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A Study on Nonlinear Budget Constraint of a Local Industrial Firm of Bangladesh: A Profit Maximization Investigation

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Abstract

Every local or global firm expects to achieve maximum profit for its survival in sustainable economic environment. This article considers that a local industrial firm of Bangladesh has achieved maximum profit during its total operation. So that this study directly tries to calculate maximum profit and also Lagrange multiplier for profit maximization investigation by the consideration of nonlinear budget constraint. In this study Cobb-Douglas production function is considered as profit function to operate mathematical procedures. Method of Lagrange multiplier is also applied here to obtain accurate results.

Keywords: profit maximization, Lagrange multiplier, nonlinear budget constraint

1. Introduction

In the 21st century much of economic theory is presented in terms of mathematical economic models (Carter, 2001). Optimization problems run through modern economics, many with explicit economic constraints, and most of them are linear (Zheng & Liu, 2022). In this paper we have considered that the budget constraint of the firm as nonlinear (Dixit, 1990). Optimization tries to find the best available element of some function and may use a variety of different computational optimization processes (Schmedders, 2008).

In mathematical economics and social sciences, profit maximization is considered as the ability of a firm to earn the maximum profit by running the firm with minimum cost (Eaton & Lipsey, 1975). Cobb-Douglas production function is a revolutionary creation in economics that helps the firms to follow profit maximization strategy (Cobb & Douglas, 1928).

The method of Lagrange multiplier is a necessary and influential practice in multivariable calculus. It is a device for transferring a lower dimensional constrained problem to a higher dimensional unconstrained problem (Islam et al., 2010a, b, 2011).

2. Literature Review

The literature review section is an introductory portion of any kind of research, which highlights the contributions of other scholars in the same field within the existing knowledge (Polit & Hungler, 2013). It deals with a secondary research sources and does not consult about future research activities (Gibbs, 2008). Cobb-Douglas production function is one of the most widely used production function in economics. In 1928, Cobb-Douglas production function is developed by the two American scholars, mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976) (Cobb & Douglas, 1928). Later in 1984, two US scholars, mathematician John V. Baxley and economist John C. Moorhouse have discussed the profit maximization method through the mathematical formulation (Baxley & Moorhouse, 1984).

Professor of mathematics Jamal Nazrul Islam (1939-2013) and his coauthors have discussed various optimization problems by providing reasonable interpretation of the Lagrange multipliers (Islam et al., 2009a, b, 2011). Devajit Mohajan and Haradhan Kumar Mohajan have consulted profit maximization policies in some details (Mohajan & Mohajan, 2022a). Pahlaj Moolio and his coworkers have considered the Cobb-Douglas production functions to explore the profit maximization strategies in mathematical economics (Moolio et al., 2009). Steven D. Levitt reveals that profit maximizing is a common method in economics, but it is rarely tested in economics (Levitt, 2006). Lia Roy and her coauthors have worked on cost minimization for the welfare of the firms to achieve maximum profit (Roy et al., 2021). Jannatul Ferdous and Haradhan Kumar Mohajan have considered profit maximization policy very briefly, but they have proceeded carefully for the sustainability of an industry (Ferdous & Mohajan, 2022).

3. Research Methodology of the Study

Research is a careful, systematic, patient study and investigation in some field of knowledge, undertaken to establish facts or principles (Grinnell, 1993; Mohajan, 2021a). Therefore, it is an essential and powerful tool for a researcher towards the progress in academic world (Pandey & Pandey, 2015). Methodology is a system of explicit rules and procedures in which research is based and against which claims of knowledge are evaluated (Ojo, 2003). Hence, research methodology is the systematic procedure approved by researchers to solve a research problem that maps out the processes, approaches, techniques, research procedures, and instruments (Kothari, 2008). It is always conscious to identify, select, process, and analyze materials related to the topics (Somekh & Lewin, 2005).

In this study we have calculated maximum profit and Lagrange multiplier by considering Cobb-Douglas production function as our profit function, which is subject to nonlinear budget constraint (Mohajan, 2018a, 2020). In this article we have tried to maintain the rules of reliability and validity as far as possible (Mohajan, 2017b). In this paper we have dependent on secondary data sources and on our own mathematical techniques. We have consulted research papers, books and handbooks of famous authors, and also, we have collected materials from internet, websites, etc. to enrich this paper (Mohajan, 2017a, 2018b).

4. Objective of the Study

The main objective of this study is to calculate maximum profit and Lagrange multiplier for profit maximization of a local firm of Bangladesh. Other minor objectives of this article are as follows:

- to show the mathematical calculations elaborately, and
- to highlight economic concepts precisely.

5. Economic Model of Profit Function

Let us consider that a local industrial firm of Bangladesh produces and supplies its product in different locations within the country. It produces its industrial products depending on the local demand and always tries to increase local utility for the profit maximization. We assume that actually the firm makes a maximum profit. In this article we will calculate maximum profit if budget constraint of the firm is nonlinear. Let the firm uses a_1 amount of capital, a_2 quantity of labor, a_3 quantity of principal raw materials, and a_4 quantity of irregular inputs. Let us consider the Cobb-Douglas production function *f* as a profit function for our model (Cobb & Douglas, 1928; Islam et al., 2010a, b; Mohajan, 2017a),

$$P = f\left(a_1, a_2, a_3, a_4\right) = A a_1^x a_2^y a_3^z a_4^w \,, \tag{1}$$

where *A* is the efficiency parameter that reflects the level of technology, i.e., technical process, economic system, etc., which represents total factor productivity. Moreover, *A* also reflects the skill and efficient level of the workforce. Here *x*, *y*, *z*, and *w* are parameters; *x* indicates the output of elasticity of capital measures the percentage change in *P* for 1% change in a_1, while a_2, a_3, and a_4 are held constants; *y* indicates the output of elasticity of labor, *z* indicates the output of elasticity of principal raw materials, and *w* indicates the output of elasticity of other inputs in the production process, are exactly parallel to *x*. The values of *x*, *y*, *z*, and *w* are determined by available technologies. Now these four parameters *x*, *y*, *z*, and *w* must satisfy the following four inequalities (Mohajan, 2021a; Mohajan & Mohajan, 2023):

$$0 < x < 1\,, 0 < y < 1\,,\ 0 < z < 1\,, \text{and}\ 0 < w < 1\,. \tag{2}$$

A strict Cobb-Douglas production function, in which $\Gamma = x + y + z + w < 1$ indicates decreasing returns to scale, $\Gamma = 1$ indicates constant returns to scale, and $\Gamma > 1$ indicates increasing returns to scale. Now we consider that the profit function is subject to a nonlinear budget constraint,

$$B(a_1, a_2, a_3, a_4) = ka_1 + la_2 + ma_3 + n(a_4)a_4, \quad (3)$$

where *k* is rate of interest or services of capital per unit of capital a_1; *l* is the wage rate per unit of labor a_2; *m* is the cost per unit of principal raw material a_3; and *n* is the cost per unit of other inputs a_4. In nonlinear budget equation (3) we consider,

$$n(a_4) = n_0 a_4 - n_0, \quad (4)$$

where n_0 being the discounted price of the irregular inputs a_4. Therefore, the nonlinear budget constraint (3) takes the form (Mohajan, 2021b, c);

$$B(a_1, a_2, a_3, a_4) = ka_1 + la_2 + ma_3 + n_0 a_4^2 - n_0 a_4. \quad (5)$$

We now formulate the maximization problem for the profit function (1) in terms of single Lagrange multiplier λ by defining the Lagrangian function $Z(a_1, a_2, a_3, a_4, \lambda)$ as (Mohajan & Mohajan, 2022b, c),

$$Z(a_1, a_2, a_3, a_4, \lambda) = Aa_1^x a_2^y a_3^z a_4^w + \lambda\{B(a_1, a_2, a_3, a_4) - ka_1 - la_2 - ma_3 - n_0 a_4^2 + n_0 a_4\}. \quad (6)$$

Relation (6) is a 5-dimensional unconstrained problem that is obtained from (1) and 4-dimensional constrained problem (3), where Lagrange multiplier λ, is considered as a device in our model.

Since the firm makes a maximum profit, first order differentiation equals to zero; then from (6) we can write,

$$Z_\lambda = B(a_1, a_2, a_3, a_4) - ka_1 - la_2 - ma_3 - n_0 a_4^2 + n_0 a_4 = 0, \quad (7a)$$

$$Z_1 = xAa_1^{x-1} a_2^y a_3^z a_4^w - \lambda k = 0, \quad (7b)$$

$$Z_2 = yAa_1^x a_2^{y-1} a_3^z a_4^w - \lambda l = 0, \quad (7c)$$

$$Z_3 = zAa_1^x a_2^y a_3^{z-1} a_4^w - \lambda m = 0, \quad (7d)$$

$$Z_4 = wAa_1^x a_2^y a_3^z a_4^{w-1} - \lambda n_0 (2a_4 - 1) = 0. \quad (7e)$$

where $\frac{\partial Z}{\partial \lambda} = Z_\lambda$, $\frac{\partial Z}{\partial a_1} = Z_1$, $\frac{\partial Z}{\partial a_2} = Z_2$, etc. indicate first order partial differentiations of multivariate Lagrangian function.

From (7a) we get,

$$B = ka_1 + la_2 + ma_3 + n_0 a_4^2 - n_0 a_4. \quad (8)$$

From (7b) we get,

$$\lambda = \frac{Axa_1^x a_2^y a_3^z a_4^w}{ka_1} \quad (9a)$$

$$ka_1 = \frac{Axa_1^x a_2^y a_3^z a_4^w}{\lambda}. \quad (9b)$$

From (7c) we get,

$$\lambda = \frac{Aya_1^x a_2^y a_3^z a_4^w}{la_2} \tag{10a}$$

$$la_2 = \frac{Aya_1^x a_2^y a_3^z a_4^w}{\lambda}. \tag{10b}$$

From (7d) we get,

$$\lambda = \frac{Aza_1^x a_2^y a_3^z a_4^w}{ma_3} \tag{11a}$$

$$ma_3 = \frac{Aza_1^x a_2^y a_3^z a_4^w}{\lambda}. \tag{11b}$$

From (7e) we get,

$$\lambda = \frac{Awa_1^x a_2^y a_3^z a_4^w}{n_0 a_4 (2a_4 - 1)} \tag{12a}$$

$$n_0 a_4^2 - n_0 a_4 = \frac{Awa_1^x a_2^y a_3^z a_4^w}{\lambda} - n_0 a_4^2. \tag{12b}$$

Now using the necessary values from (9b), (10b), (11b), and (12b) in (8) we get,

$$B = \frac{Axa_1^x a_2^y a_3^z a_4^w}{\lambda} + \frac{Aya_1^x a_2^y a_3^z a_4^w}{\lambda} + \frac{Aza_1^x a_2^y a_3^z a_4^w}{\lambda} + \frac{Awa_1^x a_2^y a_3^z a_4^w}{\lambda} - n_0 a_4^2$$

$$B = \frac{Aa_1^x a_2^y a_3^z a_4^w}{\lambda}\Gamma - n_0 a_4^2 \tag{13}$$

where $\Gamma = x + y + z + w$. Now we use the value of λ from (12a) in (13) to obtain;

$$B = \frac{Aa_1^x a_2^y a_3^z a_4^w}{\dfrac{Awa_1^x a_2^y a_3^z a_4^w}{n_0 a_4 (2a_4 - 1)}}\Gamma - n_0 a_4^2$$

$$a_4^2 (2\Gamma - w) - a_4 \Gamma - Bw / n_0 = 0$$

$$a_4 = \frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)} \tag{14}$$

where $\Gamma = x + y + z + w$. In the next step we use the value of λ from (9a) and value of a_4 from (14) in (13) we get (Mohajan, 2021a; Mohajan & Mohajan, 2022c, d);

$$B = \frac{Aa_1^x a_2^y a_3^z a_4^w}{\dfrac{Axa_1^x a_2^y a_3^z a_4^w}{ka_1}}\Gamma - n_0 \left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2$$

$$a_1 = \frac{Bx}{k\Gamma} + \frac{n_0 x}{k\Gamma}\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2. \tag{15}$$

Now we use the value of λ from (10a) and value of a_4 from (14) in (13) we get;

$$B = \frac{Aa_1^x a_2^y a_3^z a_4^w}{\frac{Aya_1^x a_2^y a_3^z a_4^w}{la_2}}\Gamma - n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2$$

$$a_2 = \frac{By}{l\Gamma} + \frac{n_0 y}{l\Gamma}\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2. \tag{16}$$

Now we use the value of λ from (11a) and value of a_4 from (14) in (13) we get;

$$B = \frac{Aa_1^x a_2^y a_3^z a_4^w}{\frac{Aza_1^x a_2^y a_3^z a_4^w}{ma_3}}\Gamma - n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2$$

$$a_3 = \frac{Bz}{m\Gamma} + \frac{n_0 z}{m\Gamma}\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2. \tag{17}$$

The stationary point for the production can be written as, (a_1, a_2, a_3, a_4) where a_1, a_2, a_3, and a_4 are given above.

Now putting the values of a_1, a_2, a_3, and a_4 from (14), (15), (16), and (17) in (9a) we get (Mohajan & Mohajan, 2022e,f),

$$\lambda = \frac{Ax^x y^y z^z}{k^x l^y m^z \Gamma^{\Gamma - w - 1}}\left\{B + n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2\right\}^{x-1}\left\{B + n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2\right\}^{y}$$

$$\times\left\{B + n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2\right\}^{z}\left\{\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right\}^{w}. \tag{18}$$

Now substituting the values of a_1, a_2, a_3, and a_4 from (14), (15), (16), and (17) in (1) we get the profit function,

$$P = \frac{Ax^x y^y z^z}{k^x l^y m^z \Gamma^{\Gamma - w}}\left\{B + n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2\right\}^{x}\left\{B + n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2\right\}^{y}$$

$$\left\{B + n_0\left[\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right]^2\right\}^{z}\left\{\frac{\Gamma \pm \sqrt{\Gamma^2 + 4Bw(2\Gamma - w)/n_0}}{2(2\Gamma - w)}\right\}^{w}. \tag{19}$$

Equation (19) is the profit function in terms of $k,l,m,n,A,B>0$, $x,y,z,w>0$, and $\Gamma=x+y+z+w>0$. All the parameters in right hand side of (19) are known to the firm and can easily calculate its maximum profit.

6. Conclusions

In this study we have calculated maximum profit of a local industrial firm of Bangladesh. We have considered that the firm achieves a maximum profit by the consideration of nonlinear budget constraint. We have also calculated Lagrange multiplier for the maximum profit. We have analyzed the Cobb-Douglas production function as our profit function with the subject to nonlinear constraint of budget.

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