1
1	2	2	0.9561	<b>1.0000</b>	<b>1.0000</b>	<b>1.0000</b>	0.9574	<b>1.0000</b>
2	4	3	0.8980	0.9561	0.9561	0.9561	0.9011	0.9561
3	8	7	0.8838	0.8981	0.9029	0.8956	0.8840	0.8979

4	16	13	0.8802	0.8840	0.8895	0.8812	0.8793	0.8835
5	32	31	0.8793	0.8805	0.8843	0.8786	0.8790	0.8799
6	64	61	0.8790	0.8796	0.8797	0.8795	0.8797	0.8789
7	128	127	0.8789	0.8794	0.8768	0.8808	0.8803	0.8786
8	256	251	0.8789	0.8794	0.8734	0.8822	0.8809	0.8785
9	512	509	0.8789	0.8793	0.8672	0.8847	0.8820	0.8785
10	1024	1021						

Table 3 also shows that as the binary digit  $i$  increases, the correlation coefficient of model (4) decreases (except for the series of composite numbers). Let's find these patterns.

Natural numbers receive (Figure. 64) a pattern of changes in the correlation coefficient in the form of a formula

$$r = 0.12064 \exp(-0.45818i^{1.99509}) + 0.87947 \tag{68}$$

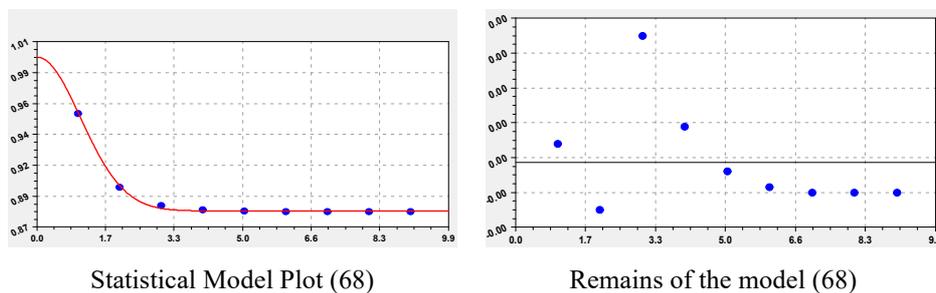


Figure 64. Influence of a binary digit on the shift of the fluctuation of a binary number in a series of natural numbers

According to the Weibull law modified by us with a positive sign, it turns out that according to formula (68) under the condition  $i=0$  we have  $r=1$ , and under the condition  $i \rightarrow \infty$  in the limit  $r=0.8795$ . Thus, regardless of the power of the series, we obtain the constant adequacy.

Odd natural numbers (Figure. 65) showed the equation

$$r = 0.12190 \exp(-0.040733i^{3.51073}) + 0.88025 \tag{69}$$

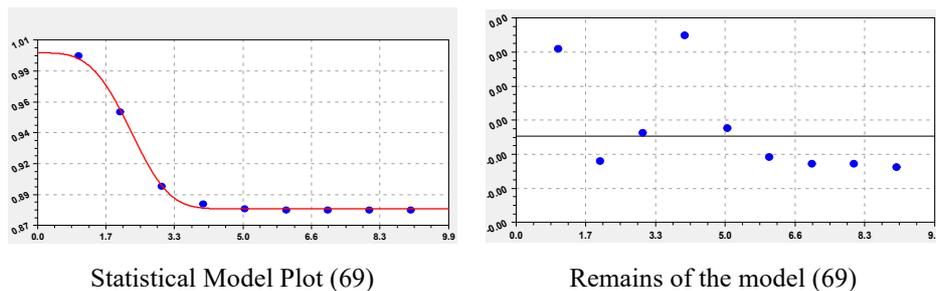


Figure 65. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of odd numbers

The exponential decay activity in formula (69) 0.040733 is less than the same parameter in equation (68) by  $0.45818 / 0.040733 = 11.25$  times. At the same time, the intensity of the decline  $3.51073 / 1.99509$  is 1.76 times greater.

Prime numbers (Figure. 66) from a series of natural numbers from 0 to 1024 gave a more complex formula

$$r = 0.10061 \exp(-0.035277i^{3.83967}) + 0.90202 \exp(-0.0026906i^{1.21298}) \tag{70}$$

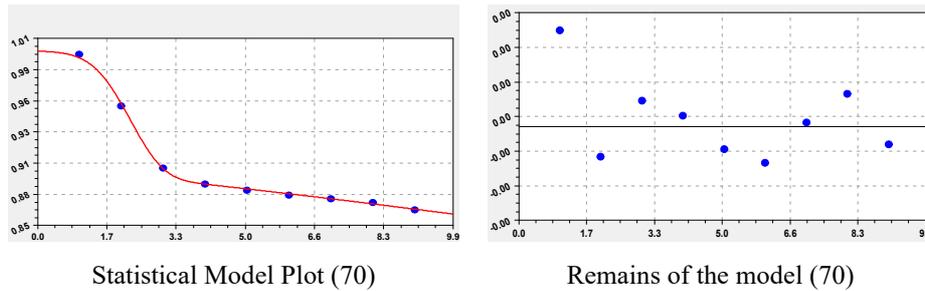


Figure 66. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of prime numbers

Here we have a pattern consisting of two laws of exponential decay according to the Mandelbrot law modified by us (in physics). In this case, both components of equation (70) in the limit decrease to the sum  $0.10061 + 0.90202 = 1.00263$ .

Is the limit  $r(i \rightarrow \infty) = 1.00263$  valid (correlation coefficient should not be greater than 1) or will the correlation coefficient be equal to one if the power of a series of prime numbers is infinite. Additional statistical studies are needed to answer.

Composite odd numbers (Figure. 67) by bits of the binary number system gave the formula

$$r = 0.13066 \exp(-0.044258i^{3.45547}) + 0.86996 \exp(0.0047971i^{0.51982}) \tag{71}$$

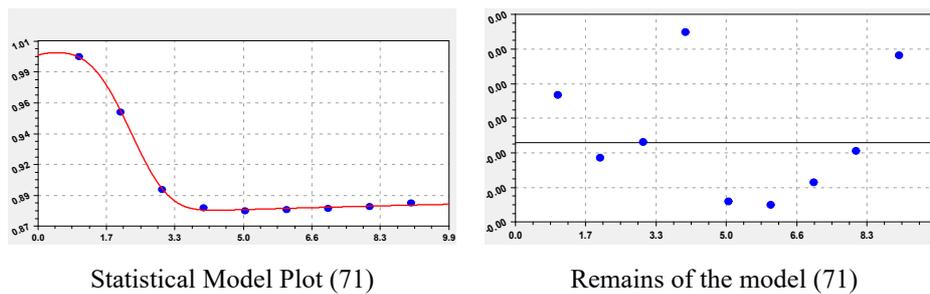


Figure 67. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of composite odd numbers

The second component of formula (71) became the law of exponential growth of prof. P.M. Mazurkin and shows the confrontation in a series of odd composite numbers. As a result, we can conclude that composite odd numbers can have some amazing properties. The amazingness of even natural numbers lies in the fact that they are increments in a series of prime numbers.

Composite natural numbers (Figure. 68) gave the same mathematical expression

$$r = 0.12234 \exp(-0.43824i^{1.91527}) + 0.87777 \exp(0.00040471i^{1.05104}) \tag{72}$$

Then it turns out that prime numbers transform composite natural numbers and, in a particular case, also composite odd numbers.

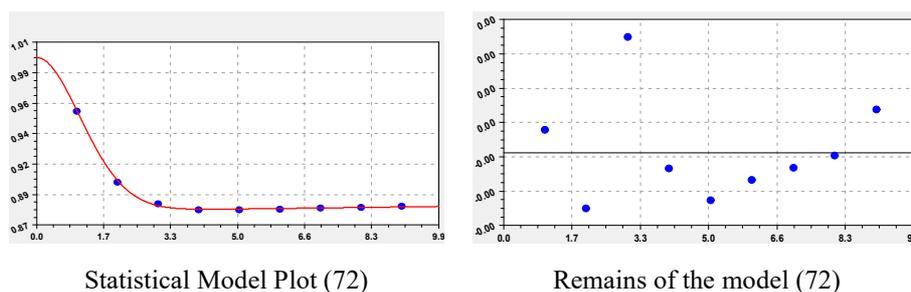


Figure 68. Influence of the digit of the binary number system on the shift of the oscillation of a binary number in a series of composite numbers

Even natural numbers (Figure. 69) showed a pattern of the form

$$r = 0.111782 \exp(-0.036960i^{3.66016}) + 0.8 + 8429 \exp(-0.00058865i^{1.14375}) \tag{73}$$

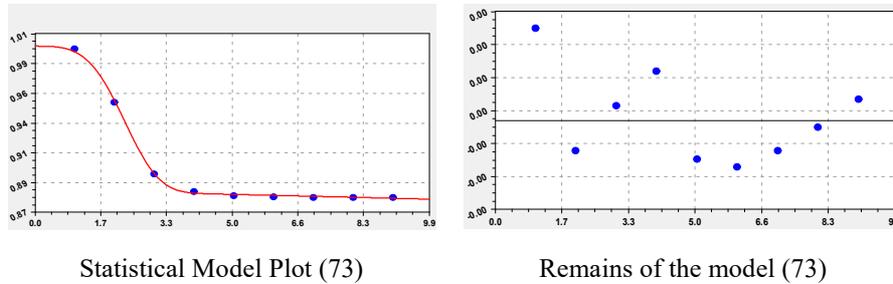


Figure 69. Influence of the digit of the binary number system on the shift of the fluctuation of a binary number in a series of even numbers

This statistical pattern is similar to the series of prime numbers, so we can conclude that prime numbers are organically related to even numbers through prime increments.

A complete statistical analysis of different types of distributions of natural numbers from 0 to 1024 according to the scheme in Figure 1 showed that they are the same according to the Riemann hypothesis. However, the rational root 1/2 is located at the zero digit of the binary number system for natural and composite natural numbers, and for the rest of the distributions (odd and prime numbers, composite odd and even natural numbers), the rational root 1/2 is located at the first digit at  $i=1$  binary number system.

It turns out that the critical Riemann line for prime numbers is on the second vertical of the binary expansion of odd natural numbers.

**20. Choice of Decomposition Vertical in the Dinary System**

Let us choose from all the previous revealed patterns that are most accurately located on the critical Riemann line:

– under the condition  $i = 0$  on the zero vertical, we obtain models of a binary number

$$z_0 = 1/2 - 1/2 \cos(\pi N + 0.0043088) \tag{74}$$

$$z_0 = 1/2 - 1/2 \cos(\pi(N - P) - 0.0034809) \tag{75}$$

– and under the condition  $i = 1$  on the first vertical we have models of a binary number

$$z_1 = 1/2 - 1/2 \cos(\pi N_1 / 2 - 1.56790) \tag{76}$$

$$z_1 = 1/2 - 1/2 \cos(\pi P / 2 - 1.56539) \tag{77}$$

$$z_1 = 1/2 - 1/2 \cos(\pi(N_1 - P) / 2 - 1.55539) \tag{78}$$

$$z_1 = 1/2 - 1/2 \cos(\pi N_0 / 2 - 0.0043041) \tag{79}$$

With an increase in the power of series of different types of numbers, we obtain limit expressions:

$$z_0 = 1/2 - 1/2 \cos(\pi N) \tag{80}$$

$$z_0 = 1/2 - 1/2 \cos(\pi(N - P)) \tag{81}$$

$$z_1 = 1/2 - 1/2 \sin(\pi N_1 / 2) \tag{82}$$

$$z_1 = 1/2 - 1/2 \sin(\pi P / 2) \tag{83}$$

$$z_1 = 1/2 - 1/2 \sin(\pi(N_1 - P)/2) \tag{84}$$

$$z_1 = 1/2 - 1/2 \cos(\pi N_0 / 2) \tag{85}$$

In all these formulas, the first term is the rational root of the 1/2 Riemann hypothesis. The critical Riemann line for natural numbers and composite natural numbers is the zero vertical of the binary expansion. For the remaining series of numbers (odd numbers, prime numbers, composite odd and whole natural numbers), the first vertical of the binary expansion of decimal numbers becomes the critical line.

**21. Conclusion**

The varieties of natural numbers  $N = \{0, 1, 2, 3, \dots\}$  are as follows:

P are simple natural numbers 0, 1, 2, 3, 5, 7, 11, 13, 17, ...

N-P - composite natural numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, ...

N0 are even natural numbers 0, 2, 4, 6, 8, 10, 12, 14, 16, ...

N1 – odd natural numbers 1, 3, 5, 7, 9, 11, 13, 15, 17, ...

N1-P are composite odd natural numbers 9, 15, 21, 25, ...

The number 0 in any row from the correspondence of numerical systems  $P \subset N \subset Z \subset Q \subset R$  is in a special decimal or binary digit  $i = -\infty$  (Table 4). With negative values of the discharge for integers  $i = \{\dots -8, -7, -6, -5, -4, -3, -2, -1, 0\}$  between 0 and 1 an infinite set of rational numbers is placed. For example, for the binary number system, we get a series of rational numbers 1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, etc. up to 0. And for the decimal number system, we have the series 1, 1/10, 1/100, 1.1000, etc.

Table 4. Comparison of beginnings for number systems

Binary system		Decimal system		prime numbers	
$i$	$2^i$	$i$	$10^i$	$j$	$P_j$
$-\infty$	0	$-\infty$	0	0	0
0	1	0	1	1	1

Prime numbers in this article are located on the positive semi-axis of the abscissa. Table 1 shows that the prime numbers 0 and 1 in the zero vertical of a binary number are, respectively, a non-trivial zero and one.

Decimal groups are set in two ways: 1) as the order 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and as the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

In the first case, the group contains six prime and four composite numbers, and this group of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 of ten natural numbers is asymmetric: the prime numbers 4 + 1 + 1, and from the end to the beginning - composite numbers according to the arrangement scheme 2 + 1 + 1. The ratio of prime numbers to the first ten natural numbers is 60%, which is very close to the golden ratio of 0.618.

In the second case, the decimal group of the number system contains five prime and composite numbers each, and this group 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 of ten natural numbers is symmetrical: at the beginning of the segment of the series are prime numbers, and from the end to the beginning - composite numbers. Since the Universe is asymmetric, we prefer the asymmetric distribution of prime numbers in the group 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

For six different series of numbers, the limiting mathematical expressions of a binary real number are revealed, which are obtained after decomposing the series from the decimal number system into the binary number system:

$$z_0 = 1/2 - 1/2 \cos(\pi N) \quad ; \quad z_0 = 1/2 - 1/2 \cos(\pi(N - P)) \quad ; \quad z_1 = 1/2 - 1/2 \sin(\pi N_1 / 2) \quad ;$$

$$z_1 = 1/2 - 1/2 \sin(\pi P / 2) \quad ; \quad z_1 = 1/2 - 1/2 \sin(\pi(N_1 - P) / 2) \quad ; \quad z_1 = 1/2 - 1/2 \cos(\pi N_0 / 2) .$$

The key point in the analysis of series of numbers is the transition from the decimal number system to the binary number system, which allowed us to prove the Riemann hypothesis about the real root 1/2.

Then the binary number system will be the beginning of a complete series of primes in the form  $P_2 = \{0;1\}$ , and the Gaussian series of primes  $P_G = \{2, 3, 5, 7, 11, 13, 17, \dots\}$  its continuation. As a result, we get  $P = P_2 + P_G$ .

The continuation of  $P_G$  can be completely collapsed in the binary system  $P_2$ . Moreover, any series  $P \subset N \subset Z \subset Q \subset R$  can also be written in binary codes.

We have proved the famous Riemann conjecture about the real root  $1/2$  for any kind of natural numbers.

In the limit, as the power of the series increases, a simple sine wave is subtracted from the real root (for series of odd, prime and odd composite numbers) and a cosine function (for series of natural, composite and integer numbers), the constant amplitude of which is also equal to  $1/2$ , and the constant half-period of the trigonometric function is two numbers: 1 - for series of natural and composite natural numbers; 2 - for series of odd natural, prime, composite odd and even numbers. At the same time, under the functions of sine and cosine, varieties of series of natural numbers are located on the critical Riemann lines:

- a) on the zero vertical of the binary expansion of series of natural and composite numbers  $\pi$ ;
- b) on the first vertical of the binary expansion of series of odd, prime, composite odd and whole natural numbers, the value  $\pi/2$ .

In the series of numbers, the idea of the nature of pure oscillations of a sinusoidal shape, which appeared in Taylor, is respected. At the same time, we considered a set of fluctuations along the verticals in binary expansions of series of numbers. In general, the residuals after the above equations make it possible to identify additional fluctuations. As a result, the binary representation, even on the critical Riemann line, is an arbitrary oscillation, which can be represented as a superposition or the sum of several pure oscillations (superposition principle).

Then it turns out that prime and other series of numbers obey Bernoulli's law with the solution of the oscillation equation in the form of the sum of a trigonometric series, and Bernoulli argued (based on physical considerations) that such a series can represent an arbitrary function. We have confirmed this assumption of Bernoulli in this article mathematically. We have proved that the series of numbers from 0 to 1024 include the sum of a trigonometric series of 9 simple vibrations. As the power of the series increases, the number of simple waves will increase. In the general case, we have previously proved that any segment of the series of numbers can be identified by a set of asymmetric wavelets with variable amplitude and half-period of oscillation.

### Mathematics Subject Classification

00A71 · 33B10 · 41A30 · 60G10 · 62G05 · 74J30 · 83C35 · 86A32 · 92D25

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