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# Importance of Total Coupon in Utility Maximization: A Sensitivity Analysis

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**Abstract**

This study takes an attempt to discuss utility maximization policies. The property of a commodity that enables it to satisfy human wants is called utility. The sensitivity analysis is included in the operation to show optimal policy of an organization. This study deals with four commodities and two constraints, such as budget constraint, and coupon constraint. The economic predictions of future production are necessary for the sustainable production. In the study Lagrange multipliers technique is applied to operate 6×6 bordered Hessian and 6×10 Jacobian appropriately. The sensitivity analysis between commodity and total budget are discussed with detail mathematical analysis.

**Keywords:** Lagrange multipliers, sensitivity analysis, total coupon, utility maximization

## 1. Introduction

In the 21st century, global economy becomes challenging, and mathematical economics plays a crucial role for the development of economic structures; especially for developing mathematical models (Carter, 2001). In mathematical economics, utility is an important portion for the organizations, and production of commodities depends on the satisfaction of the consumers (Fishburn, 1970). In the society, consumers always expect to gain maximum satisfaction from the consumption of their purchased goods (Stigler, 1950). Utility shows that individuals in the societies seek to obtain the highest level of satisfaction from their purchasing goods (Kirsh, 2017).

The concept of utility was developed in the late 18th century by the English moral philosopher, jurist, and social reformer Jeremy Bentham (1748-1832) and English philosopher, political economist, Member of Parliament (MP) and civil servant John Stuart Mill (1806-1873) (Bentham, 1780). Since economy sees its benefits and also sees welfare of human; therefore, utility maximization is a blessing both for humankind and the organization (Eaton & Lipsey, 1975). The organizations always try to maximize their profits; and they must be conscious about the utility maximization of the consumers (Dixit, 1990).

The method of Lagrange multipliers is a very useful and powerful technique in multivariate calculus, which transforms a constrained problem to a higher dimensional unconstrained problem (Islam et al., 2011; Mohajan, 2022). In this study, we consider a utility maximization problem subject to two constraints; namely, budget constraint and coupon constraint. We have stressed on the sensitivity analysis between commodity and total coupon (Mohajan & Mohajan, 2022c,d). We have used both 6×6 bordered Hessian and 6×10 Jacobian for developing utility maximization (Mohajan & Mohajan, 2022a,e).

## 2. Literature Review

The literature review section is an introductory region of research, which shows the works of previous researchers in the same field within the existing knowledge (Polit & Hungler, 2013). Two US scholars, mathematician John V. Baxley and economist John C. Moorhouse have discussed the utility maximization method through the mathematical formulation (Baxley & Moorhouse, 1984). Eminent mathematician Jamal Nazrul Islam and his coauthors have discussed utility maximization and some other optimization problems. In these studies, they have considered reasonable interpretation of the Lagrange multipliers (Islam et al., 2010, 2011). Pahlaj Moolio and his coauthors have also taken attempts to develop and solve optimization problems (Moolio et al., 2009).

Jannatul Ferdous and Haradhan Kumar Mohajan have taken attempts to calculate profit maximization problems (Ferdous & Mohajan, 2022). Devajit Mohajan and Haradhan Kumar Mohajan have discussed on profit maximization by using four variable inputs, such as capital, labor, principal raw materials, and other inputs in an industry (Mohajan & Mohajan, 2022a). Haradhan Kumar Mohajan has also used three inputs, such as capital, labor and other inputs for the sustainable production of a factory of Bangladesh (Mohajan, 2021b).

## 3. Methodology of the Study

Researchers often write a methodology section with details of the research analysis. It is considered as a way of explaining how a research work is carried out. Therefore, it is the organized and meaningful procedural works that follow scientific methods efficiently (Kothari, 2008). Research methodology shows the ways to the researchers for organizing, planning, designing and conducting a good research (Legesse, 2014). It helps to identify research areas and projects within these areas (Blessing et al., 1998). In this article we have used both qualitative and quantitative research procedures (Mohajan, 2018b, 2020). In this study we have considered four commodity variables: $Y_1$, $Y_2$, $Y_3$, and $Y_4$; and two Lagrange multipliers $\lambda_1$ and $\lambda_2$. We have used 6×6 bordered Hessian, and later 6×10 Jacobian to discuss sensitivity analysis (Mohajan & Mohajan, 2022e).

Both reliability and validity are powerful tools for doing a seminal research work. In this article we have tried to maintain them as far as possible (Mohajan, 2017b). To prepare this article we have depended on the optimization related mathematical secondary data sources (Mohajan, 2011, 2018a; Islam et al., 2012a, b). We have consulted with the published research papers, books of well-known authors, handbooks of expert researchers, research reports, internet, websites, etc. (Mohajan, 2012; Chowdhury et al., 2013; Mohajan & Mohajan, 2022f).

## 4. Objective of the Study

Main objective of this study is to discuss the usefulness of total coupon during the utility maximization, where sensitivity analysis is investigated. The other subsidiary objectives are as follows:

- to use the bordered Hessian and Jacobian for optimization,
- to explain the results properly, and
- to show the mathematical calculations in some details.

## 5. An Economic Model

In this study we consider four commodities: $Y_1$, $Y_2$, $Y_3$, and $Y_4$ (Moolio et al., 2009; Mohajan & Mohajan, 2022b). Let a consumer wants to purchase $y_1$, $y_2$, $y_3$, and $y_4$ amounts from these four commodities $Y_1$, $Y_2$, $Y_3$, and $Y_4$, respectively. The utility function can be written as (Roy et al., 2021; Mohajan & Mohajan, 2022b),

$$u(y_1, y_2, y_3, y_4) = y_1 y_2 y_3 y_4 \tag{1}$$

The budget constraint of the consumers is,

$$B = p_1 y_1 + p_2 y_2 + p_3 y_3 + p_4 y_4 \tag{2}$$

where $p_1$, $p_2$, $p_3$, and $p_4$ are the prices of per unit of commodities $y_1$, $y_2$, $y_3$, and $y_4$, respectively. Now the coupon constraint is,

$$M = m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 \tag{3}$$

where $m_1$, $m_2$, $m_3$, and $m_4$ are the coupons necessary to purchase a unit of commodity of $y_1$, $y_2$, $y_3$, and $y_4$, respectively.

Using (1), (2), and (3) we can express Lagrangian function $V(y_1, y_2, y_3, y_4, \lambda_1, \lambda_2)$ as (Baxley & Moorhouse, 1984; Ferdous & Mohajan, 2022; Mohajan & Mohajan, 2022b),

$$V(y_1, y_2, y_3, y_4, \lambda_1, \lambda_2) = y_1 y_2 y_3 y_4 + \lambda_1 (B - p_1 y_1 - p_2 y_2 - p_3 y_3 - p_4 y_4) + \lambda_2 (M - m_1 y_1 - m_2 y_2 - m_3 y_3 - m_4 y_4) \quad (4)$$

Lagrangian function (4) is a 6-dimensional unconstrained problem that maximizes utility functions; where $\lambda_1$ and $\lambda_2$ are two Lagrange multipliers.

Now we consider the bordered Hessian (Mohajan, 2021a; Mohajan & Mohajan, 2022c),

$$|H| = \begin{vmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -M_1 & -M_2 & -M_3 & -M_4 \\ -B_1 & -M_1 & V_{11} & V_{12} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{21} & V_{22} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{31} & V_{32} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{41} & V_{42} & V_{43} & V_{44} \end{vmatrix} \quad (5)$$

Now taking first and second order and cross-partial derivatives in (4) we obtain (Islam et al. 2009a, b; Mohajan & Mohajan, 2022d);

$$B_1 = p_1,\ B_2 = p_2, B_3 = p_3, B_4 = p_4.$$

$$M_1 = m_1, \quad M_2 = m_2, \quad M_3 = m_3, \quad M_4 = m_4 \quad (6)$$

$$V_{11} = 0,\ V_{12} = V_{21} = y_3 y_4,\ V_{13} = V_{31} = y_2 y_4,$$

$$V_{14} = V_{41} = y_2 y_3,\ V_{22} = 0,\ V_{23} = V_{32} = y_1 y_4,$$

$$V_{24} = V_{42} = y_1 y_3, \quad V_{33} = 0,\ V_{34} = V_{43} = y_1 y_2,\ V_{44} = 0 \quad (7)$$

We use $p_3 = p_1$ and $p_4 = p_2$, i.e., a pair of prices are same, and $m_3 = m_1$ and $m_4 = m_2$, i.e., a pair of coupon numbers are same. Now we consider that in the expansion of (5) every term contains $p_1 p_2 m_1 m_2$, then from (5) we can derive (Mohajan & Mohajan, 2022e);

$$|H| = -2 p_1 p_2 m_1 m_2 < 0 \quad (8)$$

For $y_1$, $y_2$, $y_3$, $y_4$, $\lambda_1$, and $\lambda_2$ in terms of $p_1$, $p_2$, $p_3$, $p_4$, $m_1$, $m_2$, $m_3$, $m_4$, *B*, and *M* we can calculate sixty partial derivatives, such as $\frac{\partial \lambda_1}{\partial p_1}$, $\frac{\partial \lambda_2}{\partial p_1}$, …, $\frac{\partial \lambda_1}{\partial m_1}$, $\frac{\partial \lambda_2}{\partial m_1}$, …, $\frac{\partial y_1}{\partial p_1}$, …, $\frac{\partial y_1}{\partial m_1}$, …, $\frac{\partial \lambda_1}{\partial B}$, …, $\frac{\partial \lambda_1}{\partial M}$, etc., (Islam et al., 2010; Mohajan, 2021c). Now we consider 6×6 Hessian and Jacobian matrix as (Mohajan & Mohajan, 2022a; Mohajan, 2021b)

$$J = H = \begin{bmatrix} 0 & 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ 0 & 0 & -M_1 & -M_2 & -M_3 & -M_4 \\ -B_1 & -M_1 & V_{11} & V_{12} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{21} & V_{22} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{31} & V_{32} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{41} & V_{42} & V_{43} & V_{44} \end{bmatrix} \quad (9)$$

which is non-singular at the optimum point $\left(y_1^*, y_2^*, y_3^*, y_4^*, \lambda_1^*, \lambda_2^*\right)$. Since the second order conditions have been satisfied, so the determinant of (9) does not vanish at the optimum, i.e., $|J| = |H|$ ; and we apply the implicit-function theorem. We have total 16 variables in our study, such as $\lambda_1, \lambda_2$ , $y_1, y_2, y_3, y_4$ $p_1, p_2, p_3, p_4$ , $m_1, m_2, m_3, m_4$ , $B$ , and $M$ . By the implicit function theorem, we can write (Moolio et al., 2009; Islam et al., 2011; Mohajan, 2021c);

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \mathbf{G}\left(p_1, p_2, p_3, p_4, m_1, m_2, m_3, m_4, B, M\right) \quad (10)$$

Now the 6×10 Jacobian matrix for $\mathbf{G}$, regarded as $J_G$ is given by (Mohajan et al., 2013; Mohajan, 2021a),

$$J_G = \begin{bmatrix} \frac{\partial \lambda_1}{\partial p_1} & \frac{\partial \lambda_1}{\partial p_2} & \frac{\partial \lambda_1}{\partial p_3} & \frac{\partial \lambda_1}{\partial p_4} & \frac{\partial \lambda_1}{\partial m_1} & \frac{\partial \lambda_1}{\partial m_2} & \frac{\partial \lambda_1}{\partial m_3} & \frac{\partial \lambda_1}{\partial m_4} & \frac{\partial \lambda_1}{\partial B} & \frac{\partial \lambda_1}{\partial M} \\ \frac{\partial \lambda_2}{\partial p_1} & \frac{\partial \lambda_2}{\partial p_2} & \frac{\partial \lambda_2}{\partial p_3} & \frac{\partial \lambda_2}{\partial p_4} & \frac{\partial \lambda_2}{\partial m_1} & \frac{\partial \lambda_2}{\partial m_2} & \frac{\partial \lambda_2}{\partial m_3} & \frac{\partial \lambda_2}{\partial m_4} & \frac{\partial \lambda_2}{\partial B} & \frac{\partial \lambda_2}{\partial M} \\ \frac{\partial y_1}{\partial p_1} & \frac{\partial y_1}{\partial p_2} & \frac{\partial y_1}{\partial p_3} & \frac{\partial y_1}{\partial p_4} & \frac{\partial y_1}{\partial m_1} & \frac{\partial y_1}{\partial m_2} & \frac{\partial y_1}{\partial m_3} & \frac{\partial y_1}{\partial m_4} & \frac{\partial y_1}{\partial B} & \frac{\partial y_1}{\partial M} \\ \frac{\partial y_2}{\partial p_1} & \frac{\partial y_2}{\partial p_2} & \frac{\partial y_2}{\partial p_3} & \frac{\partial y_2}{\partial p_4} & \frac{\partial y_2}{\partial m_1} & \frac{\partial y_2}{\partial m_2} & \frac{\partial y_2}{\partial m_3} & \frac{\partial y_2}{\partial m_4} & \frac{\partial y_2}{\partial B} & \frac{\partial y_2}{\partial M} \\ \frac{\partial y_3}{\partial p_1} & \frac{\partial y_3}{\partial p_2} & \frac{\partial y_3}{\partial p_3} & \frac{\partial y_3}{\partial p_4} & \frac{\partial y_3}{\partial m_1} & \frac{\partial y_3}{\partial m_2} & \frac{\partial y_3}{\partial m_3} & \frac{\partial y_3}{\partial m_4} & \frac{\partial y_3}{\partial B} & \frac{\partial y_3}{\partial M} \\ \frac{\partial y_4}{\partial p_1} & \frac{\partial y_4}{\partial p_2} & \frac{\partial y_4}{\partial p_3} & \frac{\partial y_4}{\partial p_4} & \frac{\partial y_4}{\partial m_1} & \frac{\partial y_4}{\partial m_2} & \frac{\partial y_4}{\partial m_3} & \frac{\partial y_4}{\partial m_4} & \frac{\partial y_4}{\partial B} & \frac{\partial y_4}{\partial M} \end{bmatrix} \quad (11)$$

$$= -J^{-1} \begin{bmatrix} -y_1 & -y_2 & -y_3 & -y_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -y_1 & -y_2 & -y_3 & -y_4 & 0 & 1 \\ -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 \end{bmatrix} \quad (12)$$

The inverse of Jacobian is, $J^{-1} = \frac{1}{|J|} C^T$ , where $C = \left(C_{ij}\right)$, the matrix of cofactors of $J$ , and $T$ indicates transpose, then (12) becomes (Mohajan, 2017a; Islam et al., 2009b, 2011),

$$J_G = -\frac{1}{|J|} C^T \begin{bmatrix} -y_1 & -y_2 & -y_3 & -y_4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -y_1 & -y_2 & -y_3 & -y_4 & 0 & 1 \\ -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & -\lambda_2 & 0 & 0 \end{bmatrix} \quad (13)$$

Now 6×6 transpose matrix $C^T$ can be represented by,

$$C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} & C_{61} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} & C_{62} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} & C_{63} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} & C_{64} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{65} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \quad (14)$$

Using (14) we can write (11) as a 6×10 Jacobian matrix (Mohajan & Mohajan, 2022b);

$$J_G = -\frac{1}{|J|} \left[ \begin{matrix} -y_1C_{11}-\lambda_1C_{31} & -y_2C_{11}-\lambda_1C_{41} & -y_3C_{11}-\lambda_1C_{51} & -y_4C_{11}-\lambda_1C_{61} & -y_1C_{21}-\lambda_2C_{31} \\ -y_1C_{12}-\lambda_1C_{32} & -y_2C_{12}-\lambda_1C_{42} & -y_3C_{12}-\lambda_1C_{52} & -y_4C_{12}-\lambda_1C_{62} & -y_1C_{22}-\lambda_2C_{32} \\ -y_1C_{13}-\lambda_1C_{33} & -y_2C_{13}-\lambda_1C_{43} & -y_3C_{13}-\lambda_1C_{53} & -y_4C_{13}-\lambda_1C_{63} & -y_1C_{23}-\lambda_2C_{33} \\ -y_1C_{14}-\lambda_1C_{34} & -y_2C_{14}-\lambda_1C_{44} & -y_3C_{14}-\lambda_1C_{54} & -y_4C_{14}-\lambda_1C_{64} & -y_1C_{24}-\lambda_2C_{34} \\ -y_1C_{15}-\lambda_1C_{35} & -y_2C_{15}-\lambda_1C_{45} & -y_3C_{15}-\lambda_1C_{55} & -y_4C_{15}-\lambda_1C_{65} & -y_1C_{25}-\lambda_2C_{35} \\ -y_1C_{16}-\lambda_1C_{36} & -y_2C_{16}-\lambda_1C_{46} & -y_3C_{16}-\lambda_1C_{56} & -y_4C_{16}-\lambda_1C_{66} & -y_1C_{26}-\lambda_2C_{36} \end{matrix} \right.$$

$$\left. \begin{matrix} -y_2C_{21}-\lambda_2C_{41} & -y_3C_{21}-\lambda_2C_{51} & -y_4C_{21}-\lambda_2C_{61} & C_{11} & C_{21} \\ -y_2C_{22}-\lambda_2C_{42} & -y_3C_{32}-\lambda_2C_{52} & -y_4C_{22}-\lambda_2C_{62} & C_{12} & C_{22} \\ -y_2C_{23}-\lambda_2C_{43} & -y_3C_{33}-\lambda_2C_{53} & -y_4C_{23}-\lambda_2C_{63} & C_{13} & C_{23} \\ -y_2C_{24}-\lambda_2C_{44} & -y_3C_{34}-\lambda_2C_{54} & -y_4C_{24}-\lambda_2C_{64} & C_{14} & C_{24} \\ -y_2C_{25}-\lambda_2C_{45} & -y_3C_{35}-\lambda_2C_{55} & -y_4C_{25}-\lambda_2C_{65} & C_{15} & C_{25} \\ -y_2C_{26}-\lambda_2C_{46} & -y_3C_{36}-\lambda_2C_{56} & -y_4C_{26}-\lambda_2C_{66} & C_{16} & C_{26} \end{matrix} \right] \quad (15)$$

Now we analyze the nature of consumption of commodity $y_1$ when total coupon *M* increases. Taking $T_{3(10)}$, (i.e., term of 3rd row and 10th column) from both sides of (15) we get (Islam et al., 2011; Mohajan & Mohajan, 2022e),

$$\frac{\partial y_1}{\partial M} = -\frac{1}{|J|}[C_{23}]$$

$$= -\frac{1}{|J|} \text{Cofactor of } C_{23}$$

$$= \frac{1}{|J|} \begin{vmatrix} 0 & 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & -M_1 & V_{12} & V_{13} & V_{14} \\ -B_2 & -M_2 & V_{22} & V_{23} & V_{24} \\ -B_3 & -M_3 & V_{32} & V_{33} & V_{34} \\ -B_4 & -M_4 & V_{42} & V_{43} & V_{44} \end{vmatrix}$$

$$=\frac{1}{|J|}\left\{-B_2\begin{vmatrix}-B_1 & -M_1 & V_{13} & V_{14}\\ -B_2 & -M_2 & V_{23} & V_{24}\\ -B_3 & -M_3 & V_{33} & V_{34}\\ -B_4 & -M_4 & V_{43} & V_{44}\end{vmatrix}+B_3\begin{vmatrix}-B_1 & -M_1 & V_{12} & V_{14}\\ -B_2 & -M_2 & V_{22} & V_{24}\\ -B_3 & -M_3 & V_{32} & V_{34}\\ -B_4 & -M_4 & V_{42} & V_{44}\end{vmatrix}-B_4\begin{vmatrix}-B_1 & -M_1 & V_{12} & V_{13}\\ -B_2 & -M_2 & V_{22} & V_{23}\\ -B_3 & -M_3 & V_{32} & V_{33}\\ -B_4 & -M_4 & V_{42} & V_{43}\end{vmatrix}\right\}$$

$$=\frac{1}{|J|}\left[-B_2\left\{-B_1\begin{vmatrix}-M_2 & V_{23} & V_{24}\\ -M_3 & V_{33} & V_{34}\\ -M_4 & V_{43} & V_{44}\end{vmatrix}+M_1\begin{vmatrix}-B_2 & V_{23} & V_{24}\\ -B_3 & V_{33} & V_{34}\\ -B_4 & V_{43} & V_{44}\end{vmatrix}+V_{13}\begin{vmatrix}-B_2 & -M_2 & V_{24}\\ -B_3 & -M_3 & V_{34}\\ -B_4 & -M_4 & V_{44}\end{vmatrix}\right.\right.$$

$$\left.-V_{14}\begin{vmatrix}-B_2 & -M_2 & V_{23}\\ -B_3 & -M_3 & V_{33}\\ -B_4 & -M_4 & V_{43}\end{vmatrix}\right\}+B_3\left\{-B_1\begin{vmatrix}-M_2 & V_{22} & V_{24}\\ -M_3 & V_{32} & V_{34}\\ -M_4 & V_{42} & V_{44}\end{vmatrix}+M_1\begin{vmatrix}-B_2 & V_{22} & V_{24}\\ -B_3 & V_{32} & V_{34}\\ -B_4 & V_{42} & V_{44}\end{vmatrix}+V_{12}\begin{vmatrix}-B_2 & -M_2 & V_{24}\\ -B_3 & -M_3 & V_{34}\\ -B_4 & -M_4 & V_{44}\end{vmatrix}\right.$$

$$\left.-V_{14}\begin{vmatrix}-B_2 & -M_2 & V_{22}\\ -B_3 & -M_3 & V_{32}\\ -B_4 & -M_4 & V_{42}\end{vmatrix}\right\}-B_4\left\{-B_1\begin{vmatrix}-M_2 & V_{22} & V_{23}\\ -M_3 & V_{32} & V_{33}\\ -M_4 & V_{42} & V_{43}\end{vmatrix}+M_1\begin{vmatrix}-B_2 & V_{22} & V_{23}\\ -B_3 & V_{32} & V_{33}\\ -B_4 & V_{42} & V_{43}\end{vmatrix}\right.$$

$$\left.\left.+V_{12}\begin{vmatrix}-B_2 & -M_2 & V_{23}\\ -B_3 & -M_3 & V_{33}\\ -B_4 & -M_4 & V_{43}\end{vmatrix}-V_{13}\begin{vmatrix}-B_2 & -M_2 & V_{22}\\ -B_3 & -M_3 & V_{32}\\ -B_4 & -M_4 & V_{42}\end{vmatrix}\right\}\right]$$

$$=\frac{1}{|J|}\left[B_1B_2M_2V_{34}^2-B_1B_2M_4V_{23}V_{34}-B_1B_2M_3V_{24}V_{34}-B_2^2M_1V_{34}^2+B_2B_4M_1V_{23}V_{34}+B_2B_3M_1V_{24}V_{34}\right.$$

$$+B_2^2M_4V_{13}V_{34}-B_2B_4M_2V_{13}V_{34}-B_2B_3M_4V_{13}V_{24}+B_2B_4M_3V_{13}V_{24}-B_2^2M_3V_{14}V_{34}-B_2B_3M_2V_{14}V_{34}$$

$$+B_2B_3M_4V_{14}V_{23}-B_2B_4M_3V_{14}V_{23}-B_1B_3M_2V_{24}V_{34}+B_1B_3M_3V_{24}^2-B_1B_3M_4V_{23}V_{24}+B_2B_3M_1V_{24}V_{34}$$

$$-B_3^2M_1V_{24}^2+B_3B_4M_1V_{23}V_{24}-B_2B_3M_4V_{12}V_{34}-B_3B_4M_3V_{12}V_{24}+B_3^2M_4V_{12}V_{24}-B_3B_4M_3V_{12}V_{24}$$

$$-B_2B_3M_3V_{14}V_{24}+B_2B_3M_4V_{14}V_{23}+B_3^2M_2V_{14}V_{24}-B_3B_4M_2V_{14}V_{23}-B_1B_4M_2V_{23}V_{34}-B_1B_4M_3V_{23}V_{24}$$

$$+B_1B_4M_4V_{23}^2+B_2B_4M_1V_{23}V_{34}+B_4^2M_1V_{23}^2-B_2B_4M_3V_{12}V_{34}-B_3B_4M_2V_{12}V_{34}-B_3B_4M_4V_{12}V_{23}$$

$$\left.+B_4^2M_3V_{12}V_{23}+B_2B_4M_3V_{13}V_{24}-B_2B_4M_4V_{13}V_{23}-B_3B_4M_2V_{13}V_{24}+B_4^2M_2V_{13}V_{23}\right]$$

$$=\frac{1}{|J|}\left[p_1p_2m_2y_1^2y_2^2-p_1p_2m_4y_1^2y_2y_4-p_1p_2m_3y_1^2y_2y_3-p_2^2m_1y_1^2y_2^2+p_2p_4m_1y_1^2y_2y_4\right.$$

$$+p_2p_3m_1y_1^2y_2y_3+p_2^2m_4y_1y_2^2y_4-p_2p_4m_2y_1y_2^2y_4-p_2p_3m_4y_1y_2y_3y_4+p_2p_4m_3y_1y_2y_3y_4$$

$$-p_2^2m_3y_1y_2^2y_3-p_2p_4m_2y_1y_2^2y_3+p_2p_3m_4y_1y_2y_3y_4-p_2p_4m_3y_1y_2y_3y_4-p_1p_3m_3y_1^2y_2y_3$$

$$+p_1p_3m_3y_1^2y_3^2-p_1p_3m_4y_1^2y_3y_4+p_2p_3m_1y_1^2y_2y_3-p_3^2m_1y_1^2y_3^2+p_3p_4m_1y_1^2y_3y_4$$

$$-p_2p_3m_4y_1y_2y_3y_4 \quad -p_3p_4m_3y_1y_3^2y_4 \quad +p_3^2m_4y_1y_3^2y_4 \quad -p_3p_4m_3y_1y_3^2y_4 \quad -p_2p_3m_3y_1y_2y_3^2$$

$$+p_2p_3m_4y_1y_2y_3y_4 \quad +p_3^2m_2y_1y_2y_3^2 \quad -p_3p_4m_2y_1y_2y_3y_4 \quad -p_1p_4m_2y_1^2y_2y_4 \quad -p_1p_4m_3y_1^2y_3y_4$$

$$+p_1p_4m_3y_1^2y_4^2 \quad -p_2p_4m_1y_1^2y_2y_4 \quad +p_3p_4m_1y_1^2y_3y_4 \quad +p_4^2m_1y_1^2y_4^2 \quad -p_2p_4m_3y_1y_2y_3y_4$$

$$-p_3p_4m_2y_1y_2y_3y_4 \quad -p_3p_4m_4y_1y_3y_4^2 \quad +p_4^2m_3y_1y_3y_4^2 \quad +p_2p_4m_3y_1y_2y_3y_4 \quad -p_2p_4m_4y_1y_2y_4^2$$

$$-p_3p_4m_2y_1y_2y_3y_4 \qquad +p_4^2m_2y_1y_2y_4^2\Big]$$

(16)

$$\frac{\partial y_1}{\partial M}=\frac{1}{|J|}\Big[\big(p_1p_2m_2-p_2^2m_1\big)y_1^2y_2^2+\big(p_1p_3m_3-p_3^2m_1\big)y_1^2y_3^2+\big(p_1p_4m_3+p_4^2m_1\big)y_1^2y_4^2$$
$$+\big(-p_1p_2m_4-p_1p_4y_2\big)y_1^2y_2y_4+\big(p_2p_3m_1-p_1p_2m_3-p_1p_3m_3+p_2p_3m_1\big)y_1^2y_2y_3$$
$$+\big(p_2^2m_4-p_2p_4m_2\big)y_1y_2^2y_4+\big(-p_2^2m_3-p_2p_4m_2\big)y_1y_2^2y_3+\big(p_3^2m_2-p_2p_3m_3\big)y_1y_2y_3^2$$
$$+\big(p_3^2m_4-p_3p_4m_3-p_3p_4m_3\big)y_1y_3^2y_4+\big(2p_3p_4m_1-p_1p_3m_4-p_1p_4m_3\big)y_1^2y_3y_4$$
$$-3p_3p_4m_2y_1y_2y_3y_4+\big(p_4^2m_2-p_2p_4m_4\big)y_1y_2y_4^2\Big] \tag{17}$$

Using $y_1=y_2=y_3=y_4=1$ in (17) we get,

$$\frac{\partial y_1}{\partial M}=\frac{1}{|J|}\Big[\big(p_1p_2m_2+p_4^2m_1+2p_2p_3m_1+p_2^2m_4+p_3^2m_2+2p_3p_4m_1+p_4^2m_2\big)$$
$$-\big(p_3^2m_1+p_2^2m_1+3p_3p_4m_2+p_1p_2m_4+p_1p_4m_2+p_1p_2m_3+2p_2p_4m_2+p_2^2m_3$$
$$+p_2p_3m_3+p_3^2m_4+2p_3p_4m_3$$
$$+p_1p_3m_4+p_2p_4m_4\big)\Big] \tag{18}$$

We consider, $p_1=p_2=p_3=p_4=p$, and $m_1=m_2=m_3=m_4=m$, then $|J|=-2p^2m^2$; and (18) gives,

$$\frac{\partial y_1}{\partial M}=-\frac{4}{m}<0 \tag{19}$$

where $m>0$.

Inequality (19) indicates that when the quantity of total coupon to purchase commodity $y_1$ increases, the level of consumption of the commodity $y_1$ decreases. This situation indicates that commodity $y_1$ is an inferior good.

We consider $p_3=p_1$, and $p_4=p_2$; and $m_3=m_1$, and $m_4=m_2$, then $|J|=|H|=-2p_1p_2m_1m_2$, and from (16) we get,

$$\frac{\partial m_1}{\partial M}=\frac{p_1^2 m_1+p_2^2 m_1+4p_1p_2m_2+p_1^2m_2+p_2^2m_2}{2p_1p_2m_1m_2}>0 \quad (20)$$

where $p_1, p_2, m_1, m_2 > 0$ .

Inequality (20) indicates that if the total coupon of individual/community increases, the level of consumption of commodity $y_1$ will also increases. We believe that commodity $y_1$ is not an inferior good; it may be a superior good, and it has no supplementary goods (Islam et al., 2010; Mohajan, 2021b; Mohajan & Mohajan, 2022c).

Now we analyze the nature of consumption of commodity $y_2$ when the total coupon *M* increases. Taking $T_{4(10)}$, (i.e., term of 4th row and 10th column) from both sides of (11) and (15) we get (Islam et al., 2010; Mohajan & Mohajan, 2022c,e),

$$\frac{\partial y_2}{\partial M}=-\frac{1}{|J|}[C_{24}]$$

$$=-\frac{1}{|J|}\text{Cofactor of } C_{24}$$

$$=-\frac{1}{|J|}\begin{vmatrix}0&0&-B_1&-B_3&-B_4\\-B_1&-M_1&V_{11}&V_{13}&V_{14}\\-B_2&-M_2&V_{21}&V_{23}&V_{24}\\-B_3&-M_3&V_{31}&V_{33}&V_{34}\\-B_4&-M_4&V_{41}&V_{43}&V_{44}\end{vmatrix}$$

$$=-\frac{1}{|J|}\left\{-B_1\begin{vmatrix}-B_1&-M_1&V_{13}&V_{14}\\-B_2&-M_2&V_{23}&V_{24}\\-B_3&-M_3&V_{33}&V_{34}\\-B_4&-M_4&V_{43}&V_{44}\end{vmatrix}+B_3\begin{vmatrix}-B_1&-M_1&V_{11}&V_{14}\\-B_2&-M_2&V_{21}&V_{24}\\-B_3&-M_3&V_{31}&V_{34}\\-B_4&-M_4&V_{41}&V_{44}\end{vmatrix}-B_4\begin{vmatrix}-B_1&-M_1&V_{11}&V_{13}\\-B_2&-M_2&V_{21}&V_{23}\\-B_3&-M_3&V_{31}&V_{33}\\-B_4&-M_4&V_{41}&V_{43}\end{vmatrix}\right\}$$

$$=-\frac{1}{|J|}\Bigg[-B_1\left\{-B_1\begin{vmatrix}-M_2&V_{23}&V_{24}\\-M_3&V_{33}&V_{34}\\-M_4&V_{43}&V_{44}\end{vmatrix}+M_1\begin{vmatrix}-B_2&V_{23}&V_{24}\\-B_3&V_{33}&V_{34}\\-B_4&V_{43}&V_{44}\end{vmatrix}+V_{13}\begin{vmatrix}-B_2&-M_2&V_{24}\\-B_3&-M_3&V_{34}\\-B_4&-M_4&V_{44}\end{vmatrix}\right.$$

$$\left.-V_{14}\begin{vmatrix}-B_2&-M_2&V_{23}\\-B_3&-M_3&V_{33}\\-B_4&-M_4&V_{43}\end{vmatrix}\right\}+B_3\left\{-B_1\begin{vmatrix}-M_2&V_{21}&V_{24}\\-M_3&V_{31}&V_{34}\\-M_4&V_{41}&V_{44}\end{vmatrix}+M_1\begin{vmatrix}-B_2&V_{21}&V_{24}\\-B_3&V_{31}&V_{34}\\-B_4&V_{41}&V_{44}\end{vmatrix}\right.$$

$$\left.-V_{14}\begin{vmatrix}-B_2&-M_2&V_{21}\\-B_3&-M_3&V_{31}\\-B_4&-M_4&V_{41}\end{vmatrix}\right\}-B_4\left\{-B_1\begin{vmatrix}-M_2&V_{21}&V_{23}\\-M_3&V_{31}&V_{33}\\-M_4&V_{41}&V_{43}\end{vmatrix}+M_1\begin{vmatrix}-B_2&V_{21}&V_{23}\\-B_3&V_{31}&V_{33}\\-B_4&V_{41}&V_{43}\end{vmatrix}-V_{13}\begin{vmatrix}-B_2&-M_2&V_{21}\\-B_3&-M_3&V_{31}\\-B_4&-M_4&V_{41}\end{vmatrix}\right\}$$

$$= -\frac{1}{|J|}\Big[B_1^2M_2V_{34}^2 \quad -B_1^2M_4V_{23}V_{34} \quad -B_1^2M_3V_{24}V_{34} \quad -B_1B_2M_1V_{34}^2 \quad +B_1B_4M_1V_{23}V_{34} \quad +B_1B_3M_1V_{23}V_{34}$$

$$-B_1B_2M_4V_{13}V_{34} \quad -B_1B_4M_2V_{13}V_{34} \quad +B_1B_3M_4V_{13}V_{24} \quad -B_1B_4M_3V_{13}V_{24} \quad +B_1B_2M_3V_{14}V_{34}$$

$$-B_1B_3M_2V_{14}V_{34} \quad +B_1B_3M_4V_{14}V_{23} \quad +B_1B_4M_3V_{14}V_{23} \quad -B_1B_3M_2V_{14}V_{34} \quad +B_1B_3M_4V_{12}V_{34}$$

$$+B_1B_3M_3V_{14}V_{24} \quad -B_1B_3M_4V_{13}V_{24} \quad +B_2B_3M_1V_{14}V_{34} \quad -B_3B_4M_1V_{12}V_{34} \quad -B_3^2M_1V_{14}V_{24} \quad -B_1B_4M_1V_{13}V_{24}$$

$$-B_2B_3M_3V_{14}^2 \quad +B_2B_3M_4V_{13}V_{14} \quad +B_3^2M_2V_{14}^2 \quad -B_3B_4M_2V_{13}V_{14} \quad -B_1B_4M_2V_{13}V_{34} \quad +B_1B_4M_3V_{12}V_{34}$$

$$-B_1B_4M_3V_{14}V_{23} \quad +B_1B_4M_4V_{13}V_{23} \quad +B_2B_4M_1V_{13}V_{34} \quad -B_3B_4M_1V_{12}V_{34} \quad +B_3B_4M_1V_{14}V_{23} \quad -B_4^2M_1V_{13}V_{23}$$

$$+B_2B_4M_3V_{13}V_{14} - B_2B_4M_4V_{13}^2 - B_3B_4M_2V_{13}V_{14} + B_4^2M_2V_{13}^2 + B_3B_4M_4V_{12}V_{13} - B_4^2M_3V_{12}V_{13}\Big]$$

$$= -\frac{1}{|J|}\Big[p_1^2m_2y_1^2y_2^2 \quad -p_1^2m_4y_1^2y_2y_4 \quad -p_1p_2m_1y_1^2y_2^2 \quad +p_1p_4m_1y_1^2y_2y_4 \quad +p_1p_3m_1y_1^2y_2y_4$$

$$-p_1p_2m_4y_1y_2^2y_4 \quad -p_1p_4m_2y_1y_2^2y_4 \quad +p_1p_3m_4y_1^2y_2y_3 \quad -p_1p_4m_3y_1y_2y_3y_4 \quad +p_1p_2m_3y_1y_2^2y_3$$

$$+p_1p_3k_2y_1y_2^2y_3 \quad +p_1p_3m_4y_1y_2y_3y_4 \quad -p_1p_3m_2y_1y_2^2y_3 \quad +p_1p_4m_3y_1y_2y_3y_4 \quad +p_1p_3m_4y_1y_2y_3y_4$$

$$+p_1p_3m_3y_1y_2y_3^2 \quad -p_1p_3m_4y_1y_2y_3y_4 \quad +p_2p_3m_1y_1y_2^2y_3 \quad -p_3p_4m_1y_1y_2y_3y_4 \quad -p_3^2m_1y_1y_2y_3^2$$

$$-p_1p_4m_1y_1y_2y_3y_4 \quad -p_2p_3m_3y_2^2y_3^2 \quad +p_2p_3m_4y_2^2y_3y_4 \quad +p_3^2m_2y_2^2y_3^2 \quad -p_3p_4m_2y_2^2y_3y_4$$

$$-p_1p_4m_2y_1y_2y_3y_4 \quad +p_1p_4m_3y_1y_2y_3y_4 \quad -p_1p_4m_3y_1y_2y_3y_4 \quad +p_1p_4m_4y_1y_2y_4^2 \quad +p_2p_4m_1y_1y_2^2y_4$$

$$-p_3p_4m_1y_1y_2y_3y_4 \quad +p_3p_4m_1y_1y_2y_3y_4 \quad -p_4^2m_1y_1y_2y_4^2 \quad +p_2p_4m_3y_2^2y_3y_4 \quad -p_2p_4m_4y_2^2y_4^2$$

$$-p_3p_4m_2y_2^2y_3y_4 + p_4^2m_2y_2^2y_4^2 + p_3p_4m_4y_2y_3y_4^2 - p_4^2m_3y_2y_3y_4^2\Big]$$

$$= -\frac{1}{|J|}\Big[\left(p_1^2m_2 - p_1p_2m_1\right)y_1^2y_2^2 + \left(p_4^2m_2 - p_2p_4m_4\right)y_2^2y_4^2 + \left(p_1p_3m_4 - p_1^2m_3 - p_1^2m_4\right)y_1^2y_2y_3$$

$$+\left(p_1p_3m_1 + p_1p_4m_1\right)y_1^2y_2y_4 + \left(p_3^2m_2 - p_2p_3m_3\right)y_2^2y_3^2$$

$$+\left(p_1p_3m_4 - p_3p_4m_1 - p_1p_4m_2 - p_1p_4m_1\right)y_1y_2y_3y_4$$

$$+\left(p_2p_4m_1 - p_1p_4m_2 - p_1p_2m_4\right)y_1y_2^2y_4 + \left(p_1p_2m_3 + p_2p_3m_1\right)y_1y_2^2y_3 + \left(p_1p_3m_3 - p_3^2m_1\right)y_1y_2y_3^2$$

$$+\left(p_2p_3m_4 - p_3p_4m_2\right)y_2^2y_3y_4 + \left(p_1p_4m_4 - p_4^2m_1\right)y_1y_2y_4^2 + \left(p_2p_4m_3 - p_3p_4m_2\right)y_2^2y_3y_4$$

$$+\left(p_3 p_4 m_4 - p_4^2 m_3\right) y_2 y_3 y_4^2\Big]$$

(21)

Now we use $p_3 = p_1$, and $p_4 = p_2$ where pair of prices are same, and $m_3 = m_1$, and $m_4 = m_2$, i.e., two types of coupon numbers are same, $|J| = |H| = -2p_1 p_2 m_1 m_2$. We put $y_1 = y_2 = y_3 = y_4 = 1$ then (21) becomes (Mohajan & Mohajan, 2022b);

$$\frac{\partial y_2}{\partial M} = \frac{3p_1^2 m_2 + p_2^2 m_1 - p_1 p_2 m_1 - 2p_1 p_2 m_2}{2p_1 p_2 m_1 m_2} \tag{22}$$

Now we use, $m_1 = m_2 = m$ in (22), and then we get,

$$\frac{\partial y_2}{\partial M} = \frac{(p_1 - p_2)^2 + p_1(2p_1 - p_2)}{2mp_1 p_2} \tag{23}$$

where $p_1, p_2, m > 0$.

Now if $2p_1 \geq p_2$ in (23) we get,

$$\frac{\partial y_2}{\partial M} > 0 \tag{24}$$

Inequality (24) indicates that if the total coupon of individual/community increases, the level of consumption of commodity $y_2$ will also increases. We believe that commodity $y_2$ is not an inferior good; it may be a superior good, and it has no supplementary goods (Islam et al., 2010; Mohajan, 2021b; Mohajan & Mohajan, 2022c).

For $\frac{\partial y_2}{\partial M} < 0$ we have;

$$(p_1 - p_2)^2 + p_1(2p_1 - p_2) < 0 \tag{25}$$

Now let, $\frac{p_2}{p_1} = x > 0$ then from (25) we get,

$$x^2 - 3x + 3 < 0 \tag{26}$$

where there is no real root. Therefore, $\frac{\partial y_2}{\partial M} \not< 0$. From this study we have realized that $\frac{\partial y_2}{\partial M} \neq 0$, so that, we face one possible situation as in (24), i.e., $\frac{\partial y_2}{\partial M} > 0$.

## 6. Conclusions

In this study we have tried to discuss sensitivity analysis between commodity and total coupon during utility maximization analysis. Thinking for the novel researchers we have stressed on detail mathematical calculations. We have used two constraints: budget constraint and coupon constraint to perform the research efficiently. We have used four commodity variables to operate the mathematical formulation properly. We have observed that the method of Lagrange multipliers is a very useful and powerful technique in multivariate calculus, and we have applied this device to investigate the optimization problems.

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