

The Four Necessary and Sufficient Canonical Tests of the Special Theory of Relativity

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Abstract

The present paper is a continuation of our previous published work where we dispelled the misconception that the three canonical experiments mentioned by H.P. Robertson in his 1949 seminal paper, are all that is necessary and sufficient in deciding if and when an alternative theory of relativity is equivalent to Einstein's special relativity.

Keywords: special relativity, Selleri transformations, Lorentz transformations, relativistic Doppler effect, relativistic optical aberration, Fizeau experiment, Michelson-Morley experiment in non-vacuum medium, Robertson 1949 paper

1. Introduction

In our previously published paper (Sfarti, A., 2025), we have shown that one produces contradictory outcomes by inserting the Seller transforms versus the Lorentz transforms into the plane wave formula. We remind you that inserting the Lorentz (Einstein, A., 1905; Lorentz, H. A., 1904) transformations into the plane wave equation Einstein obtained through identification of the coefficients multiplying x, y, z, t respectively:

$$\omega = \omega' \gamma \left(1 + \frac{l'v}{c}\right)$$

$$l = \frac{l' - \frac{v}{c}}{1 + \frac{l'v}{c}} \quad (1.1)$$

On the other hand, inserting the Selleri transformation the plane wave equation one

obtains:

$$\omega = \omega' \gamma(v) \left(1 + \frac{l'v}{c}\right)$$

$$l = \frac{l'}{1 + \frac{l'v}{c}} \quad (1.2)$$

The Selleri derivation of the relativistic Doppler effect is correct but the relativistic aberration formula is not correct as we can see by comparison with Einstein, and, more importantly, by comparison with experiment. There is an obvious difference in the cosine for the x direction while the predictions for the y and z directions coincide between the two theories. Today, we know that Selleri transformation would pass the Ives-Stilwell experiment that tests the transverse Doppler effect (Saathoff, G. et al., 2003; Müller, H. et al., 2003a; 2003b; 2003c). The

above derivation shows that the Selleri theory would give an incorrect prediction for all optical experiments involving relativistic aberration. In addition, Selleri's transformations give the wrong answer for the speed composition formula:

$$u'_x = \frac{dx'}{dt'} = \frac{dx - vdt}{dt} = u_x - v \quad (1.3)$$

$$u'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma dt} = \frac{u_y}{\gamma} \quad (1.4)$$

The expression (1.3) above means that the Selleri theory gives the wrong prediction for the Fizeau experiment while (1.3) and (1.4) combined show that the Selleri theory gives the wrong prediction for all Michelson-Morley experiments in refractive (non-vacuum) media (Sfarti, A., 2011). According to H.P. Robertson's seminal paper (Robertson, H.P., 1949), Selleri's theory should be equivalent with the special theory of relativity since it gives the same answers for the canonical tests of special relativity, the Michelson-Morley, Kennedy-Thorndike and Ives-Stilwell experiments (Saathoff, G. et al., 2003; Müller, H. et al., 2003a; 2003b; 2003c; Sfarti, A., 2011; Sheng, S., 2024; The KM3NeT Collaboration, 2025; Jin, T., 2024; Wei, J. J., 2025; Wolf, W., 2024; Kepuladze, Z., 2025; Amram, D., 2024; Brout, D., 2022; Guerrero, M., 2025; Li, J.R., 2025). Yet, we see that the answers diverge when it comes to the tests of relativistic aberration. This brings us to the realization that the three canonical optical tests are necessary but not sufficient in establishing the equivalence between the special theory of relativity and alternative interpretations, tests of relativistic aberration need to be added to the set. Alternatively, tests of the Michelson-Morley in refractive medium should be added. This conclusion follows naturally from the fact that the Michelson-Morley experiment deals with space transformations, the Ives-Stilwell experiment is an expression of time transformation (time dilation) while the Kennedy-Thorndike experiment is an expression of both time and space transformations while aberration is an expression of angle transformation, so it is not covered by any of the three tests mentioned by Robertson and this is how he missed the issue.

2. Relationships with the Test Theories of Special Relativity

There are two main types of tests of special

relativity: the Robertson-Mansouri-Sexl (Mansouri R. & Sexl R.U., 1977) (in short, RMS) and the Standard Model Extension (Kostelecký, V. A. & Russell, N. 2011) (in short, SME). Both theories are parametrized theories of special relativity (SME including RMS by virtue of having much more parameters). For example, RMS has three parameters, α , β , δ that relate respectively to tests of time dilation, measurement of length in the direction of motion and measuring length in the direction perpendicular on the direction of motion. The combination of the three canonical experiments is necessary to obtain the complete Lorentz transformation. Michelson-Morley only tests the combination between β and δ , while Kennedy-Thorndike tests the combination between α and β . To obtain the individual values, it's necessary to measure one of these quantities directly. This was achieved by Ives-Stilwell experiment that measured α . So β can be determined using Kennedy-Thorndike, and subsequently δ using Michelson-Morley. Deviations from the two-way (round-trip) speed of light are given by:

$$\frac{c}{c'} = 1 + (\beta - \delta - 0.5) \frac{v^2}{c^2} \sin^2 \theta + (\alpha - \beta + 1) \frac{v^2}{c^2} \quad (2.1)$$

where c is the speed of light in the "preferred" frame, and c' is the speed of light measured in the moving frame at an angle θ from the direction in which the frame is moving. To verify that special relativity is correct, the expected values of the parameters are $\alpha = -0.5$, $\beta = 0.5$, $\delta = 0$ and thus $c' = c$ (Sfarti, A., 2008a; 2008b; 2014).

In the previous section we have shown that a fourth parameter, say φ , is necessary to be associated with a fourth canonical test, the one that measures angle. In other words, the current three canonical experimental tests of special relativity are necessary but not sufficient, a fourth test, relating to measuring aberration angles is necessary. An example as how the RMS theory could be extended via the addition of the extra parameter is by adding a parametrized angle transformation:

$$l = \frac{l' - \varphi \frac{v}{c}}{1 + \frac{l' v}{c}} \quad (2.2)$$

Using the notation from (2.1) the above can be re-written as:

$$\cos \theta = \frac{\cos \theta' - \frac{v}{c} \phi}{1 + \frac{v}{c} \cos \theta'} \quad (2.3)$$

For $\phi = 1$ one recovers special relativity. Note that we need that **both** the speed **and** the angle of light ray constrain **both** the parameters α, β, δ **and** ϕ . The constraint of parameter ϕ is by far the strongest due to the dependency in first order of the speed v .

3. Important Contemporary Reviews of the Selleri Relativity

An excellent paper (Capria, M., 2022) by Marco Capria does an in-depth analysis of Selleri's "Weak Relativity" concluding with the previous observations that while Selleri theory is "experimentally indistinguishable" from special relativity, its explanations of certain experiments, most notably of the Sagnac experiment, are downright incorrect, bordering on fringe, in the style of Ives or Dingle (see his paper conclusions). Like everybody before him, Capria follows the path set by H.P. Robertson (Robertson, H.P., 1949) in considering only the Michelson-Morley, Kennedy-Thorndike and Ives-Stilwell experiments as canonical experiments in determining equivalence between various theories and the special theory of relativity, thus missing the fact that experiments in the class of Fizeau, stellar aberration or Michelson-Morley in non-vacuum media provide the "separation" between Selleri's and Einstein's relativity.

4. Conclusion

We have discovered that the three canonical optical tests are necessary but not sufficient in establishing the equivalence between the special theory of relativity and alternative interpretations, the tests of relativistic aberration need to be added to the list.

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