Paradigm Academic Press
Law and Economy
ISSN 2788-7049
JAN. 2023 VOL.2, NO.1

Utility Maximization Analysis of an Organization: A Mathematical Economic Procedure

Devajit Mohajan[1] & Haradhan Kumar Mohajan[2]

[1] Department of Civil Engineering, Chittagong University of Engineering & Technology, Chittagong, Bangladesh

[2] Assistant Professor, Department of Mathematics, Premier University, Chittagong, Bangladesh

Correspondence: Haradhan Kumar Mohajan, Assistant Professor, Department of Mathematics, Premier University, Chittagong, Bangladesh.

doi:10.56397/LE.2023.01.01

Abstract

In the society utility is the vital concept, especially in mathematical economics. It is considered as the tendency of an object or action that increases or decreases overall happiness. In social sciences, the property of a commodity that enables to satisfy human wants is called utility. This paper has tried to operate utility maximization policy of an organization by considering two constraints: budget constraint and coupon constraint. To develop the maximization policy of utility function, the techniques of multivariate calculus are used. In this study four commodity variables are used to operate the mathematical analysis efficiently. In this article Lagrange multiplier technique is applied to achieve optimal result throughout the study.

Keywords: commodities, Lagrange multipliers, utility maximization, budget and coupon constraints

1. Introduction

Mathematical modeling in economics is the application of mathematics in economics, where algebra, geometry, set theory, calculus, etc. are used to explain economic behavior of optimization (Samuelson, 1947). In mathematical economics, utility is an important concept; because it directly influences the demand and supply of the organizations (Fishburn, 1970). In the society there are two types of utility: positive utility and negative utility. Pleasure, happiness, benefit, advantage, good, etc. are considered as positive utility. On the other hand, opposite of these, such as pain, evil, unhappiness, bad, etc. are considered as negative utility (Bentham, 1780; Stigler, 1950; Mohajan, 2021a). Utility indicates that individuals seek to obtain the highest level of satisfaction from their purchasing goods (Kirsh, 2017). The concept of utility was developed in the late 18th century by the English moral philosopher, jurist, and social reformer Jeremy Bentham (1748-1832) and English philosopher, political economist, Member of Parliament (MP) and civil servant John Stuart Mill (1806-1873) (Bentham, 1780; Read, 2004).

The method of Lagrange multipliers is a very useful and powerful technique in multivariate calculus. In this article we have used this for transforming a constrained problem to a higher dimensional unconstrained problem (Islam et al., 2009a, b, 2010, 2011; Mohajan, 2021 b, c). We consider that all the consumers are rational so that they find the most value for their spender money, and they purchase the necessary commodities within their budget. As they are rational, always want to maximize their utility. In this study we consider two constraints; budget constraint and coupon constraint (Islam et al., 2010; Mohajan, 2017a).

Actually, utility maximization is the capability of an organization to earn the maximum profit within its budget. It directly effects on the organization and indirectly plays a role in economy and social well-being. Since economy sees its benefits and also sees welfare of human; therefore, utility maximization is a blessing both for humankind and the organization (Eaton & Lipsey, 1975). To increase utility, we have tried to provide coupons among the consumers. They can buy coupons with a stipulated price and purchase the essential commodities on priority basis. As a result, utility of the goods will increase and the producers can produce their commodities with full enthusiasm. Moreover, in the study we have introduced some theorems with proof where necessary (Mohajan, 2021a).

2. Literature Review

In mathematical economics, the literature review section is considered as an introductory portion of research that shows the works of previous researchers in the same field within the existing knowledge (Polit & Hungler, 2013). David Gauthier believes that economic man seeks to maximize utility. According to him, “the rational and moral individual seeks the maximum happiness of mankind, with which he identifies his own maximum happiness” (Gauthier, 1975). Ivan Moscati has investigated how William Stanley *Jevons,* Carl *Menger, and* Leon *Walras* have taken attempts to measure utility. Ivan Moscati also shows the contrast between ordinal and cardinal views of utility during the period 1870-1960 (Moscati, 2013). John V. Baxley and John C. Moorhouse have analyzed the utility maximization through the mathematical formulation by illustrating an explicit example (Baxley & Moorhouse, 1984). Qi Zhao and his coauthors have proposed multi-product utility maximization as a general approach to the recommendation driven by economic principles (Zhao et al., 2017).

Famous mathematician and physicist Jamal Nazrul Islam and his coauthors have given reasonable interpretation of the Lagrange multipliers and they have examined the behavior of the firm by analyzing comparative static results (Islam et al., 2009a, b, 2010). Pahlaj Moolio and his coauthors have stressed on optimization of output in an organization. They have used Lagrange multiplier to form and solve economic models (Moolio et al., 2009). Lia Roy and her coauthors have worked on optimization to develop cost minimization of an industry (Roy et al., 2021).

Devajit Mohajan and Haradhan Kumar Mohajan has discussed profit maximization policies by using four variable inputs, such as capital, labor, principal raw materials, and other inputs in an industry, where mathematical economic models are applied by considering budget constraint. They have studied Cobb-Douglas production function with detail mathematical analysis (Mohajan & Mohajan, 2022). Haradhan Kumar Mohajan has considered three inputs, such as capital, labor and other inputs for the sustainable production of a factory of Bangladesh (Mohajan, 2021a). In a published book he and his coauthors have displayed the optimization operations and have analyzed economics behaviors for global social welfare (Mohajan et al., 2013).

3. Methodology of the Study

In any research, methodology is the organized and meaningful procedural works that follow scientific methods efficiently (Kothari, 2008). In this study we have analyzed mathematical economic model of utility function for four commodities by introducing two Lagrange multipliers μ_1 and μ_2 , where we have considered 6-dimensional unconstrained problem that maximized utility function (Mohajan, 2021a). In this article we have used both qualitative and quantitative research procedures (Mohajan, 2018, 2020). In the research analysis we have introduced some theorems with proof to increase the concept and interest of the article among the readers (Mohajan, 2017b; 2021 b).

In our paper we have depended on the secondary data. These are collected from research articles of renowned journals, books and handbooks of famous authors, internet and websites, etc. (Mohajan, 2022a, b).

4. Objective of the Study

The main objective of this study is to provide utility maximization policy of an organization with mathematical procedures. The other minor objectives of the study are as follows:

- to show mathematical calculations accurately,
- to introduce and prove the theorems for the better achievement, and
- to develop mathematical formulation efficiently.

5. Utility in Economic Model

Consumers expect maximum satisfaction from consuming the purchased goods. If they receive desired satisfaction from the commodities they are using, in future they may try to spend their total income for buying these commodities again (Stigler, 1950).

Marginal Utility: Marginal utility is defined as the extra utility gained from the consumption of one additional unit of a commodity. If an individual consumes more of a good per time period, his/her total utility increases, but marginal utility diminishes (Castro & Araujo, 2019). The concept of marginal utility (MU) was proposed by Italian economist Ferdinando Galiani (1728-1787) (Galiani, 1751). Three economists William Stanley *Jevons,* Carl *Menger, and* Leon *Walras have* developed the idea of marginal utility, and consider them the founders of the marginal revolution in economics (Gauthier, 1975). The MU of commodity *X* is,

$$MU = \frac{\Delta U}{\Delta X}$$

If $MU > 0$, the commodity brings additional happiness; if $MU = 0$, there is no extra happiness of consumption of commodity; and if $MU < 0$ the consumption of commodity is harmful (Lin & Peng, 2019*)*.

Cardinal Utility: Cardinal utility is dominated until the 20th century (Dominick, 2008). Cardinal utility is first successfully introduced by English economist, Alfred Marshel (1842-1924). It indicates that the utilities obtained from consumption can be measured and ranked objectively, and can be represented by numbers, such as 1, 2, 3, …, *n* (Moscati, 2013). Two utility functions $u(x)$ and $v(x)$ can be related by the equation,

$$u(x) = av(x) + b$$

where *a* and *b* are constants (Strotz, 1953).

Ordinal Utility: In economics, an ordinal utility indicates the preference relation that identifies which option is better than the other. The ordinal utility concept was first introduced by Vilfredo Federico Damaso Pareto (1848-1923) in 1906 (Pareto, 1906). Later, it was developed by British economist Sir John Hicks (1904-1989) and English economist, mathematician and statistician, Sir Roy George Douglas Allen (1906-1983). It is a multi-good approach and only ranks the utility received from consuming various amounts of a commodity or a bundle of commodities (Dominick, 2008).

6. An Economic Model

In mathematical economics, a commodity is an economic good and the price of a commodity good is typically determined as a function of its market as a whole. Three types of commodities available in the global market are: i) Soft and non-durable commodities, which are grown in the cultivable field, such as wheat, rice, sugar, etc., ii) Hard and durable commodities (e.g., metallic), which are collected from mines, such as gold, silver, bronze, etc., iii) Energy commodities, which are also collected from mines or produced, such as electricity, gas, coal, oil, etc. (Mas-Colell et al., 1995; Alvino et al., 2018). Let us consider an economic world where there are only four commodities. We consider these four commodities as; X_1, X_2, X_3, and X_4, which are available sufficiently in the markets depending on the local and global demand (Moolio et al., 2009; Roy et al., 2021; Mohajan & Mohajan, 2022). The consumers have enough money to purchase these within their budget. Let a wise consumer wants to purchase only α_1, α_2, α_3, and α_4 amounts from these four commodities X_1, X_2, X_3, and X_4 respectively. In this model we consider that the consumer spends all of his/her income to purchase these four commodities, and also submits all of his/her coupons. Let us consider a utility function as follows (Islam et al., 2010; Mohajan & Mohajan, 2022):

$$u = u(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \tag{1}$$

In our model we consider two constraints: budget constraint and coupon constraint. The budget constraint of the consumer can be represented as,

$$B = p_1\alpha_1 + p_2\alpha_2 + p_3\alpha_3 + p_4\alpha_4 \tag{2}$$

where p_1, p_2, p_3, and p_4 are the prices (in dollars) of per unit of commodity of X_1, X_2, X_3, and X_4, respectively. Now the coupon constraint is given by,

$$C = c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 + c_4\alpha_4 \tag{3}$$

where c_1, c_2, c_3, and c_4 are the coupons necessary to purchase a unit of commodity of α_1, α_2, α_3, and α_4, respectively.

In our economic model there are two types of constraints, such as budget constraint and coupon constraint. Therefore, we use two Lagrange multipliers μ_1 and μ_2 as devices of mathematical procedures. Using (1), (2), and (3) we can express Lagrangian function *v* as (Mohajan, 2017a),

$$v(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\mu_1,\mu_2)=u(\alpha_1,\alpha_2,\alpha_3,\alpha_4)+\mu_1(B-p_1\alpha_1-p_2\alpha_2-p_3\alpha_3-p_4\alpha_4)$$
$$+\mu_2(C-c_1\alpha_1-c_2\alpha_2-c_3\alpha_3-c_4\alpha_4) \tag{4}$$

Lagrangian function (4) is a 6-dimensional unconstrained problem that maximizes utility function. By taking partial derivatives of (4) and using optimization techniques for utility maximization we set them equal to zero. That is, we use necessary conditions of maximization in multivariate calculus as follows (Mohajan, 2017a):

$$v_{\mu_1}=\frac{\partial v}{\partial \mu_1}=B-p_1\alpha_1-p_2\alpha_2-p_3\alpha_3-p_4\alpha_4=0 \tag{5a}$$

$$v_{\mu_2}=\frac{\partial v}{\partial \mu_2}=C-c_1\alpha_1-c_2\alpha_2-c_3\alpha_3-c_4\alpha_4=0 \tag{5b}$$

$$v_1=\frac{\partial v}{\partial \alpha_1}=u_1-\mu_1 p_1-\mu_2 c_1=0 \tag{5c}$$

$$v_2=\frac{\partial v}{\partial \alpha_2}=u_2-\mu_1 p_2-\mu_2 c_2=0 \tag{5d}$$

$$v_3=\frac{\partial v}{\partial \alpha_3}=u_3-\mu_1 p_3-\mu_2 c_3=0 \tag{5e}$$

$$v_4=\frac{\partial v}{\partial \alpha_4}=u_4-\mu_1 p_4-\mu_2 c_4=0 \tag{5f}$$

where $v_1=\frac{\partial v}{\partial \alpha_1}$, $v_2=\frac{\partial v}{\partial \alpha_2}$, etc.; and $v_{\mu_1}=\frac{\partial v}{\partial \mu_1}$, $v_{\mu_2}=\frac{\partial v}{\partial \mu_2}$, etc. indicate first order partial differentiation. We observe that in our study three targeted functions $u(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, $B(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, and $C(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ are functions of four variables α_1, α_2, α_3, and α_4. We consider that in the market there is an infinitesimal increase in *U*, *B*, and *R*, i.e., the increased amounts are dU, dB, and dR, respectively; then we can write,

$$du=u_1d\alpha_1+u_2d\alpha_2+u_3d\alpha_3+u_4d\alpha_4 \tag{6a}$$

$$dB=p_1d\alpha_1+p_2d\alpha_2+p_3d\alpha_3+p_4d\alpha_4 \tag{6b}$$

$$dC=c_1d\alpha_1+c_2d\alpha_2+c_3d\alpha_3+c_4d\alpha_4 \tag{6c}$$

Theorem 1: The Lagrange multipliers, μ_1 and μ_2; have provided in equation (4) indicate the changes in the utility resulting to one of the constraints being operative, but not the other.

Proof: Now we consider that the budget of an economic scheme is non-changeable, then $dB=0$, we consider for this scheme $\mu_1=0$ (Islam et al., 2011). From (5c) we get,

$$u_1 - \mu_2 c_1 = 0 \Rightarrow \frac{u_1}{c_1} = \mu_2 \text{, and similarly; } \frac{u_2}{c_2} = \mu_2 \text{, } \frac{u_3}{c_3} = \mu_2 \text{, and } \frac{u_4}{c_4} = \mu_2 \tag{7}$$

Dividing (6a) by (6c) we get,

$$\frac{du}{dC} = \frac{u_1 d\alpha_1 + u_2 d\alpha_2 + u_3 d\alpha_3 + u_4 d\alpha_4}{c_1 d\alpha_1 + c_2 d\alpha_2 + c_3 d\alpha_3 + c_4 d\alpha_4} \tag{8}$$

In our economic model we consider that our budget is fixed, i.e., $B = \text{constant} \Rightarrow dB = 0$, moreover, $\alpha_2 = \text{constant}, \Rightarrow d\alpha_2 = 0$; $\alpha_3 = \text{constant} \Rightarrow d\alpha_3 = 0$; and $\alpha_4 = \text{constant} \Rightarrow d\alpha_4 = 0$ in (8), then we get from (8);

$$\frac{du}{dC} = \frac{u_1 d\alpha_1}{c_1 d\alpha_1} = \frac{u_1}{c_1} = \mu_2 \text{, by (7)} \tag{9a}$$

Similarly,

$$\frac{du}{dC} = \frac{u_2 d\alpha_2}{c_2 d\alpha_2} = \frac{u_2}{c_2} = \mu_2 \tag{9b}$$

$$\frac{du}{dC} = \frac{u_3 d\alpha_3}{c_3 d\alpha_3} = \frac{u_3}{c_3} = \mu_2 \text{, and} \tag{9c}$$

$$\frac{du}{dC} = \frac{u_4 d\alpha_4}{c_4 d\alpha_4} = \frac{u_4}{c_4} = \mu_2 \tag{9d}$$

Hence, from (9a-d) we see that the Lagrange multiplier μ_2 can be interpreted as,

$$\left(\frac{\partial u}{\partial C}\right)_{B=\text{constant}} = \mu_2 \tag{10}$$

Again, in our mathematical model we consider that the total cost of coupons is fixed, i.e., $C = \text{constant} \Rightarrow dC = 0$, and also, we consider for this scheme $\mu_2 = 0$. From (5c) we get,

$$u_1 - \mu_1 p_1 = 0 \quad \Rightarrow \frac{u_1}{p_1} = \mu_1 \text{, and similarly; } \frac{u_2}{p_2} = \mu_1 \text{, } \frac{u_3}{p_3} = \mu_1 \text{, and } \frac{u_4}{p_4} = \mu_1 \tag{11}$$

Dividing (6a) by (6b) we get,

$$\frac{du}{dB} = \frac{u_1 d\alpha_1 + u_2 d\alpha_2 + u_3 d\alpha_3 + u_4 d\alpha_4}{p_1 d\alpha_1 + p_2 d\alpha_2 + p_3 d\alpha_3 + p_4 d\alpha_4} \tag{12}$$

In our model we consider that the total cost of coupons is fixed, i.e., $C = \text{constant} \Rightarrow dC = 0$, moreover, $\alpha_2 = \text{constant}, \Rightarrow d\alpha_2 = 0$; $\alpha_3 = \text{constant} \Rightarrow d\alpha_3 = 0$; and $\alpha_4 = \text{constant} \Rightarrow d\alpha_4 = 0$ in (12), then we get from (12);

$$\frac{du}{dB} = \frac{u_1 d\alpha_1}{p_1 d\alpha_1} = \frac{u_1}{p_1} = \mu_1 \text{ by (11)} \tag{13a}$$

Similarly,

$$\frac{du}{dB} = \frac{u_2 d\alpha_2}{p_2 d\alpha_2} = \frac{u_2}{p_2} = \mu_1 \tag{13b}$$

$$\frac{du}{dB} = \frac{u_3 d\alpha_3}{p_3 d\alpha_3} = \frac{u_3}{p_3} = \mu_1 \text{, and} \tag{13c}$$

$$\frac{du}{dB} = \frac{u_4 d\alpha_4}{p_4 d\alpha_4} = \frac{u_4}{p_4} = \mu_1 \tag{13d}$$

Combining equation (13a-d) we see that the Lagrange multiplier μ_1 can be interpreted as,

$$\left(\frac{\partial u}{\partial B}\right)_{C=\text{constant}} = \mu_1 \tag{14}$$

We can conclude that the two Lagrange multipliers μ_1 and μ_2 in this mathematical model, give the changes in the utility consequent to one of the constraints being operative, but not the other (Mohajan, 2021a). Hence, the theorem is proved.

7. Mathematical Economic Formulation

Now we consider that in our economic model the utility function is given by,

$$u(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \tag{15}$$

Using utility function from (15) in Lagrangian function (4) we get,

$$v(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu_1, \mu_2) = \alpha_1 \alpha_2 \alpha_3 \alpha_4 + \mu_1 (B - p_1\alpha_1 - p_2\alpha_2 - p_3\alpha_3 - p_4\alpha_4)$$

$$+ \mu_2 (C - c_1\alpha_1 - c_2\alpha_2 - c_3\alpha_3 - c_4\alpha_4) \tag{16}$$

Using the necessary conditions of multivariate calculus for maximization in equation (16) we yield;

$$v_{\mu_1} = B - p_1\alpha_1 - p_2\alpha_2 - p_3\alpha_3 - p_4\alpha_4 = 0, \tag{17a}$$

$$v_{\mu_2} = C - c_1\alpha_1 - c_2\alpha_2 - c_3\alpha_3 - c_4\alpha_4 = 0, \tag{17b}$$

$$v_1 = \alpha_2 \alpha_3 \alpha_4 - \mu_1 p_1 - \mu_2 c_1 = 0, \tag{17c}$$

$$v_2 = \alpha_1 \alpha_3 \alpha_4 - \mu_1 p_2 - \mu_2 c_2 = 0, \tag{17d}$$

$$v_3 = \alpha_1 \alpha_2 \alpha_4 - \mu_1 p_3 - \mu_2 c_3 = 0 \text{, and} \tag{17e}$$

$$v_4 = \alpha_1 \alpha_2 \alpha_3 - \mu_1 p_4 - \mu_2 c_4 = 0. \tag{17f}$$

Now we are in a position to provide a theorem related to optimization. We use the maximization techniques of multivariate calculus. First, we try for the estimation of the amount of four commodities (Mohajan & Mohajan, 2022).

Theorem 2: a) In the economic model, the amount of four commodities can be expressed as;

$$\text{i)}\quad \alpha_1 = \left[\frac{(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_3 + \mu_2 c_3)(\mu_1 p_4 + \mu_2 c_4)}{(\mu_1 p_1 + \mu_2 c_1)^2}\right]^{\frac{1}{3}}$$

$$\text{ii)}\quad \alpha_2 = \left[\frac{(\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_3 + \mu_2 c_3)(\mu_1 p_4 + \mu_2 c_4)}{(\mu_1 p_2 + \mu_2 c_2)^2}\right]^{\frac{1}{3}}$$

$$\text{iii)}\quad \alpha_3 = \left[\frac{(\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_4 + \mu_2 c_4)}{(\mu_1 p_3 + \mu_2 c_3)^2}\right]^{\frac{1}{3}}$$

$$\text{iv)}\quad \alpha_4 = \left[\frac{(\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_3 + \mu_2 c_3)}{(\mu_1 p_4 + \mu_2 c_4)^2}\right]^{\frac{1}{3}}.$$

b) The two Lagrange multipliers of the scheme can be expressed as;

$$\text{i)}\quad \mu_1 = \alpha_3\alpha_4 \frac{\alpha_2 c_2 - \alpha_1 c_1}{c_1 p_2 - c_2 p_1} = \alpha_1\alpha_2 \frac{\alpha_4 c_4 - \alpha_3 c_3}{c_4 p_3 - c_3 p_4}$$

$$\text{ii)}\quad \mu_2 = \alpha_3\alpha_4 \frac{\alpha_2 p_2 - \alpha_1 p_1}{c_1 p_2 - c_2 p_1} = \alpha_1\alpha_2 \frac{\alpha_4 p_4 - \alpha_3 p_3}{c_3 p_4 - c_4 p_3}$$

where $c_1 p_2 \neq c_2 p_1$ and $c_4 p_3 \neq c_3 p_4$.

Proof: From (17c) we get,

$$\alpha_2\alpha_3\alpha_4 = \mu_1 p_1 + \mu_2 c_1 \tag{18a}$$

From (17d) we get,

$$\alpha_1\alpha_3\alpha_4 = \mu_1 p_2 + \mu_2 c_2 \tag{18b}$$

From (17e) we get,

$$\alpha_1\alpha_2\alpha_4 = \mu_1 p_3 + \mu_2 c_3 \tag{18c}$$

From (17f) we get,

$$\alpha_1\alpha_2\alpha_3 = \mu_1 p_4 + \mu_2 c_4 \tag{18d}$$

Multiplying equations (18a-d) we get,

$$\alpha_1^3\alpha_2^3\alpha_3^3\alpha_4^3 = (\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_3 + \mu_2 c_3)(\mu_1 p_4 + \mu_2 c_4) \tag{19}$$

$$\alpha_1^3(\mu_1 p_1 + \mu_2 c_1)^3 = (\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_3 + \mu_2 c_3)(\mu_1 p_4 + \mu_2 c_4)$$

$$\alpha_1 = \left[\frac{(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_3 + \mu_2 c_3)(\mu_1 p_4 + \mu_2 c_4)}{(\mu_1 p_1 + \mu_2 c_1)^2}\right]^{\frac{1}{3}} \tag{20a}$$

where $\mu_1 p_1 + \mu_2 c_1 \neq 0$. Similarly,

$$\alpha_2 = \left[\frac{(\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_3 + \mu_2 c_3)(\mu_1 p_4 + \mu_2 c_4)}{(\mu_1 p_2 + \mu_2 c_2)^2}\right]^{\frac{1}{3}} \tag{20b}$$

where $\mu_1 p_2 + \mu_2 c_2 \neq 0$.

$$\alpha_3 = \left[\frac{(\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_4 + \mu_2 c_4)}{(\mu_1 p_3 + \mu_2 c_3)^2}\right]^{\frac{1}{3}} \tag{20c}$$

where $\mu_1 p_3 + \mu_2 c_3 \neq 0$.

$$\alpha_4 = \left[\frac{(\mu_1 p_1 + \mu_2 c_1)(\mu_1 p_2 + \mu_2 c_2)(\mu_1 p_3 + \mu_2 c_3)}{(\mu_1 p_4 + \mu_2 c_4)^2}\right]^{\frac{1}{3}} \tag{20d}$$

where $\mu_1 p_4 + \mu_2 c_4 \neq 0$. Therefore, the theorem for amount of commodities is proved.
From (17c) we get,

$$\mu_1 = \frac{\alpha_2 \alpha_3 \alpha_4 - \mu_2 c_1}{p_1} \tag{21a}$$

$$\mu_2 = \frac{\alpha_2 \alpha_3 \alpha_4 - \mu_1 p_1}{c_1} \tag{21b}$$

From (17d) we get,

$$\mu_1 = \frac{\alpha_1 \alpha_3 \alpha_4 - \mu_2 c_2}{p_2} \tag{22a}$$

$$\mu_2 = \frac{\alpha_1 \alpha_3 \alpha_4 - \mu_1 p_2}{c_2} \tag{22b}$$

From (17e) we get,

$$\mu_1 = \frac{\alpha_1 \alpha_2 \alpha_4 - \mu_2 c_3}{p_3} \tag{23a}$$

$$\mu_2 = \frac{\alpha_1 \alpha_2 \alpha_4 - \mu_1 p_3}{c_3} \tag{23b}$$

From (17f) we get,

$$\mu_1 = \frac{\alpha_1 \alpha_2 \alpha_3 - \mu_2 c_4}{p_4} \tag{24a}$$

$$\mu_2 = \frac{\alpha_1 \alpha_2 \alpha_3 - \mu_1 p_4}{c_4} \tag{24b}$$

Combining (21a) and (22a) we get,

$$\frac{\alpha_2\alpha_3\alpha_4 - \mu_2 c_1}{p_1} = \frac{\alpha_1\alpha_3\alpha_4 - \mu_2 c_2}{p_2}$$

$$\frac{\alpha_2\alpha_3\alpha_4}{p_1} - \mu_2\frac{c_1}{p_1} = \frac{\alpha_1\alpha_3\alpha_4}{p_2} - \mu_2\frac{c_2}{p_2}$$

$$\mu_2\left(\frac{c_1}{p_1} - \frac{c_2}{p_2}\right) = \alpha_3\alpha_4\left(\frac{\alpha_2}{p_1} - \frac{\alpha_1}{p_2}\right)$$

$$\mu_2 = \alpha_3\alpha_4\frac{\alpha_2 p_2 - \alpha_1 p_1}{c_1 p_2 - c_2 p_1} \tag{25a}$$

where $c_1 p_2 \neq c_2 p_1$.
Combining (21b) and (22b) we get,

$$\frac{\alpha_2\alpha_3\alpha_4 - \mu_1 p_1}{c_1} = \frac{\alpha_1\alpha_3\alpha_4 - \mu_1 p_2}{c_2}$$

$$\mu_1 = \alpha_3\alpha_4\frac{\alpha_2 c_2 - \alpha_1 c_1}{c_2 p_1 - c_1 p_2} \tag{25b}$$

where $c_1 p_2 \neq c_2 p_1$.
Combining (23a) and (24a) we get,

$$\frac{\alpha_1\alpha_2\alpha_4 - \mu_2 c_3}{p_3} = \frac{\alpha_1\alpha_2\alpha_3 - \mu_2 c_4}{p_4}$$

$$\mu_2 = \alpha_1\alpha_2\frac{\alpha_4 p_4 - \alpha_3 p_3}{c_3 p_4 - c_4 p_3} \tag{26a}$$

where $c_3 p_4 \neq c_4 p_3$.
Combining (23b) and (24b) we get,

$$\frac{\alpha_1\alpha_2\alpha_4 - \mu_1 p_3}{c_3} = \frac{\alpha_1\alpha_2\alpha_3 - \mu_1 p_4}{c_4}$$

$$\mu_1 = \alpha_1\alpha_2\frac{\alpha_4 c_4 - \alpha_3 c_3}{c_4 p_3 - c_3 p_4} \tag{26b}$$

where $c_4 p_3 \neq c_3 p_4$.
Combining (25b) and (26b) we get,

$$\mu_1 = \alpha_3\alpha_4\frac{\alpha_2 c_2 - \alpha_1 c_1}{c_2 p_1 - c_1 p_2} = \alpha_1\alpha_2\frac{\alpha_4 c_4 - \alpha_3 c_3}{c_4 p_3 - c_3 p_4} \tag{27a}$$

where $c_1 p_2 \neq c_2 p_1$ and $c_4 p_3 \neq c_3 p_4$.
Combining (25a) and (26a) we get,

$$\mu_2 = \alpha_3\alpha_4 \frac{\alpha_2 p_2 - \alpha_1 p_1}{c_1 p_2 - c_2 p_1} = \alpha_1\alpha_2 \frac{\alpha_4 p_4 - \alpha_3 p_3}{c_3 p_4 - c_4 p_3} \tag{27b}$$

where $c_1 p_2 \neq c_2 p_1$ and $c_3 p_4 \neq c_4 p_3$. Hence, the theorem for Lagrangian multipliers is proved.

In the above Theorem 2, we have observed that commodities are in terms of Lagrangian multipliers and vice-versa. Now we take an attempt to represent them independently.

From (17b) we get,

$$c_2\alpha_2 = C - c_1\alpha_1 - c_3\alpha_3 - c_4\alpha_4 \tag{28}$$

From (17a) we get,

$$p_1\alpha_1 = B - p_2\alpha_2 - p_3\alpha_3 - p_4\alpha_4$$

$$\alpha_1 = \frac{B}{p_1} - \frac{p_2}{p_1}\alpha_2 - \frac{p_3}{p_1}\alpha_3 - \frac{p_4}{p_1}\alpha_4 \tag{29}$$

From (28) and (29) we can write,

$$c_2\alpha_2 = C - \frac{Bc_1}{p_1} - \frac{p_2 c_1}{p_1}\alpha_2 - \frac{p_3 c_1}{p_1}\alpha_3 - \frac{p_4 c_1}{p_1}\alpha_4 - c_3\alpha_3 - c_4\alpha_4$$

$$\alpha_2 = \frac{Cp_1 - Bc_1}{c_2 p_1 + c_1 p_2} - \frac{c_1 p_3 + c_3 p_1}{c_2 p_1 + c_1 p_2}\alpha_3 - \frac{c_1 p_4 + c_4 p_1}{c_2 p_1 + c_1 p_2}\alpha_4 \tag{30}$$

Equation (29) gives;

$$\alpha_1 = \frac{B}{p_1} - \frac{p_2}{p_1}\left(\frac{Cp_1 - Bc_1}{c_2 p_1 + c_1 p_2} - \frac{c_1 p_3 + c_3 p_1}{c_2 p_1 + c_1 p_2}\alpha_3 - \frac{c_1 p_4 + c_4 p_1}{c_2 p_1 + c_1 p_2}\alpha_4\right) - \frac{p_3}{p_1}\alpha_3 - \frac{p_4}{p_1}\alpha_4$$

$$\alpha_1 = \frac{Bc_2 - Cp_2}{c_2 p_1 + c_1 p_2} + \frac{c_3 p_2 - c_2 p_3}{c_2 p_1 + c_1 p_2}\alpha_3 + \frac{c_4 p_2 - c_2 p_4}{c_2 p_1 + c_1 p_2}\alpha_4 \tag{31}$$

Theorem 3: For simplicity let, $\alpha_3 = \alpha_4 = 1$, i.e., these commodities have one unit each, the amount of other two commodities can be accounted as;

a) $\alpha_1 = \dfrac{-(C - c_3 - c_4)p_2 + (B - p_3 - p_4)c_2}{c_2 p_1 + c_1 p_2}$

b) $\alpha_2 = \dfrac{(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1}{c_2 p_1 + c_1 p_2}$.

Proof: For $\alpha_3 = \alpha_4 = 1$ in (31) we get,

$$\alpha_1 = \frac{Bc_2 - Cp_2}{c_2 p_1 + c_1 p_2} + \frac{c_3 p_2 - c_2 p_3}{c_2 p_1 + c_1 p_2} + \frac{c_4 p_2 - c_2 p_4}{c_2 p_1 + c_1 p_2}$$

$$\alpha_1 = \frac{(B - p_3 - p_4)c_2 - (C - c_3 - c_4)p_2}{c_2 p_1 + c_1 p_2} \tag{32}$$

For $\alpha_3 = \alpha_4 = 1$ in (30) we get,

$$\alpha_2 = \frac{Cp_1 - Bc_1}{c_2 p_1 + c_1 p_2} - \frac{c_1 p_3 + c_3 p_1}{c_2 p_1 + c_1 p_2} - \frac{c_1 p_4 + c_4 p_1}{c_2 p_1 + c_1 p_2}$$

$$\alpha_2 = \frac{(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1}{c_2 p_1 + c_1 p_2} \quad (33)$$

From equations (32) and (33) we see that the amounts of commodities are in terms of prices, coupon numbers, total budget, and total cost of coupons. Consequently, these are free from Lagrangian multipliers. Hence, the theorem is proved.

Now we want to represent Lagrangian multipliers, utility, total budget, and total cost of coupon with free of the commodity terms (Mohajan et al., 2013). The following theorems help in this regard.

Theorem 4: Optimized Lagrangeian multipliers can be expressed as;

a) $$\mu_1^* = \frac{(C - c_3 - c_4)(c_2 p_1 + c_1 p_2) - 2Bc_1 c_2}{c_2^2 p_1^2 - c_1^2 p_2^2}$$

b) $$\mu_1^* = \frac{(C-c_3-c_4)\{B(c_2p_1+c_1p_2)+(c_1p_2-c_2p_1)(p_3+p_4)\}-(C-c_3-c_4)^2 p_1p_2-\left(B^2-(p_3+p_4)^2\right)c_1c_2}{(c_2p_1+c_1p_2)^2(c_4p_3-c_3p_4)(c_4-c_3)^{-1}}$$

c) $$\mu_2^* = \mu_2 = \frac{-B(c_2 p_1 + c_1 p_2) + (c_2 p_1 - c_1 p_2)(p_3 + p_4)}{c_1^2 p_2^2 - c_2^2 p_1^2}$$

d) $$\mu_2^* = \frac{(C-c_3-c_4)\{B(c_2p_1+c_1p_2)+(c_2p_1-c_1p_2)(p_3+p_4)\}-(C-c_3-c_4)^2 p_1p_2-\left(B^2-(p_3+p_4)^2\right)c_1c_2}{(c_2p_1+c_1p_2)^2(c_4p_3-c_3p_4)(p_4-p_3)^{-1}}.$$

Proof: For $\alpha_3 = \alpha_4 = 1$ in (27a) we get,

$$\mu_1 = \alpha_1 \alpha_2 \frac{c_4 - c_3}{c_4 p_3 - c_3 p_4} \quad (34)$$

Now using the values of α_1 and α_2 from (32) and (33) in (34) we get,

$$\mu_1^* = \frac{-(C - c_3 - c_4)p_2 + (B - p_3 - p_4)c_2}{c_2 p_1 + c_1 p_2} \cdot \frac{(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1}{c_2 p_1 + c_1 p_2} \cdot \frac{c_4 - c_3}{c_4 p_3 - c_3 p_4}$$

$$\mu_1^* = \frac{(C-c_3-c_4)\{B(c_2p_1+c_1p_2)+(c_1p_2-c_2p_1)(p_3+p_4)\}-(C-c_3-c_4)^2 p_1p_2-\left(B^2-(p_3+p_4)^2\right)c_1c_2}{(c_2p_1+c_1p_2)^2(c_4p_3-c_3p_4)(c_4-c_3)^{-1}} \quad (35)$$

Again for $\alpha_3 = \alpha_4 = 1$ in (27a) we get,

$$\mu_1 = \frac{\alpha_2 c_2 - \alpha_1 c_1}{c_2 p_1 - c_1 p_2} \quad (36)$$

Now putting the values of α_1 and α_2 from (32) and (33) in (36) we get,

$$\mu_1 = \frac{[(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1]c_2 - [-(C - c_3 - c_4)p_2 + (B - p_3 - p_4)c_2]c_1}{c_2^2 p_1^2 - c_1^2 p_2^2}$$

$$\mu_1^* = \mu_1 = \frac{(C - c_3 - c_4)(c_2 p_1 + c_1 p_2) - 2Bc_1c_2}{c_2^2 p_1^2 - c_1^2 p_2^2} \tag{37}$$

Again for $\alpha_3 = \alpha_4 = 1$ in (27b) we get,

$$\mu_2 = \alpha_1 \alpha_2 \frac{p_4 - p_3}{c_3 p_4 - c_4 p_3} \tag{38}$$

Now putting the values of α_1 and α_2 from (32) and (33) in (38) we get,

$$\mu_2 = \frac{-(C - c_3 - c_4)p_2 + (B - p_3 - p_4)c_2}{c_2 p_1 + c_1 p_2} \cdot \frac{(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1}{c_2 p_1 + c_1 p_2} \cdot \frac{p_4 - p_3}{c_3 p_4 - c_4 p_3}$$

$$\mu_2^* = \frac{(C - c_3 - c_4)\{B(c_2 p_1 + c_1 p_2) + (c_2 p_1 - c_1 p_2)(p_3 + p_4)\} - (C - c_3 - c_4)^2 p_1 p_2 - \left(B^2 - (p_3 + p_4)^2\right)c_1 c_2}{(c_2 p_1 + c_1 p_2)^2 (c_4 p_3 - c_3 p_4)(p_4 - p_3)^{-1}} \tag{39}$$

Again for $\alpha_3 = \alpha_4 = 1$ in (27b) we get,

$$\mu_2 = \frac{\alpha_2 p_2 - \alpha_1 p_1}{c_1 p_2 - c_2 p_1} \tag{40}$$

Now putting the values of α_1 and α_2 from (32) and (33) in (40) we get,

$$\mu_2 = \frac{[(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1]p_2 - [-(C - c_3 - c_4)p_2 + (B - p_3 - p_4)c_2]p_1}{c_1^2 p_2^2 - c_2^2 p_1^2}$$

$$\mu_2^* = \mu_2 = \frac{-B(c_2 p_1 + c_1 p_2) + (c_2 p_1 - c_1 p_2)(p_3 + p_4)}{c_1^2 p_2^2 - c_2^2 p_1^2} \tag{41}$$

From (35) and (37) we have obtained optimum values of Lagrangian multiplier μ_1^*. On the other hand, from (39) and (41) we have obtained optimum values of Lagrangian multiplier μ_2^*. We have observed that the Lagrangian multipliers μ_1^* and μ_2^* are free from α_1 and α_2, and also free from α_3 and α_4. We have obtained expected results; and hence, the theorem is proved.

Now we take an attempt to obtain optimum values of utility, total budget, and total cost of coupons. Following theorem will provide the maximum values of utility, total budget and total cost of coupons for utility maximization.

Theorem 5: a) The maximized utility function can be expressed as,

$$u^* = \frac{(C - c_3 - c_4)\{B(c_2 p_1 + c_1 p_2) + (c_1 p_2 - c_2 p_1)(p_3 + p_4)\} - (C - c_3 - c_4)^2 p_1 p_2 - \left(B^2 - (p_3 + p_4)^2\right)c_1 c_2}{(c_2 p_1 + c_1 p_2)^2}.$$

b) The amount of budget for utility maximization of the model can be expressed as,

$$B^* = (p_3 + p_4)\left(1 + \frac{1}{2}\frac{c_2 p_1}{c_1 p_2}\right).$$

c) The total number of coupons for utility maximization of the model can be expressed as,

$$C = \frac{1}{2p_2}\left[2p_2(c_3 + c_4) + (p_3 + p_4)c_2\right].$$

Proof: From (15) we have the utility function for $\alpha_3 = \alpha_4 = 1$,

$$u = \alpha_1 \alpha_2 \tag{42}$$

Now using the values of α_1 and α_2 from (32) and (33) in (42) we get,

$$u = \frac{-(C - c_3 - c_4)p_2 + (B - p_3 - p_4)c_2}{c_2 p_1 + c_1 p_2} \cdot \frac{(C - c_3 - c_4)p_1 - (B + p_3 + p_4)c_1}{c_2 p_1 + c_1 p_2}$$

$$u^* = \frac{(C - c_3 - c_4)\{B(c_2 p_1 + c_1 p_2) + (c_1 p_2 - c_2 p_1)(p_3 + p_4)\} - (C - c_3 - c_4)^2 p_1 p_2 - \left(B^2 - (p_3 + p_4)^2\right)c_1 c_2}{(c_2 p_1 + c_1 p_2)^2} \tag{43}$$

From (2) for $\alpha_3 = \alpha_4 = 1$ we get the budget of the scheme,

$$B = p_1 \alpha_1 + p_2 \alpha_2 + p_3 + p_4 \tag{44}$$

Now substituting the values of α_1 and α_2 from (32) and (33) in (44) we get,

$$B = \frac{(B - p_3 - p_4)c_2 p_1 - (B + p_3 + p_4)c_1 p_2 + (p_3 + p_4)(c_2 p_1 + c_1 p_2)}{c_2 p_1 + c_1 p_2}$$

$$B^* = (p_3 + p_4)\left(1 + \frac{1}{2}\frac{c_2 p_1}{c_1 p_2}\right) \tag{45}$$

From (2) for $\alpha_3 = \alpha_4 = 1$ we get the total coupon of the scheme,

$$C = c_1 \alpha_1 + c_2 \alpha_2 + c_3 + c_4 \tag{46}$$

Now substituting the values of α_1 and α_2 from (32) and (33) in (46) we get,

$$C = \frac{(C - c_3 - c_4)(c_2 p_1 - c_1 p_2) - (p_3 + p_4)c_1 c_2 + (c_2 p_1 + c_1 p_2)(c_3 + c_4)}{c_2 p_1 + c_1 p_2}$$

$$C^* = \frac{1}{2 p_2}\left[2 p_2 (c_3 + c_4) + (p_3 + p_4)c_2\right] \tag{47}$$

We have obtained the maximum utility, total budget, and total number of coupons for utility maximization. Hence, the theorem is proved.

8. Conclusions and Recommendations

In this study we have tried to discuss utility maximization policy of an organization. We have considered that the scheme of the organization has followed the policy of maximization, and hence we have taken steps to obtain the value of utility by considering the maximization. We have observed that Lagrange multipliers play an important role in mathematical economics. In this article we have used two Lagrange multipliers to perform the job properly. So that we have used two constraints: budget constraint and coupon constraint in our research analysis. In this study we have provided some theorems with proof. So that the readers will find interest when they go through this study. Throughout the paper we have tried to introduce mathematical calculations in some details.

References

Alvino, L., Constantinides, E., & Franco, M. (2018). Towards a Better Understanding of Consumer Behavior: Marginal Utility as a Parameter in Neuromarketing Research. *International Journal of Marketing Studies*, *10*(1), 90–106.

Baxley, J. V., & Moorhouse, J. C. (1984). Lagrange Multiplier Problems in Economics. *The American Mathematical Monthly*, *91*(7), 404–412.

Bentham, J. (1780). *An Introduction to the Principles of Morals and Legislation*. CreateSpace Independent Publishing Platform.

Castro, L. C., & Araujo, A. S. (2019). Marginal Utility and its Diminishing Methods. *International Journal of*

Tax Economics and Management, *2*(6), 36–47.

Dominick, S. (2008). *Principles of Microeconomics*. Oxford University Press, New Delhi.

Eaton, B., & Lipsey, R. (1975). The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition. *Review of Economic Studies*, *42*(1), 27–49.

Fishburn, P. C. (1970). *Utility Theory for Decision Making*. Huntington, NY: Robert E. Krieger.

Galiani, F. (1751). *Della Moneta*. Reprinted in: Collezioni Custodi Scrittori Classici Italiani di Economia Politica, 10.

Gauthier, D. (1975). Reason and Maximization. *Canadian Journal of Philosophy*, *4*(3), 411–433.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009a). Preference of Social Choice in Mathematical Economics. *Indus Journal of Management & Social Sciences*, *3*(1), 17–38.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009b). Political Economy and Social Welfare with Voting Procedure. *KASBIT Business Journal*, *2*(1), 42–66.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2010). Utility Maximization Subject to Multiple Constraints. *Indus Journal of Management & Social Sciences*, *4*(1), 15–29.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2011). Output Maximization Subject to a Nonlinear Constraint. *KASBIT Business Journal*, *4*(1), 116–128.

Kirsh, Y. (2017). Utility and Happiness in a Prosperous Society. Working Paper Series, No. 37-2017, Institute for Policy Analysis, The Open University of Israel.

Kothari, C. R. (2008). *Research Methodology: Methods and Techniques* (2nd Ed.). New Delhi: New Age International (P) Ltd.

Lin, C.-C., & Peng, S.-S. (2019). The Role of Diminishing Marginal Utility in the Ordinal and Cardinal Utility Theories. *Australian Economic Papers*, *58*(3), 233–246.

Mas-Colell, A., Whinston, M.D., & Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press, Oxford.

Mohajan, D., & Mohajan, H. K. (2022). Profit Maximization Strategy in an Industry: A Sustainable Procedure. *Law and Economy*, *1*(3), 17–43. https://doi:10.56397/LE.2022.10.02.

Mohajan, H. K. (2017a). Optimization Models in Mathematical Economics. *Journal of Scientific Achievements*, *2*(5), 30–42.

Mohajan, H. K. (2017b). Two Criteria for Good Measurements in Research: Validity and Reliability. *Annals of Spiru Haret University. Economic Series*, *17*(3), 58–82.

Mohajan, H. K. (2018). Qualitative Research Methodology in Social Sciences and Related Subjects. *Journal of Economic Development, Environment and People*, *7*(1), 23–48.

Mohajan, H. K. (2020). Quantitative Research: A Successful Investigation in Natural and Social Sciences. *Journal of Economic Development, Environment and People*, *9*(4), 52–79.

Mohajan, H. K. (2021a). Utility Maximization of Bangladeshi Consumers within Their Budget: A Mathematical Procedure. Journal of Economic Development, Environment and People, *10*(3), 60–85.

Mohajan, H. K. (2021b). Product Maximization Techniques of a Factory of Bangladesh: A Sustainable Procedure. *American Journal of Economics, Finance and Management*, *5*(2), 23–44.

Mohajan, H. K. (2021c). Estimation of Cost Minimization of Garments Sector by Cobb-Douglass Production Function: Bangladesh Perspective. *Annals of Spiru Haret University. Economic Series*, *21*(2), 267–299.

Mohajan, H. K. (2022a). Four Waves of Feminism: A Blessing for Global Humanity. *Studies in Social Science & Humanities*, *1*(2), 1–8. https://doi:10.56397/SSSH.2022.09.01.

Mohajan, H. K. (2022b). An Overview on the Feminism and Its Categories. *Research and Advances in Education*, *1*(3), 11–26. https://doi.org/10.56397/RAE.2022.09.02.

Mohajan, H. K., Islam, J. N., & Moolio, P. (2013). *Optimization and Social Welfare in Economics*. Lambert Academic Publishing, Germany.

Moolio, P., Islam, J. N., & Mohajan, H. K. (2009). Output Maximization of an Agency. *Indus Journal of Management and Social Sciences*, *3*(1), 39–51.

Moscati, J. (2013). Were Jevons, Menger and Walras Really Cardinalists? On the Notion of Measurement in Utility Theory, Psychology, Mathematics and other Disciplines, 1870–1910. *History of Political Economy*, *45*(3), 373–414.

Pareto, V. (1906). *Manuale di Economia Politica, con una Introduzione alla Scienza Sociale*. Societa Editrice Libraria, Milano.

Polit, D. F., & Hungler, B. P. (2013). *Essentials of Nursing Research: Methods, Appraisal, and Utilization* (8^{th} Ed.). Philadelphia: Wolters Kluwer/Lippincott Williams and Wilkins.

Read, D. (2004). Utility Theory from Jeremy Bentham to Daniel Kahneman. Working Paper No. LSEOR 04-64, London School of Economics and Political Science.

Roy, L., Molla, R., & Mohajan, H. K. (2021). Cost Minimization is Essential for the Sustainable Development of an Industry: A Mathematical Economic Model Approach. *Annals of Spiru Haret University. Economic Series*, *21*(1), 37–69.

Samuelson, P. A. (1947). *Foundations of Economic Analysis*. Harvard University Press, Cambridge, MA.

Stigler, G. J. (1950). The Development of Utility Theory. I. *Journal of Political Economy*, *58*(4), 307–327.

Strotz, R. (1953). Cardinal Utility. *American Economic Review*, *43*(2), 384–397.

Zhao, Q., Zhang, Y., & Friedman, D. (2017). Multi-Product Utility Maximization for Economic Recommendation. *WSDM*, 435–443.