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Various Problems Arise in Industrial Economics If Wage Rate Increases: A Study for Nonlinear Budget Constraint

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Abstract

This study tries to discuss the economic effects of various inputs if the wage rate of a firm is increased. Cobb-Douglas production function, 6×6 bordered Hessian matrix, and 6×6 Jacobian are used here during the mathematical calculations to investigate economic predictions. Adjustment of various inputs and outputs in future production are essential for the profit maximization. Therefore, appropriate decisions can make the firm stronger to face the various challenges of the twenty first century. In the study profit maximization is considered with subject to the nonlinear budget constraint.

Keywords: profit maximization, nonlinear budget constraint, wage rate

1. Introduction

In economics mathematical modeling is a popular method to operate optimization problems (Samuelson, 1947). At present use of it is common to the mathematical economists (Carter, 2001). It plays a leading role in modern economics for the development of local and global financial structures (Ferdous & Mohajan, 2022). It influences the economists to develop the economic area and to increase the welfare of the society (Eaton & Lipsey, 1975).

In crude sense, profit is defined as the difference between total revenue and economic costs (Tripathi, 2019). Profit maximization practice is essential for the sustainability of an industrial firm. In multivariable calculus, the method of Lagrange multiplier is a very useful and powerful technique (Islam et al., 2010). A firm needs to be sincere during the production, financing, marketing, transportation, and overall management system for the sustainability in the competitive global economy (Mohajan & Mohajan, 2023g).

At the start we have used Cobb-Douglas production function, and then we have used the determinant of 6×6 bordered Hessian matrix, and 6×6 Jacobian (Cobb & Douglas, 1928). We have analyzed four input variables, such as capital, labor, principal raw materials, and irregular input. Throughout the study we have tried to provide mathematical calculation in simple forms.

2. Literature Review

Literature review is an opening section of a research. It deals with a secondary research source, where a researcher shows the works of previous researchers in the same field (Polit & Hungler, 2013). Two US scholars Charles W. Cobb (1875-1949) and Paul H. Douglas (1892-1976) for the first time have established the functional distribution of income between capital and labor (Cobb & Douglas, 1928). Other two US professors John V. Baxley and John C. Moorhouse have developed the mathematical formulation of optimization problem (Baxley & Moorhouse, 1984). Professor Jamal Nazrul Islam and his coauthors have revised the optimization problems properly giving interpretation of the Lagrange multipliers (Islam et al., 2009a, b, 2010, 2011).

Lia Roy and her coauthors have discussed the cost minimization of an industry (Roy et al., 2021). Jannatul Ferdous and Haradhan Kumar Mohajan have reviewed on a profit maximization problem (Mohajan, 2022; Ferdous & Mohajan, 2022). In a series of papers Devajit Mohajan and Haradhan Kumar Mohajan have discussed profit maximization and utility maximization with sensitivity analysis (Mohajan & Mohajan, 2022a-f, 2023 a-g). Pahlaj Moolio and his coauthors have worked on output maximization structure (Moolio et al., 2009).

3. Research Methodology of the Study

Research is a vital part for professors to develop their academic area (Pandey & Pandey, 2015).

On the other hand, methodology is a guideline to prepare a good research (Kothari, 2008). Therefore, in brief, a research methodology is the collection of a set of principles for planning, designing, organizing, and conducting a good research (Legesse, 2014).

In this study we have taken an attempt to discuss the future economic outcomes of an industrial firm when wage rate is increased (Mohajan, 2018a). Usually, economic firms do not want to increase the wage rate. But, sometimes these firms compel to increase the wage rate (Mohajan, 2017b, 2020). In this study we have tried to show some results on four inputs if wage rate of the firm is really increased.

We have started our main section with the Cobb-Douglas production function, which we have considered as our profit function. To show the economic effects of four inputs we have used Lagrange multiplier, 6×6 bordered Hessian matrix, and 6×6 Jacobian. In this study we have taken helps from the secondary data sources of optimization, such as published and unpublished articles, books, conference papers, and also from internet, websites, etc. (Mohajan, 2018b).

4. Objective of the Study

The main objective of this study is to discuss the effects of various inputs when wage rate of the workers is increased. In a firm efficient use of expert workers can develop the firm to the profit maximization atmosphere. In this study we have taken a nonlinear budget constraint. Other subsidiary objectives of the study are as follows:

- to explain the mathematical terms properly, and
- to provide the economic predictions precisely.

5. Lagrangian Function

Let us consider that an industrial firm is willing to make a maximum profit from its products. Let the firm uses ζ_1 amount of capital, ζ_2 quantity of labor, ζ_3 quantity of principal raw materials, and ζ_4 quantity of irregular input for its annual production. Let us consider the Cobb-Douglas production function $f(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ as a profit function for our model (Cobb & Douglas, 1928; Mohajan & Mohajan, 2022a),

$$P(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = f(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta}, \tag{1}$$

where *A* is the efficiency parameter that reflects the level of technology, i.e., technical process, economic system, etc., which represents total factor productivity. Moreover, *A* reflects the skill and efficient level of the workforce. Here α , β , γ , and δ are parameters; α indicates the output of elasticity of capital measures the percentage change in $P(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ for 1% change in ζ_1, while ζ_2, ζ_3, and ζ_4 are held constants. Similarly, β indicates the output of elasticity of labor, γ indicates the output of elasticity of principal raw materials, and δ indicates the output of elasticity of irregular input. Now these four parameters α , β , γ , and δ must satisfy the following four inequalities (Islam et al., 2010; Mohajan, 2021c):

$$0 < \alpha < 1,\ 0 < \beta < 1,\ 0 < \gamma < 1, \text{ and } 0 < \delta < 1. \tag{2}$$

A strict Cobb-Douglas production function, in which $\exists = \alpha + \beta + \gamma + \delta < 1$ indicates decreasing returns to scale, $\exists = 1$ indicates constant returns to scale, and $\exists > 1$ indicates increasing returns to scale. Now we consider that the profit function is subject to a nonlinear budget constraint as (Mohajan & Mohajan, 2022b),

$$B(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = k\zeta_1 + l\zeta_2 + m\zeta_3 + n(\zeta_4)\zeta_4, \tag{3}$$

where *k* is rate of interest or services of capital per unit of capital ζ_1; *l* is the wage rate per unit of labor ζ_2; *m* is the cost per unit of principal raw material ζ_3; and *n* is the cost per unit of irregular input ζ_4. In nonlinear budget equation (3) we consider (Mohajan & Mohajan, 2023a),

$$n(\zeta_4) = n_0\zeta_4 - n_0\,, \tag{4}$$

where n_0 being the discounted price of the irregular input ζ_4. Therefore, the nonlinear budget constraint (3) takes the form (Mohajan, 2021b; Mohajan & Mohajan, 2023b);

$$B(\zeta_1,\zeta_2,\zeta_3,\zeta_4) = k\zeta_1 + l\zeta_2 + m\zeta_3 + n_0\zeta_4^2 - n_0\zeta_4\,. \tag{5}$$

We now formulate the maximization problem for the profit function (1) in terms of single Lagrange multiplier λ by defining the Lagrangian function $u(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\lambda)$ as (Mohajan & Mohajan, 2023f),

$$u(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\lambda) = A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta} + \lambda\{B(\zeta_1,\zeta_2,\zeta_3,\zeta_4) - k\zeta_1 - l\zeta_2 - m\zeta_3 - n_0\zeta_4^2 + n_0\zeta_4\}. \tag{6}$$

Relation (6) is a 5-dimensional unconstrained problem that is obtained from (1) and 4-dimensional constrained problem (3), where Lagrange multiplier λ, is considered as a device in our profit maximization model.

6. Analysis on Four Inputs

For maximization, first order differentiation equals to zero; then from (6) we can write (Mohajan, 2022; Mohajan & Mohajan, 2023e),

$$u_\lambda = B - k\zeta_1 - l\zeta_2 - m\zeta_3 - n_0\zeta_4^2 + n_0\zeta_4 = 0\,, \tag{7a}$$

$$u_1 = \alpha A\zeta_1^{\alpha-1}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta} - \lambda k = 0\,, \tag{7b}$$

$$u_2 = \beta A\zeta_1^{\alpha}\zeta_2^{\beta-1}\zeta_3^{\gamma}\zeta_4^{\delta} - \lambda l = 0\,, \tag{7c}$$

$$u_3 = \gamma A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma-1}\zeta_4^{\delta} - \lambda m = 0\,, \tag{7d}$$

$$u_4 = \delta A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta-1} - \lambda n_0(2\zeta_4 - 1) = 0\,, \tag{7e}$$

where, $\frac{\partial u}{\partial \lambda} = u_\lambda$, $\frac{\partial u}{\partial \zeta_1} = u_1$, $\frac{\partial u}{\partial \zeta_2} = u_2$, etc. indicate first-order partial differentiations of multivariate Lagrangian function.

Using equations (2) to (7) we can determine the values of ζ_1, ζ_2, ζ_3, and ζ_4 as follows (Ferdous & Mohajan 2022; Mohajan & Mohajan, 2022a):

$$\zeta_1 = \frac{\alpha B}{k\,\exists}\,, \tag{8a}$$

$$\zeta_2 = \frac{\beta B}{l\,\exists}\,, \tag{8b}$$

$$\zeta_3 = \frac{\gamma B}{m\,\exists}\,, \tag{8c}$$

$$\zeta_4 = \frac{\delta B}{n\,\exists}\,. \tag{8d}$$

7. Bordered Hessian Matrix Analysis

Let us consider the determinant of the 5×5 bordered Hessian matrix as (Islam et al. 2011; Mohajan & Mohajan, 2022g),

$$|H| = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & u_{11} & u_{12} & u_{13} & u_{14} \\ -B_2 & u_{21} & u_{22} & u_{23} & u_{24} \\ -B_3 & u_{31} & u_{32} & u_{33} & u_{34} \\ -B_4 & u_{41} & u_{42} & u_{43} & u_{44} \end{vmatrix}. \quad (9)$$

Taking first-order partial differentiations of (5) we get,

$$B_1 = k,\ B_2 = l,\ B_3 = m, \text{ and } B_4 = 2n_0\zeta_4 - n_0. \quad (10)$$

Taking second-order and cross-partial derivatives of (6) we get (Roy et al., 2021; Mohajan & Mohajan, 2023c),

$$u_{11} = \alpha(\alpha-1)A\zeta_1^{\alpha-2}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta},$$

$$u_{22} = \beta(\beta-1)A\zeta_1^{\alpha}\zeta_2^{\beta-2}\zeta_3^{\gamma}\zeta_4^{\delta},$$

$$u_{33} = \gamma(\gamma-1)A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma-2}\zeta_4^{\delta},$$

$$u_{44} = \delta(\delta-1)A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta-2},$$

$$u_{12} = u_{21} = \alpha\beta A\zeta_1^{\alpha-1}\zeta_2^{\beta-1}\zeta_3^{\gamma}\zeta_4^{\delta},$$

$$u_{13} = u_{31} = \alpha\gamma A\zeta_1^{\alpha-1}\zeta_2^{\beta}\zeta_3^{\gamma-1}\zeta_4^{\delta},$$

$$u_{14} = u_{41} = \alpha\delta A\zeta_1^{\alpha-1}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta-1}, \quad (11)$$

$$u_{23} = u_{32} = \beta\gamma A\zeta_1^{\alpha}\zeta_2^{\beta-1}\zeta_3^{\gamma-1}\zeta_4^{\delta},$$

$$u_{24} = u_{42} = \beta\delta A\zeta_1^{\alpha}\zeta_2^{\beta-1}\zeta_3^{\gamma}\zeta_4^{\delta-1},$$

$$u_{34} = u_{43} = \gamma\delta A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma-1}\zeta_4^{\delta-1}.$$

where $\frac{\partial^2 u}{\partial\zeta_1\partial\zeta_2} = u_{12} = u_{21}$, $\frac{\partial^2 u}{\partial\zeta_2^2} = u_{22}$, etc. indicate cross-partial, second order differentiations of multivariate Lagrangian function, respectively, etc.

Now we expand the Hessian (9) as $|H| > 0$ (Mohajan et al., 2013; Mohajan & Mohajan, 2022d),

$$|H| = \frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B^2}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2\exists^2}(\alpha+\beta+\gamma+\delta)(\delta+3) > 0, \quad (12)$$

where efficiency parameter, $A > 0$, and budget of the firm, $B > 0$; ζ_1, ζ_2, ζ_3, and ζ_4 are four different types of inputs; and consequently, $\zeta_1, \zeta_2, \zeta_3, \zeta_4 > 0$. Parameters, $\alpha, \beta, \gamma, \delta > 0$; also in the model either $0 < \exists = \alpha + \beta + \gamma + \delta < 1$, $\exists = 1$ or $\exists > 1$. Hence, equation (12) gives; $|H| > 0$ (Islam et al., 2011; Mohajan & Mohajan, 2022d).

8. Determination of Lagrange Multiplier λ

Now using the necessary values from (8) in (7a) we get (Islam et al., 2010; Mohajan & Mohajan, 2023g),

$$B=\frac{\alpha A\zeta_1^{\alpha}\zeta_1^{\beta}\zeta_1^{\gamma}\zeta_1^{\delta}}{\lambda}+\frac{\beta A\zeta_1^{\alpha}\zeta_1^{\beta}\zeta_1^{\gamma}\zeta_1^{\delta}}{\lambda}+\frac{\gamma A\zeta_1^{\alpha}\zeta_1^{\beta}\zeta_1^{\gamma}\zeta_1^{\delta}}{\lambda}+\frac{\delta A\zeta_1^{\alpha}\zeta_1^{\beta}\zeta_1^{\gamma}\zeta_1^{\delta}}{\lambda}$$

$$\lambda=\frac{A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta}\exists}{B}. \tag{13}$$

9. Jacobian Matrix Analysis

We have observed that the second-order condition is satisfied, so that the determinant of (5) survives at the optimum, i.e., $|J|=|H|$; and hence, we can apply the implicit function theorem. Now we compute twenty-five partial derivatives, such as $\frac{\partial\lambda}{\partial k}$, $\frac{\partial\zeta_1}{\partial k}$, $\frac{\partial\zeta_3}{\partial l}$, $\frac{\partial\zeta_4}{\partial B}$, etc. that are referred to as the comparative statics of the model (Chiang, 1984; Mohajan & Mohajan, 2023e).

Let $\mathbf{G}$ be the vector-valued function of ten variables $\lambda^*,\zeta_1^*,\zeta_2^*,\zeta_3^*,\zeta_4^*,k,l,m,n$, and B, and we define the function $\mathbf{G}$ for the point $(\lambda^*,\zeta_1^*,\zeta_2^*,\zeta_3^*,\zeta_4^*,k,l,m,n,B)\in R^{10}$, and take the values in R^5. By the Implicit Function Theorem of multivariable calculus, the equation (Mohajan & Mohajan, 2022e),

$$F(\lambda^*,\zeta_1^*,\zeta_2^*,\zeta_3^*,\zeta_4^*,k,l,m,n,B)=0, \tag{14}$$

may be solved in the form of

$$\begin{bmatrix}\lambda\\ \zeta_1\\ \zeta_2\\ \zeta_3\\ \zeta_4\end{bmatrix}=\mathbf{G}(k,l,m,n,B). \tag{15}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}(k,l,m,n,B)$; regarded as $J_G=\frac{\partial(\lambda,\zeta_1,\zeta_2,\zeta_3,\zeta_4)}{\partial(k,l,m,n_0,B)}$, and is represented by;

$$J_G=\begin{bmatrix}\frac{\partial\lambda}{\partial k} & \frac{\partial\lambda}{\partial l} & \frac{\partial\lambda}{\partial m} & \frac{\partial\lambda}{\partial n_0} & \frac{\partial\lambda}{\partial B}\\ \frac{\partial\zeta_1}{\partial k} & \frac{\partial\zeta_1}{\partial l} & \frac{\partial\zeta_1}{\partial m} & \frac{\partial\zeta_1}{\partial n_0} & \frac{\partial\zeta_1}{\partial B}\\ \frac{\partial\zeta_2}{\partial k} & \frac{\partial\zeta_2}{\partial l} & \frac{\partial\zeta_2}{\partial m} & \frac{\partial\zeta_2}{\partial n_0} & \frac{\partial\zeta_2}{\partial B}\\ \frac{\partial\zeta_3}{\partial k} & \frac{\partial\zeta_3}{\partial l} & \frac{\partial\zeta_3}{\partial m} & \frac{\partial\zeta_3}{\partial n_0} & \frac{\partial\zeta_3}{\partial B}\\ \frac{\partial\zeta_4}{\partial k} & \frac{\partial\zeta_4}{\partial l} & \frac{\partial\zeta_4}{\partial m} & \frac{\partial\zeta_4}{\partial n_0} & \frac{\partial\zeta_4}{\partial B}\end{bmatrix}. \tag{16}$$

$$= -J^{-1}\begin{bmatrix} -\zeta_1 & -\zeta_2 & -\zeta_3 & -\zeta_4^2+\zeta_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda\zeta_4+\lambda & 0 \end{bmatrix}.$$

The inverse of Jacobian matrix is, $J^{-1} = \frac{1}{|J|}C^T$, where $C = \left(C_{ij}\right)$, the matrix of cofactors of J, where T for transpose, then (16) becomes (Mohajan, 2021c; Moolio et al., 2009),

$$= -\frac{1}{|J|}\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} -\zeta_1 & -\zeta_2 & -\zeta_3 & -\zeta_4^2+\zeta_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda\zeta_4+\lambda & 0 \end{bmatrix}$$

$$J_G = -\frac{1}{|J|}\begin{bmatrix} -\zeta_1C_{11}-\lambda C_{21} & -\zeta_2C_{11}-\lambda C_{31} & -\zeta_3C_{11}-\lambda C_{41} & -\zeta_4^2C_{11}+\zeta_4C_{11}-2\lambda\zeta_4C_{51}+\lambda C_{51} & C_{11} \\ -\zeta_1C_{12}-\lambda C_{22} & -\zeta_2C_{12}-\lambda C_{32} & -\zeta_3C_{12}-\lambda C_{42} & -\zeta_4^2C_{12}+\zeta_4C_{12}-2\lambda\zeta_4C_{52}+\lambda C_{52} & C_{12} \\ -\zeta_1C_{13}-\lambda C_{23} & -\zeta_2C_{13}-\lambda C_{33} & -\zeta_3C_{13}-\lambda C_{43} & -\zeta_4^2C_{13}+\zeta_4C_{13}-2\lambda\zeta_4C_{53}+\lambda C_{53} & C_{13} \\ -\zeta_1C_{14}-\lambda C_{24} & -\zeta_2C_{14}-\lambda C_{34} & -\zeta_3C_{14}-\lambda C_{44} & -\zeta_4^2C_{14}+\zeta_4C_{14}-2\lambda\zeta_4C_{54}+\lambda C_{54} & C_{14} \\ -\zeta_1C_{15}-\lambda C_{25} & -\zeta_2C_{15}-\lambda C_{35} & -\zeta_3C_{15}-\lambda C_{45} & -\zeta_4^2C_{15}+\zeta_4C_{15}-2\lambda\zeta_4C_{55}+\lambda C_{55} & C_{15} \end{bmatrix}. \quad (17)$$

In (17) total 25 comparative statics are available, and in this study, we deal only with four of them when wage rate is increased. The firm always attempts for the profit maximization production (Baxley & Moorhouse, 1984; Islam et al., 2010).

10. Sensitivity Analysis

Now we observe the effects on capital ζ_1 when the wage rate, *l* increases. Taking T_{22} (i.e., term of 2nd row and 2nd column) from both sides of (17) we get (Mohajan, 2021b; Wiese, 2021),

$$\frac{\partial\zeta_1}{\partial l} = \frac{\zeta_2}{|J|}\left[C_{12}\right] + \frac{\lambda}{|J|}\left[C_{32}\right]$$

$$= \frac{\zeta_2}{|J|}\text{Cofactor of } C_{12} + \frac{\lambda}{|J|}\text{Cofactor of } C_{32}$$

$$= -\frac{\zeta_2}{|J|}\begin{vmatrix} -B_1 & u_{12} & u_{13} & u_{14} \\ -B_2 & u_{22} & u_{23} & u_{24} \\ -B_3 & u_{32} & u_{33} & u_{34} \\ -B_4 & u_{42} & u_{43} & u_{44} \end{vmatrix} - \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & u_{12} & u_{13} & u_{14} \\ -B_3 & u_{32} & u_{33} & u_{34} \\ -B_4 & u_{42} & u_{43} & u_{44} \end{vmatrix}$$

$$= -\frac{\zeta_2}{|J|}\left\{ -B_1\begin{vmatrix} u_{22} & u_{23} & u_{24} \\ u_{32} & u_{33} & u_{34} \\ u_{42} & u_{43} & u_{44} \end{vmatrix} - u_{12}\begin{vmatrix} -B_2 & u_{23} & u_{24} \\ -B_3 & u_{33} & u_{34} \\ -B_4 & u_{43} & u_{44} \end{vmatrix} + u_{13}\begin{vmatrix} -B_2 & u_{22} & u_{24} \\ -B_3 & u_{32} & u_{34} \\ -B_4 & u_{42} & u_{44} \end{vmatrix} - u_{14}\begin{vmatrix} -B_2 & u_{22} & u_{23} \\ -B_3 & u_{32} & u_{33} \\ -B_4 & u_{42} & u_{43} \end{vmatrix} \right\}$$

$$-\frac{\lambda}{|J|}\left\{B_2\begin{vmatrix}-B_1 & u_{13} & u_{14}\\ -B_3 & u_{33} & u_{34}\\ -B_4 & u_{43} & u_{44}\end{vmatrix}-B_3\begin{vmatrix}-B_1 & u_{12} & u_{14}\\ -B_3 & u_{32} & u_{34}\\ -B_4 & u_{42} & u_{44}\end{vmatrix}-B_4\begin{vmatrix}-B_1 & u_{12} & u_{13}\\ -B_3 & u_{32} & u_{33}\\ -B_4 & u_{42} & u_{43}\end{vmatrix}\right\}$$

$$=-\frac{\zeta_2}{|J|}\left[-B_1\{u_{22}(u_{33}u_{44}-u_{43}u_{34})+u_{23}(u_{42}u_{34}-u_{32}u_{44})+u_{24}(u_{32}u_{43}-u_{42}u_{33})\}\right.$$

$$-u_{12}\{-B_2(u_{33}u_{44}-u_{43}u_{34})+u_{23}(-B_4u_{34}+B_3u_{44})+u_{24}(-B_3u_{43}+B_4u_{33})\}$$

$$+u_{13}\{-B_2(u_{32}u_{44}-u_{42}u_{34})+u_{22}(-B_4u_{34}+B_3u_{44})+u_{24}(-B_3u_{42}+B_4u_{32})\}$$

$$\left.-u_{14}\{-B_2(u_{32}u_{43}-u_{42}u_{33})+u_{22}(-B_4u_{33}+B_3u_{43})+u_{23}(-B_3u_{42}+B_du_{32})\}\right]$$

$$-\frac{\lambda}{|J|}\left[B_2\{-B_1(u_{33}u_{44}-u_{43}u_{34})-u_{13}(-B_3u_{44}+B_4u_{34})+u_{14}(-B_3u_{43}+B_4u_{33})\}\right.$$

$$-B_3\{-B_1(u_{32}u_{44}-u_{42}u_{34})-u_{12}(-B_3u_{44}+B_4u_{34})+u_{14}(-B_3u_{42}+B_4u_{32})\}$$

$$\left.-B_4\{-B_1(u_{32}u_{43}-u_{42}u_{33})-u_{12}(-B_3u_{43}+B_4u_{33})+u_{13}(-B_3u_{42}+B_4u_{32})\}\right]$$

$$=-\frac{\zeta_2}{|J|}\{-B_1u_{22}u_{33}u_{44}+B_1u_{22}u_{43}u_{34}-B_1u_{23}u_{42}u_{34}+B_1u_{23}u_{32}u_{44}-B_1u_{24}u_{32}u_{43}+B_1u_{24}u_{42}u_{33}$$

$$+B_2u_{12}u_{33}u_{44}-B_2u_{12}u_{43}u_{34}+B_4u_{12}u_{23}u_{34}-B_3u_{12}u_{23}u_{44}+B_3u_{12}u_{24}u_{43}-B_4u_{12}u_{24}u_{33}-B_2u_{13}u_{32}u_{44}$$

$$+B_2u_{13}u_{42}u_{34}-B_4u_{13}u_{22}u_{34}+B_3u_{13}u_{22}u_{44}-B_3u_{13}u_{24}u_{42}+B_4u_{13}u_{24}u_{32}+B_2u_{14}u_{32}u_{43}-B_2u_{14}u_{42}u_{33}$$

$$+B_4u_{14}u_{22}u_{33}-B_3u_{14}u_{22}u_{43}+B_3u_{14}u_{23}u_{42}-B_4u_{14}u_{23}u_{32}\}-\frac{\lambda}{|J|}\{-B_1B_2u_{33}u_{44}+B_1B_2u_{43}u_{34}$$

$$-B_2B_3u_{13}u_{44}-B_2B_4u_{13}u_{34}-B_2B_3u_{14}u_{43}+B_2B_4u_{14}u_{33}+B_1B_3u_{32}u_{44}-B_1B_3u_{42}u_{34}-B_3^2u_{12}u_{44}$$

$$+B_3B_4u_{12}u_{34}+B_3^2u_{14}u_{42}-B_3B_4u_{14}u_{32}+B_1B_4u_{32}u_{43}-B_1B_4u_{42}u_{33}-B_3B_4u_{12}u_{43}+B_4^2u_{12}u_{33}$$

$$+B_3B_4u_{13}u_{42}-B_4^2u_{13}u_{32}\}$$

$$=-\frac{\zeta_2}{|J|}\{-B_1u_{22}u_{33}u_{44}+B_1u_{22}u_{34}^2-B_1u_{23}u_{42}u_{34}+B_1u_{23}^2u_{44}-B_1u_{24}u_{32}u_{43}+B_1u_{24}^2u_{33}+B_2u_{12}u_{33}u_{44}$$

$$-B_2u_{12}u_{34}^2+B_4u_{12}u_{23}u_{34}-B_3u_{12}u_{23}u_{44}+B_3u_{12}u_{24}u_{43}-B_4u_{12}u_{24}u_{33}-B_2u_{13}u_{32}u_{44}+B_2u_{13}u_{42}u_{34}$$

$$-B_4u_{13}u_{22}u_{34}+B_3u_{13}u_{22}u_{44}-B_3u_{13}u_{24}^2+B_4u_{13}u_{24}u_{32}+B_2u_{14}u_{32}u_{43}-B_2u_{14}u_{42}u_{33}+B_4u_{14}u_{22}u_{33}$$

$$-B_3u_{14}u_{22}u_{43}+B_3u_{14}u_{23}u_{42}-B_4u_{14}u_{23}^2\}-\frac{\lambda}{|J|}\{-B_1B_2u_{33}u_{44}+B_1B_2u_{34}^2-B_2B_3u_{13}u_{44}-B_2B_4u_{13}u_{34}$$

$$+B_2B_3u_{14}u_{43} + B_2B_4u_{14}u_{33} + B_1B_3u_{32}u_{44} - B_1B_3u_{42}u_{34} - B_3^2u_{12}u_{44} + B_3B_4u_{12}u_{34} + B_3^2u_{14}u_{42}$$

$$-B_3B_4u_{14}u_{32} + B_1B_4u_{32}u_{43} - B_1B_4u_{42}u_{33} - B_3B_4u_{12}u_{43} + B_4^2u_{12}u_{33} + B_3B_4u_{13}u_{42} - B_4^2u_{13}u_{32}\}$$

$$= -\frac{1}{|J|}\frac{A^3\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}}{\zeta_1^2\zeta_2\zeta_3^2\zeta_4^2} \{-k\zeta_1^2\beta(\beta-1)\gamma(\gamma-1)\delta(\delta-1) + k\zeta_1^2\beta(\beta-1)\gamma^2\delta^2 - k\zeta_1^2\beta^2\gamma^2\delta^2$$

$$+k\zeta_1^2\beta^2\gamma^2\delta(\delta-1) - k\zeta_1^2\beta^2\gamma^2\delta^2 + k\zeta_1^2\beta^2\gamma(\gamma-1)\delta^2 + l\zeta_1\zeta_2\alpha\beta\gamma(\gamma-1)\delta(\delta-1) - l\zeta_1\zeta_2\alpha\beta\gamma^2\delta^2$$

$$+n\zeta_1\zeta_4\alpha\beta^2\gamma^2\delta - m\zeta_1\zeta_3\alpha\beta^2\gamma\delta(\delta-1) + m\zeta_1\zeta_3\alpha\beta^2\gamma\delta^2 - n\zeta_1\zeta_4\alpha\beta^2\gamma(\gamma-1)\delta$$

$$-l\zeta_1\zeta_2\alpha\beta\gamma^2\delta(\delta-1) + l\zeta_1\zeta_2\alpha\beta\gamma^2\delta^2 - n\zeta_1\zeta_4\alpha\beta(\beta-1)\gamma^2\delta + m\zeta_1\zeta_3\alpha\beta(\beta-1)\gamma\delta(\delta-1)$$

$$-m\zeta_1\zeta_3\alpha\beta^2\gamma\delta^2 + n\zeta_1\zeta_4\alpha\beta^2\gamma^2\delta + l\zeta_1\zeta_2\alpha\beta\gamma^2\delta^2 - l\zeta_1\zeta_2\alpha\beta\gamma(\gamma-1)\delta^2$$

$$+n\zeta_1\zeta_4\alpha\beta(\beta-1)\gamma(\gamma-1)\delta - m\zeta_1\zeta_3\alpha\beta(\beta-1)\gamma\delta^2 + m\zeta_1\zeta_3\alpha\beta^2\gamma\delta^2 - n\zeta_1\zeta_4\alpha\beta^2\gamma^2\delta\}$$

$$-\frac{\lambda}{|J|}\frac{A^2\zeta_1^{2\alpha}\zeta_2^{2\beta}\zeta_3^{2\gamma}\zeta_4^{2\delta}}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2} \{-kl\zeta_1^2\zeta_2^2\gamma(\gamma-1)\delta(\delta-1) + kl\zeta_1^2\zeta_2^2\gamma^2\delta^2 - lm\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta(\delta-1)$$

$$-nl\zeta_1\zeta_2^2\zeta_4\alpha\gamma^2\delta^2 + lm\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta^2 + nl\zeta_1\zeta_2^2\zeta_4\alpha\gamma(\gamma-1)\delta^2 + km\zeta_1^2\zeta_2\zeta_3\gamma\delta(\delta-1)$$

$$-km\zeta_1^2\zeta_2\zeta_3\gamma\delta(\delta-1) - m^2\zeta_1\zeta_2\zeta_3^2\alpha\beta\delta(\delta-1) + mn\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\delta^2 + m^2\zeta_1\zeta_2\zeta_3^2\alpha\beta\delta^2$$

$$-mn\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta - kn\zeta_1^2\zeta_2\zeta_4\beta\gamma(\gamma-1)\delta + kn\zeta_1^2\zeta_2\zeta_4\beta\gamma^2\delta - mn\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta$$

$$+n^2\zeta_1\zeta_2\zeta_4^2\alpha\beta\gamma(\gamma-1) + mn\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta - n^2\zeta_1\zeta_2\zeta_4^2\alpha\beta\gamma^2\}$$

$$= -\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}}{\zeta_1^2\zeta_2\zeta_3^2\zeta_4^2} \{-k\zeta_1^2\alpha^{-1}(\beta-1)(\gamma-1)(\delta-1) + k\zeta_1^2\alpha^{-1}(\beta-1)\gamma\delta - k\zeta_1^2\alpha^{-1}\beta\gamma\delta$$

$$+k\zeta_1^2\alpha^{-1}\beta\gamma(\delta-1) - k\zeta_1^2\alpha^{-1}\beta\gamma\delta + k\zeta_1^2\alpha^{-1}\beta(\gamma-1)\delta + l\zeta_1\zeta_2(\gamma-1)(\delta-1) - l\zeta_1\zeta_2\gamma\delta + n\zeta_1\zeta_4\beta\gamma$$

$$-m\zeta_1\zeta_3\beta(\delta-1) + m\zeta_1\zeta_3\beta\delta - n\zeta_1\zeta_4\gamma(\delta-1) - l\zeta_1\zeta_2\gamma(\delta-1) + l\zeta_1\zeta_2\gamma\delta - n\zeta_1\zeta_4(\beta-1)\gamma$$

$$+m\zeta_1\zeta_3(\beta-1)(\delta-1) - m\zeta_1\zeta_3\beta\delta + n\zeta_1\zeta_4\beta\gamma + l\zeta_1\zeta_2\gamma\delta - l\zeta_1\zeta_2(\gamma-1)\delta + n\zeta_1\zeta_4(\beta-1)(\gamma-1)$$

$$+m\zeta_1\zeta_3(\beta-1)\delta + m\zeta_1\zeta_3\beta\delta - n\zeta_1\zeta_4\beta\gamma\} -\frac{1}{|J|}\frac{A^2\gamma\delta\zeta_1^{2\alpha}\zeta_2^{2\beta}\zeta_3^{2\gamma}\zeta_4^{2\delta}}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2} \frac{A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta}\exists}{B}$$

$$\{-kl\zeta_1^2\zeta_2^2(\gamma-1)(\delta-1) + kl\zeta_1^2\zeta_2^2\gamma\delta - lm\zeta_1\zeta_2^2\zeta_3\alpha(\delta-1) - nl\zeta_1\zeta_2^2\zeta_4\alpha\gamma + lm\zeta_1\zeta_2^2\zeta_3\alpha\delta$$

$$+nl\zeta_1\zeta_2^2\zeta_4\alpha(\gamma-1) - m^2\zeta_1\zeta_2\zeta_3^2\alpha\beta\gamma^{-1}(\delta-1) + m^2\zeta_1\zeta_2\zeta_3^2\alpha\beta\gamma^{-1}\delta - kn\zeta_1^2\zeta_2\zeta_4\beta(\gamma-1)$$

$$-kn\zeta_1^2\zeta_2\zeta_4\beta\gamma + n^2\zeta_1\zeta_2\zeta_4^2\alpha\beta(\gamma-1)\delta^{-1} - n^2\zeta_1\zeta_2\zeta_4^2\alpha\beta\gamma\delta^{-1}\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1\zeta_2\zeta_3^2\zeta_4^2\exists}\{-(\beta-1)(\gamma-1)(\delta-1)+\beta(\gamma-1)(\delta-1)-\beta\gamma(\delta-1)$$

$$+(\beta-1)\gamma(\delta-1)+(2\zeta_4-1)\beta\gamma\delta+2(\beta-1)\gamma\delta-(2\zeta_4-1)\gamma\beta(\delta-1)-(2\zeta_4-1)(\beta-1)\gamma\delta$$

$$+(2\zeta_4-1)(\beta-1)(\gamma-1)\delta\}-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1\zeta_2\zeta_3^2\zeta_4^2\exists}\{-(\gamma-1)(\delta-1)+3\gamma\delta-2\gamma(\delta-1)$$

$$-2(2\zeta_4-1)\gamma\delta+(2\zeta_4-1)^2(\gamma-1)\delta-(2\zeta_4-1)^2\gamma\delta\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1\zeta_2\zeta_3^2\zeta_4^2\exists}\{-(\beta-1)(\gamma-1)(\delta-1)+\beta(\gamma-1)(\delta-1)-\beta\gamma(\delta-1)$$

$$+(\beta-1)\gamma(\delta-1)+2(\beta-1)\gamma\delta+(2\zeta_4-1)\beta\gamma\delta-(2\zeta_4-1)\gamma\beta(\delta-1)-(2\zeta_4-1)(\beta-1)\gamma\delta$$

$$+(2\zeta_4-1)(\beta-1)(\gamma-1)\delta\}-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1\zeta_2\zeta_3^2\zeta_4^2\exists}\{-(\gamma-1)(\delta-1)+3\gamma\delta-2\gamma(\delta-1)$$

$$-2(2\zeta_4-1)\gamma\delta+(2\zeta_4-1)^2(\gamma-1)\delta-(2\zeta_4-1)^2\gamma\delta\}$$

$$\frac{\partial\zeta_1}{\partial l}=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1\zeta_2\zeta_3^2\zeta_4^2\exists}\{-4\zeta_4^2+\zeta_4(6\delta+2\beta\gamma-2\beta\delta-4\gamma\delta)+(3\gamma-\beta\gamma\delta+\beta\delta-2\delta)\}. \tag{18}$$

Now we consider $\alpha=\beta=\gamma=\delta=\frac{1}{4}$ then we get, $\exists=1$, i.e., for constant returns to scale, we get from (18),

$$\frac{\partial\zeta_1}{\partial l}=\frac{1}{|J|}\frac{A^3B}{4^7\zeta_1^{1/4}\zeta_2^{1/4}\zeta_3^{5/4}\zeta_3^{5/4}}\{(8\zeta_4-5)^2-44\}. \tag{19}$$

In (19) if $(8\zeta_4-5)^2>44$, i.e., $\zeta_4>\left(5\pm2\sqrt{11}\right)/8$, then we get,

$$\frac{\partial\zeta_1}{\partial l}>0. \tag{20}$$

From inequality (20) we observe that when the wage rate increases, the capital ζ_1 of the firm also increases. It seems that due to increase of wage rate, total labor cost also increases. But the demand of the products also increases in parallel; the firm achieves maximum revenue, and moves to profit maximization. As a result, the firm may enjoy sustainable economic environment.

In (19) if $(8\zeta_4-5)^2<44$, i.e., $\zeta_4>\left(5\pm2\sqrt{11}\right)/8$, then we get,

$$\frac{\partial\zeta_1}{\partial l}<0. \tag{21}$$

From inequality (21) we observe that when the wage rate increases, the capital ζ_1 of the firm decreases. It seems that due to increase of wage rate, total labor cost of the firm also increases. Consequently, total production cost of the firm increases. On the other hand, the laborers work for fewer hours due to income effect. In this situation the firm may reduce its capital for the sustainability in the economic world.

Now we study the effect on labor ζ_2 when the wage rate of per unit of labor, *l* increases. Taking T_{32} (i.e., term of 3rd row and 2nd column) from both sides of (17) we get (Mohajan, 2021c; Mohajan & Mohajan, 2023d),

$$\frac{\partial \zeta_2}{\partial l} = \frac{\zeta_2}{|J|}[C_{13}] + \frac{\lambda}{|J|}[C_{33}]$$

$$= \frac{\zeta_2}{|J|}\text{Cofactor of } C_{13} + \frac{\lambda}{|J|}\text{Cofactor of C}_{33}$$

$$= \frac{\zeta_2}{|J|}\begin{vmatrix} -B_1 & u_{11} & u_{13} & u_{14} \\ -B_2 & u_{21} & u_{23} & u_{24} \\ -B_3 & u_{31} & u_{33} & u_{34} \\ -B_4 & u_{41} & u_{43} & u_{44} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_3 & -B_4 \\ -B_1 & u_{11} & u_{13} & u_{14} \\ -B_3 & u_{31} & u_{33} & u_{34} \\ -B_4 & u_{41} & u_{43} & u_{44} \end{vmatrix}$$

$$= \frac{\zeta_2}{|J|}\left\{ -B_1\begin{vmatrix} u_{21} & u_{23} & u_{24} \\ u_{31} & u_{33} & u_{34} \\ u_{41} & u_{43} & u_{44} \end{vmatrix} - u_{11}\begin{vmatrix} -B_2 & u_{23} & u_{24} \\ -B_3 & u_{33} & u_{34} \\ -B_4 & u_{43} & u_{44} \end{vmatrix} + u_{13}\begin{vmatrix} -B_2 & u_{21} & u_{24} \\ -B_3 & u_{31} & u_{34} \\ -B_4 & u_{41} & u_{44} \end{vmatrix} - u_{14}\begin{vmatrix} -B_2 & u_{21} & u_{23} \\ -B_3 & u_{31} & u_{33} \\ -B_4 & u_{41} & u_{43} \end{vmatrix} \right\}$$

$$+ \frac{\lambda}{|J|}\left\{ -B_1\begin{vmatrix} -B_1 & u_{13} & u_{14} \\ -B_3 & u_{33} & u_{34} \\ -B_4 & u_{43} & u_{44} \end{vmatrix} + B_3\begin{vmatrix} -B_1 & u_{11} & u_{14} \\ -B_3 & u_{31} & u_{34} \\ -B_4 & u_{41} & u_{44} \end{vmatrix} - B_4\begin{vmatrix} -B_1 & u_{11} & u_{13} \\ -B_3 & u_{31} & u_{33} \\ -B_4 & u_{41} & u_{43} \end{vmatrix} \right\}$$

$$= \frac{\zeta_2}{|J|}\left[-B_1\{u_{21}(u_{33}u_{44} - u_{43}u_{34}) + u_{23}(u_{41}u_{34} - u_{31}u_{44}) + u_{24}(u_{31}u_{43} - u_{41}u_{33})\} \right.$$

$$-u_{11}\{-B_2(u_{33}u_{44} - u_{43}u_{34}) + u_{23}(-B_4u_{34} + B_3u_{44}) + u_{24}(-B_3u_{43} + B_4u_{33})\}$$

$$+u_{13}\{-B_2(u_{31}u_{44} - u_{41}u_{34}) + u_{21}(-B_4u_{34} + B_3u_{44}) + u_{24}(-B_3u_{41} + B_4u_{31})\}$$

$$\left. -u_{14}\{-B_2(u_{31}u_{43} - u_{41}u_{33}) + u_{21}(-B_4u_{33} + B_3u_{43}) + u_{23}(-B_3u_{41} + B_4u_{31})\} \right]$$

$$+ \frac{\lambda}{|J|}\left[\{-B_1\{-B_1(u_{33}u_{44} - u_{43}u_{34}) + u_{13}(-B_4u_{34} + B_3u_{44}) + u_{14}(-B_3u_{43} + B_4u_{33})\} \right.$$

$$+B_3\{-B_1(u_{31}u_{44} - u_{41}u_{34}) + u_{11}(-B_4u_{34} + B_3u_{44}) + u_{14}(-B_3u_{41} + B_4u_{31})\}$$

$$\left. -B_4\{-B_1(u_{31}u_{43} - u_{41}u_{33}) + u_{11}(-B_4u_{33} + B_3u_{43}) + u_{13}(-B_3u_{41} + B_4u_{31})\} \right]$$

$$= -\frac{\zeta_2}{|J|}\{B_1u_{21}u_{33}u_{44} - B_1u_{21}u_{43}u_{34} + B_1u_{23}u_{41}u_{24} - B_1u_{23}u_{31}u_{44} + B_1u_{24}u_{31}u_{43} - B_1u_{24}u_{41}u_{33}$$

$$-B_2u_{11}u_{33}u_{44} + B_2u_{11}u_{43}u_{34} - B_4u_{11}u_{23}u_{34} + B_3u_{11}u_{23}u_{44} - B_3u_{11}u_{24}u_{43} + B_4u_{11}u_{24}u_{33} + B_2u_{13}u_{31}u_{44}$$

$$-B_2u_{13}u_{41}u_{34} + B_4u_{13}u_{21}u_{34} - B_3u_{13}u_{21}u_{44} + B_3u_{13}u_{24}u_{41} - B_4u_{13}u_{24}u_{31} - B_2u_{14}u_{31}u_{43} + B_2u_{14}u_{41}u_{33}$$

$$-B_4u_{14}u_{21}u_{33} + B_3u_{14}u_{21}u_{43} - B_3u_{14}u_{23}u_{41} + B_4u_{14}u_{23}u_{31}\} + \frac{\lambda}{|J|}\{B_1^2u_{33}u_{44} - B_1^2u_{43}u_{34} + B_1B_4u_{13}u_{34}$$

$$- B_1B_3u_{13}u_{44} + B_1B_3u_{14}u_{43} - B_1B_4u_{14}u_{33} - B_1B_3u_{31}u_{44} + B_1B_3u_{41}u_{34} - B_3B_4u_{11}u_{34} + B_3^2u_{11}u_{44}$$

$$- B_3^2u_{14}u_{41} + B_3B_4u_{14}u_{31} + B_1B_4u_{31}u_{43} - B_1B_4u_{41}u_{33} + B_4^2u_{11}u_{33} - B_3B_4u_{11}u_{43} + B_3B_4u_{13}u_{41}$$

$$- B_4^2u_{13}u_{31}\}$$

$$= -\frac{\zeta_2}{|J|}\frac{A^3\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2}\{k\zeta_1\zeta_2\alpha\beta\gamma(\gamma-1)\delta(\delta-1) - k\zeta_1\zeta_2\alpha\beta\gamma^2\delta^2 + k\zeta_1\zeta_2\alpha\beta\gamma\delta^2$$

$$- k\zeta_1\zeta_2\alpha\beta\gamma^2\delta(\delta-1) + k\zeta_1\zeta_2\alpha\beta\gamma^2\delta^2 - k\zeta_1\zeta_2\alpha\beta\gamma(\gamma-1)\delta^2 - l\zeta_2^2\alpha(\alpha-1)\gamma(\gamma-1)\delta(\delta-1)$$

$$+ l\zeta_2^2\alpha(\alpha-1)\gamma^2\delta^2 - n\zeta_2\zeta_4\alpha(\alpha-1)\beta\gamma^2\delta + m\zeta_2\zeta_3\alpha(\alpha-1)\beta\gamma\delta(\delta-1) - m\zeta_2\zeta_3\alpha(\alpha-1)\beta\gamma\delta^2$$

$$+ n\zeta_2\zeta_4\alpha(\alpha-1)\beta\gamma(\gamma-1)\delta + l\zeta_2^2\alpha^2\gamma^2\delta(\delta-1) - l\zeta_2^2\alpha^2\gamma^2\delta^2 + n\zeta_2\zeta_4\alpha^2\beta\gamma^2\delta$$

$$- m\zeta_2\zeta_3\alpha(\alpha-1)\beta\gamma\delta(\delta-1) + m\zeta_2\zeta_3\alpha^2\beta\gamma\delta^2 - n\zeta_2\zeta_4\alpha^2\beta\gamma^2\delta - l\zeta_2^2\alpha^2\gamma^2\delta^2 + l\zeta_2^2\alpha^2\gamma(\gamma-1)\delta^2$$

$$- n\zeta_2\zeta_4\alpha^2\beta\gamma(\gamma-1)\delta + m\zeta_2\zeta_3\alpha^2\beta\gamma\delta^2 - m\zeta_2\zeta_3\alpha^2\beta\gamma\delta^2 + n\zeta_2\zeta_4\alpha^2\beta\gamma^2\delta\}$$

$$+ \frac{\lambda}{|J|}\{k^2\zeta_1^2\zeta_2^2\gamma(\gamma-1)\delta(\delta-1) - k^2\zeta_1^2\zeta_2^2\gamma^2\delta^2 + kn\zeta_1\zeta_2^2\zeta_4\alpha\gamma^2\delta + km\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta(\delta-1)$$

$$+ km\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta^2 - kn\zeta_1\zeta_2^2\zeta_4\alpha\gamma(\gamma-1)\delta - km\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta(\delta-1) + km\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta^2$$

$$- nl\zeta_2^2\zeta_3\zeta_4\alpha(\alpha-1)\gamma\delta + m^2\zeta_2^2\zeta_3^2\alpha(\alpha-1)\delta(\delta-1) - m^2\zeta_2^2\zeta_3^2\alpha^2\delta^2 + nl\zeta_2^2\zeta_3\zeta_4\alpha^2\gamma\delta$$

$$+ kn\zeta_1\zeta_2^2\zeta_4\alpha\gamma^2\delta - kn\zeta_1\zeta_2^2\zeta_4\alpha\gamma(\gamma-1)\delta + n^2\zeta_2^2\zeta_4^2\alpha(\alpha-1)\gamma(\gamma-1) + n^2h_2^2h_4^2a(a-1)c(c-1)$$

$$- mn\zeta_2^2\zeta_3\zeta_4\alpha(\alpha-1)\gamma\delta + mn\zeta_2^2\zeta_3\zeta_4\alpha^2\gamma\delta - n^2\zeta_2^2\zeta_4^2\alpha^2\gamma^2\}$$

$$= -\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}}{\zeta_1^2\zeta_3^2\zeta_4^2}\{k\zeta_1(\gamma-1)(\delta-1) - k\zeta_1\gamma(\delta-1) + k\zeta_1\gamma\delta - k\zeta_1(\gamma-1)\delta$$

$$+ l\zeta_2\alpha\beta^{-1}\gamma(\delta-1) - 2l\zeta_2\alpha\beta^{-1}\gamma\delta + l\zeta_2\alpha\beta^{-1}(\gamma-1)\delta - l\zeta_2(\alpha-1)\beta^{-1}(\gamma-1)(\delta-1)$$

$$+ l\zeta_2(\alpha-1)\beta^{-1}\gamma\delta + m\zeta_3(\alpha-1)(\delta-1) + 2m\zeta_3\alpha\delta - 2m\zeta_3\alpha(\delta-1) + n\zeta_4(\alpha-1)(\gamma-1)$$

$$- n\zeta_4(\alpha-1)\gamma - n\zeta_4\alpha(\gamma-1) + n\zeta_4\alpha\gamma\} + \frac{1}{|J|}\frac{A^2\gamma\delta\zeta_1^{2\alpha}\zeta_2^{2\beta}\zeta_3^{2\gamma}\zeta_4^{2\delta}}{\zeta_1^2\zeta_3^2\zeta_4^2}\frac{A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta}\exists}{B}$$

$$\{k^2\zeta_1^2(\gamma-1)(\delta-1) - k^2\zeta_1^2\gamma\delta + kn\zeta_1\zeta_4\alpha\gamma - km\zeta_1\zeta_3\alpha(\delta-1) + km\zeta_1\zeta_3\alpha\delta - kn\zeta_1\zeta_4\alpha(\gamma-1)$$

$$- km\zeta_1\zeta_3\alpha(\delta-1) + km\zeta_1\zeta_3\alpha\delta - nl\zeta_2\zeta_4\alpha(\alpha-1) + m^2\zeta_3^2\alpha(\alpha-1)\gamma^{-1}(\delta-1) - m^2\zeta_3^2\alpha^2\gamma^{-1}\delta$$

$$+nl\zeta_2\zeta_4\alpha^2 \quad +kn\zeta_1\zeta_4\alpha\gamma \quad -kn\zeta_1\zeta_4\alpha(\gamma-1) \quad +n^2\zeta_4^2\alpha(\alpha-1)\gamma(\gamma-1)\delta^{-1} \quad -mn\zeta_3\zeta_4\alpha(\alpha-1)$$

$$+mn\zeta_3\zeta_4\alpha^2 \quad -n^2\zeta_4^2\alpha\gamma\delta^{-1}\}$$

$$\frac{\partial\zeta_2}{\partial l}=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_3^2\zeta_4^2\exists} \quad \{\alpha(\gamma-1)(\delta-1) \quad -(\alpha-1)(\gamma-1)(\delta-1) \quad +(\alpha-1)\gamma(\delta-1)$$

$$-2\alpha\gamma(\delta-1)+\alpha\gamma\delta+(\alpha-1)\gamma\delta+(2\zeta_4-1)(\alpha-1)(\gamma-1)\delta-(2\zeta_4-1)(\alpha-1)\gamma\delta-(2\zeta_4-1)\alpha(\gamma-1)\delta$$

$$+(2\zeta_4-1)\alpha\gamma\delta\} \quad \frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_3^2\zeta_4^2\exists} \quad \{\alpha(\gamma-1)(\delta-1) \quad +(\alpha-1)\gamma(\delta-1) \quad -2\alpha\gamma(\delta-1)$$

$$+4(2\zeta_4-1)\alpha\gamma\delta-2(2\zeta_4-1)\alpha(\gamma-1)\delta-2(2\zeta_4-1)(\alpha-1)\gamma\delta \quad +(2\zeta_4-1)^2(\alpha-1)(\gamma-1)\delta$$

$$-(2\zeta_4-1)^2\alpha\gamma\delta\}. \tag{22}$$

For $\beta=\frac{1}{2}$ in (22) we get,

$$\frac{\partial\zeta_2}{\partial l}=\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_3^2\zeta_4^2\exists} \quad \{(-4\alpha\delta-\alpha\gamma\delta-\alpha\gamma+2\alpha+2\gamma+4\delta-3\gamma\delta) \quad +\zeta_4(16\alpha\delta+16\gamma\delta-10\delta)$$

$$-\zeta_4^2(8\alpha\delta+8\gamma\delta-8\delta)\}. \tag{23}$$

Now we use $\alpha=\gamma=\delta=\frac{1}{4}$ then, $\exists=\frac{5}{4}$, i.e., for increasing returns to scale in (23) we get,

$$\frac{\partial\zeta_2}{\partial l}=\frac{1}{|J|}\frac{A^3\alpha\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{128\zeta_1^2\zeta_3^2\zeta_4^2\exists}\{99-4(4\zeta_4+1)^2\}. \tag{24}$$

If $4(4\zeta_4+1)^2>99$, i.e., $\zeta_4>\left(-2+3\sqrt{11}\right)/8$, in (24) we get,

$$\frac{\partial\zeta_2}{\partial l}<0. \tag{25}$$

From inequality (25) we see that if the wage rate l of the organization increases, the total labor ζ_2 of the organization decreases, which is not reasonable. It seems that due to income effect laborers need less total labor hours to manage their daily expenditure. Consequently, some laborers remain absent, because they have enough money to maintain their families during absent periods.

If $4(4\zeta_4+1)^2<99$, i.e., $\zeta_4<\left(-2+3\sqrt{11}\right)/8$, in (24) we get,

$$\frac{\partial\zeta_2}{\partial l}>0. \tag{26}$$

The inequality (26) shows that if the wage rate l of the organization increases, the total labor ζ_2 of the organization also increases, which is reasonable. Due to substitution effect for more income the total labor hours among the laborers increase and also more new workers may join in the firm.

Now we see the effect on principal raw materials ζ_3 when the wage rate per unit of labor, l increases. Taking T_{42} (i.e., term of 4th row and 2nd column) from both sides of (17) we get (Mohajan, 2021a, 2022; Mohajan & Mohajan, 2022b, 2023a),

$$\frac{\partial \zeta_3}{\partial l} = \frac{\zeta_2}{|J|}[C_{14}] + \frac{\lambda}{|J|}[C_{34}]$$

$$= \frac{\zeta_2}{|J|}\text{Cofactor of } C_{14} + \frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{34}$$

$$= -\frac{\zeta_2}{|J|}\begin{vmatrix} -B_1 & u_{11} & u_{12} & u_{14} \\ -B_2 & u_{21} & u_{22} & u_{24} \\ -B_3 & u_{31} & u_{32} & u_{34} \\ -B_4 & u_{41} & u_{42} & u_{44} \end{vmatrix} - \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_4 \\ -B_1 & u_{11} & u_{12} & u_{14} \\ -B_3 & u_{31} & u_{32} & u_{34} \\ -B_4 & u_{41} & u_{42} & u_{44} \end{vmatrix}$$

$$= -\frac{\zeta_2}{|J|}\left\{ -B_1\begin{vmatrix} u_{21} & u_{22} & u_{24} \\ u_{31} & u_{32} & u_{34} \\ u_{41} & u_{42} & u_{44} \end{vmatrix} - u_{11}\begin{vmatrix} -B_2 & u_{22} & u_{24} \\ -B_3 & u_{32} & u_{34} \\ -B_4 & u_{42} & u_{44} \end{vmatrix} + u_{12}\begin{vmatrix} -B_2 & u_{21} & u_{24} \\ -B_3 & u_{31} & u_{34} \\ -B_4 & u_{41} & u_{44} \end{vmatrix} - u_{14}\begin{vmatrix} -B_2 & u_{21} & u_{22} \\ -B_3 & u_{31} & u_{32} \\ -B_4 & u_{41} & u_{42} \end{vmatrix} \right\}$$

$$-\frac{\lambda}{|J|}\left\{ B_1\begin{vmatrix} -B_1 & u_{12} & u_{14} \\ -B_3 & u_{32} & u_{34} \\ -B_4 & u_{42} & u_{44} \end{vmatrix} - B_2\begin{vmatrix} -B_1 & u_{11} & u_{14} \\ -B_3 & u_{31} & u_{34} \\ -B_4 & u_{41} & u_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_1 & u_{11} & u_{12} \\ -B_3 & u_{31} & v_{32} \\ -B_4 & u_{41} & u_{42} \end{vmatrix} \right\}$$

$$= -\frac{\zeta_2}{|J|}\left[-B_1\{u_{21}(u_{32}u_{44} - u_{42}u_{34}) + u_{22}(u_{41}u_{34} - u_{31}u_{44}) + u_{24}(u_{31}u_{42} - u_{41}u_{32})\}\right.$$

$$-u_{11}\{-B_2(u_{32}u_{44} - u_{42}u_{34}) + u_{22}(-B_4u_{34} + B_3u_{44}) + u_{24}(-B_3u_{42} + B_4u_{32})\}$$

$$+u_{12}\{-B_2(u_{31}u_{44} - u_{41}u_{34}) + u_{21}(-B_4u_{34} + B_3u_{44}) + u_{24}(-B_3u_{41} + B_4u_{31})\}$$

$$\left. -u_{14}\{-B_2(u_{31}u_{42} - u_{41}u_{32}) + u_{21}(-B_4u_{32} + B_3u_{42}) + u_{22}(-B_3u_{41} + B_4u_{31})\}\right]$$

$$-\frac{\lambda}{|J|}\left[-B_1\{-B_1(u_{32}u_{44} - u_{42}u_{34}) + u_{12}(-B_4u_{34} + B_3u_{44}) + u_{14}(-B_3u_{42} + B_4u_{32})\}\right.$$

$$-B_2\{-B_1(u_{31}u_{44} - u_{41}u_{34}) + u_{11}(-B_4u_{34} + B_3u_{44}) + u_{14}(-B_3u_{41} + B_4u_{31})\}$$

$$\left. +B_4\{-B_1(u_{31}u_{42} - u_{41}u_{32}) + u_{11}(-B_4u_{32} + B_3u_{42}) + u_{12}(-B_3u_{41} + B_4u_{31})\}\right]$$

$$= -\frac{\zeta_2}{|J|}\{-B_1u_{21}u_{32}u_{44} + B_1u_{21}u_{42}u_{34} - B_1u_{22}u_{41}u_{34} + B_1u_{22}u_{31}u_{44} - B_1u_{24}u_{31}u_{42} + B_1u_{24}u_{41}u_{32}$$

$$+B_2u_{11}u_{32}u_{44} - B_2u_{11}u_{42}u_{34} + B_4u_{11}u_{22}u_{34} - B_3u_{11}u_{22}u_{44} + B_3u_{11}u_{24}u_{42} - B_4u_{11}u_{24}u_{32}$$

$$-B_2u_{12}u_{31}u_{44} + B_2u_{12}u_{41}u_{34} - B_4u_{12}u_{21}u_{34} + B_3u_{12}u_{21}u_{44} - B_3u_{12}u_{24}u_{41} + B_4u_{12}u_{24}u_{31}$$

$$+B_2u_{14}u_{31}u_{42} - B_2u_{14}u_{41}u_{32} + B_4u_{14}u_{21}u_{32} - B_3u_{14}u_{21}u_{42} + B_3u_{14}u_{22}u_{41} - B_4u_{14}u_{22}u_{31}\}$$

$$-\frac{\lambda}{|J|}\left\{B_1^2u_{32}u_{44}-B_1^2u_{42}u_{34}+B_1B_4u_{12}u_{34}-B_1B_3u_{12}u_{44}+B_1B_3u_{14}u_{42}-B_1B_4u_{14}u_{32}\right.$$

$$+B_1B_2u_{31}u_{44}-B_1B_2u_{41}u_{34}+B_2B_4u_{11}u_{34}-B_2B_3u_{11}u_{44}+B_2B_3u_{14}u_{41}-B_2B_4u_{14}u_{31}$$

$$\left.-B_1B_4u_{31}u_{42}+B_1B_4u_{41}u_{32}-B_4^2u_{11}u_{32}+B_3B_4u_{11}u_{42}-B_3B_4u_{12}u_{41}+B_4^2u_{12}u_{31}\right\}$$

$$=-\frac{\zeta_2}{|J|}\frac{A^3\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2}\left\{-k\zeta_1\zeta_3\alpha\beta^2\gamma\delta(\delta-1)+k\zeta_1\zeta_3\alpha\beta^2\gamma\delta^2-k\zeta_1\zeta_3\alpha\beta(\beta-1)\gamma\delta^2\right.$$

$$-k\zeta_1\zeta_3\alpha\beta^2\gamma\delta^2+k\zeta_1\zeta_3\alpha\beta(\beta-1)\gamma\delta(\delta-1)+k\zeta_1\zeta_3\alpha\beta^2\gamma\delta^2+l\zeta_2\zeta_3\alpha(\alpha-1)\beta\gamma\delta(\delta-1)$$

$$-l\zeta_2\zeta_3\alpha(\alpha-1)\beta\gamma\delta^2+n\zeta_3\zeta_4\alpha(\alpha-1)\beta(\beta-1)\gamma\delta-m\zeta_3^2\alpha(\alpha-1)\beta(\beta-1)\delta(\delta-1)$$

$$+m\zeta_3^2\alpha(\alpha-1)\beta^2\delta^2-n\zeta_3\zeta_4\alpha(\alpha-1)\beta^2\gamma\delta-l\zeta_2\zeta_3\alpha^2\beta\gamma\delta(\delta-1)+l\zeta_2\zeta_3\alpha^2\beta\gamma\delta^2$$

$$-n\zeta_3\zeta_4\alpha^2\beta^2\gamma\delta+m\zeta_3^2\alpha^2\beta^2\delta(\delta-1)-m\zeta_3^2\alpha^2\beta^2\delta^2+n\zeta_3\zeta_4\alpha^2\beta^2\gamma\delta+l\zeta_2\zeta_3\alpha^2\beta\gamma\delta^2$$

$$\left.-l\zeta_2\zeta_3\alpha^2\beta\gamma\delta^2+n\zeta_3\zeta_4\alpha^2\beta^2\gamma\delta-m\zeta_3^2\alpha^2\beta^2\delta^2+m\zeta_3^2\alpha^2\beta(\beta-1)\delta^2-n\zeta_3\zeta_4\alpha^2\beta(\beta-1)\gamma\delta\right\}$$

$$-\frac{\lambda}{|J|}\frac{A^2\zeta_1^{2\alpha}\zeta_2^{2\beta}\zeta_3^{2\gamma}\zeta_4^{2\delta}}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2}\left\{k^2\zeta_1^2\zeta_2\zeta_3\beta\gamma\delta(\delta-1)-k^2\zeta_1^2\zeta_2\zeta_3\beta\gamma\delta^2+kn\zeta_1\zeta_2\zeta_3\alpha\beta\gamma\delta\right.$$

$$-km\zeta_1\zeta_2\zeta_3^2\beta\gamma\delta(\delta-1)+km\zeta_1\zeta_2\zeta_3^2\beta\gamma\delta^2-kn\zeta_1\zeta_2\zeta_3\alpha\beta\gamma\delta+kl\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta(\delta-1)$$

$$+kl\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta(\delta-1)-kl\zeta_1\zeta_2^2\zeta_3\alpha\gamma\delta^2+nl\zeta_2^2\zeta_3\zeta_4\alpha(\alpha-1)\gamma\delta+lm\zeta_2^2\zeta_3^2\alpha^2\delta^2$$

$$-lm\zeta_2^2\zeta_3^2\alpha(\alpha-1)\delta(\delta-1)-nl\zeta_2^2\zeta_3\zeta_4\alpha^2\gamma\delta-kn\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta+kn\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta$$

$$\left.-n^2\zeta_2\zeta_3\zeta_4^2\alpha(\alpha-1)\beta\gamma+mn\zeta_2\zeta_3^2\zeta_4\alpha(\alpha-1)\beta\delta-mn\zeta_2\zeta_3^2\zeta_4\alpha^2\beta\delta+n^2\zeta_2\zeta_3\zeta_4^2\alpha^2\beta\gamma\right\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_2\zeta_3\zeta_4^2\exists}\left\{-\alpha\beta(\delta-1)+\alpha(\beta-1)(\delta-1)+(\alpha-1)\beta(\delta-1)\right.$$

$$-(\alpha-1)(\beta-1)(\delta-1)+(2\zeta_4-1)(\alpha-1)(\beta-1)\delta-(2\zeta_4-1)(\alpha-1)\beta\delta+(2\zeta_4-1)\alpha\beta\delta$$

$$\left.-(2\zeta_4-1)\alpha(\beta-1)\delta\right\}-\frac{1}{|J|}\frac{A^2\zeta_1^{2\alpha}\zeta_2^{2\beta}\zeta_3^{2\gamma}\zeta_4^{2\delta}B^2}{\zeta_1^2\zeta_2\zeta_3\zeta_4^2\exists}\frac{A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta}\exists}{B}\left\{\alpha(\delta-1)-(\alpha-1)(\delta-1)\right.$$

$$\left.+(2\zeta_4-1)\alpha(\delta-1)+2(2\zeta_4-1)(\alpha-1)\delta-2(2\zeta_4-1)\alpha\delta-(2\zeta_4-1)^2(\alpha-1)\delta+(2\zeta_4-1)^2\alpha\delta\right\}$$

$$\frac{\partial\zeta_3}{\partial l}=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_2\zeta_3\zeta_4^2\exists}\left\{4\zeta_4^2\delta+2\zeta_4(\alpha\delta-\alpha-4\delta)+4\delta-\alpha\delta+\alpha-1\right\}. \quad (27)$$

Now using $\alpha = \beta = \gamma = \delta = \dfrac{1}{4}$ then we get, $\exists = 1$, i.e., for constant returns to scale, in (27) we get,

$$\frac{\partial \zeta_3}{\partial l} = \frac{1}{|J|}\frac{A^3 \alpha\beta\gamma\delta \zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta} B}{\zeta_1^2 \zeta_2 \zeta_3 \zeta_4^2 \exists}\left\{\frac{313}{16} - \left(4\zeta_4 - \frac{19}{4}\right)^2\right\}. \tag{28}$$

If $\left(4\zeta_4 - \dfrac{19}{4}\right)^2 > \dfrac{313}{16}$, i.e., $\zeta_4 > \left(19 \pm \sqrt{313}\right)/16$, then (28) we get,

$$\frac{\partial \zeta_3}{\partial l} < 0. \tag{29}$$

The inequality (29) expresses that if the wage rate l of the firm increases, purchasing of principle raw material ζ_3 decreases. In this situation, the laborers earn more money and some laborers remain absent frequently, as they have enough money to maintain their daily expenditures. Moreover, for paying more wages, the firm cannot manage sufficient money to buy principle raw material. Consequently, the firm decreases purchasing of principle raw material.

If $\left(4\zeta_4 - \dfrac{19}{4}\right)^2 < \dfrac{313}{16}$, i.e., $\zeta_4 < \left(19 \pm \sqrt{313}\right)/16$, then (27) we get,

$$\frac{\partial \zeta_3}{\partial l} > 0. \tag{30}$$

The inequality (30) expresses that if the wage rate l of the firm increases, purchasing of principle raw material ζ_3 also increases. This seems that for more earnings the laborers works more working hours. Consequently, the firm increases purchasing of principle raw material.

Now we observe the effect on irregular input ζ_4 when the wage rate per unit of labor, l increases. Taking T_{52} (i.e., term of 5th row and 2nd column) from both sides of (17) we get (Islam et al., 2011; Mohajan, 2017a; Ferdous & Mohajan, 2022),

$$\frac{\partial \zeta_4}{\partial l} = \frac{\zeta_2}{|J|}[C_{15}] + \frac{\lambda}{|J|}[C_{35}]$$

$$= \frac{\zeta_2}{|J|}\text{Cofactor of } C_{15} + \frac{\lambda}{|J|}\text{Cofactor of } \text{C}_{35}$$

$$= \frac{\zeta_2}{|J|}\begin{vmatrix} -B_1 & u_{11} & u_{12} & u_{13} \\ -B_2 & u_{21} & u_{22} & u_{23} \\ -B_3 & u_{31} & u_{32} & u_{33} \\ -B_4 & u_{41} & u_{42} & u_{43} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 \\ -B_1 & u_{11} & u_{12} & u_{13} \\ -B_3 & u_{31} & u_{32} & u_{33} \\ -B_4 & u_{41} & u_{42} & u_{43} \end{vmatrix}$$

$$= \frac{\zeta_2}{|J|}\left\{ -B_1\begin{vmatrix} u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \\ u_{41} & u_{42} & u_{43} \end{vmatrix} - u_{11}\begin{vmatrix} -B_2 & u_{22} & u_{23} \\ -B_3 & u_{32} & u_{33} \\ -B_4 & u_{42} & u_{43} \end{vmatrix} + u_{12}\begin{vmatrix} -B_2 & u_{21} & u_{23} \\ -B_3 & u_{31} & u_{33} \\ -B_4 & u_{41} & u_{43} \end{vmatrix} - u_{13}\begin{vmatrix} -B_2 & u_{21} & u_{22} \\ -B_3 & u_{31} & u_{32} \\ -B_4 & u_{41} & u_{42} \end{vmatrix}\right\}$$

$$+\frac{\lambda}{|J|}\left\{B_1\begin{vmatrix}-B_1 & u_{12} & u_{13}\\ -B_3 & u_{32} & u_{33}\\ -B_4 & u_{42} & u_{43}\end{vmatrix}-B_2\begin{vmatrix}-B_1 & u_{11} & u_{13}\\ -B_3 & u_{31} & u_{33}\\ -B_4 & u_{41} & u_{43}\end{vmatrix}+B_3\begin{vmatrix}-B_1 & u_{11} & u_{12}\\ -B_3 & u_{31} & u_{32}\\ -B_4 & u_{41} & u_{42}\end{vmatrix}\right\}$$

$$=\frac{\zeta_2}{|J|}\left[-B_1\left\{u_{21}\left(u_{32}u_{43}-u_{42}u_{33}\right)+u_{22}\left(u_{41}u_{33}-u_{31}u_{43}\right)+u_{23}\left(u_{31}u_{42}-u_{41}u_{32}\right)\right\}\right.$$

$$-u_{11}\left\{-B_2\left(u_{32}u_{43}-u_{42}u_{33}\right)+u_{22}\left(-B_4u_{33}+B_3u_{43}\right)+u_{23}\left(-B_3u_{42}+B_4u_{32}\right)\right\}$$

$$+u_{12}\left\{-B_2\left(u_{31}u_{43}-u_{41}u_{33}\right)+u_{21}\left(-B_4u_{33}+B_3u_{43}\right)+u_{23}\left(-B_3u_{41}+B_4u_{31}\right)\right\}$$

$$\left.-u_{13}\left\{-B_2\left(u_{31}u_{42}-u_{41}u_{32}\right)+u_{21}\left(-B_4u_{32}+B_3u_{42}\right)+u_{22}\left(-B_3u_{41}+B_4u_{31}\right)\right\}\right]$$

$$+\frac{\lambda}{|J|}\left[B_1\left\{-B_1\left(u_{32}u_{43}-u_{42}u_{33}\right)+u_{12}\left(-B_4u_{33}+B_3u_{43}\right)+u_{13}\left(-B_3u_{42}+B_4u_{32}\right)\right\}\right.$$

$$-B_2\left\{-B_1\left(u_{31}u_{43}-u_{41}u_{33}\right)+u_{11}\left(-B_4u_{33}+B_3u_{43}\right)+u_{13}\left(-B_3u_{41}+B_4u_{31}\right)\right\}$$

$$\left.+B_3\left\{-B_1\left(u_{31}u_{42}-u_{41}u_{32}\right)+u_{11}\left(-B_4u_{32}+B_3u_{42}\right)+u_{12}\left(-B_3u_{41}+B_4u_{31}\right)\right\}\right]$$

$$=\frac{\zeta_2}{|J|}\left\{-B_1u_{21}u_{32}u_{43}+B_1u_{21}u_{42}u_{33}-B_1u_{22}u_{41}u_{33}+B_1u_{22}u_{31}u_{43}-B_1u_{23}u_{31}u_{42}+B_1u_{23}u_{41}u_{32}\right.$$

$$+B_2u_{11}u_{32}u_{43}-B_2u_{11}u_{42}u_{33}+B_4u_{11}u_{22}u_{33}-B_3u_{11}u_{22}u_{43}+B_3u_{11}u_{23}u_{42}-B_4u_{11}u_{23}u_{32}$$

$$-B_2u_{12}u_{31}u_{43}+B_2u_{12}u_{41}u_{33}-B_4u_{12}u_{21}u_{33}+B_3u_{12}u_{21}u_{43}-B_3u_{12}v_{23}u_{41}+B_4u_{12}u_{23}u_{31}$$

$$\left.+B_2u_{13}u_{31}u_{42}-B_2u_{13}u_{41}u_{32}+B_4u_{13}u_{21}u_{32}-B_3u_{13}u_{21}u_{42}+B_3u_{13}u_{22}u_{41}-B_4u_{13}u_{22}u_{31}\right\}$$

$$+\frac{\lambda}{|J|}\left\{-B_1^2u_{32}u_{43}+B_1^2u_{42}u_{33}-B_1B_4u_{12}u_{33}+B_1B_3u_{12}u_{43}-B_1B_3u_{13}u_{42}+B_1B_4u_{13}u_{32}\right.$$

$$+B_1B_2u_{31}u_{43}-B_1B_2u_{41}uu_{33}+B_2B_4u_{11}u_{33}-B_2B_3u_{11}u_{43}+B_2B_3u_{13}u_{41}-B_2B_4u_{13}u_{31}$$

$$\left.-B_1B_3u_{31}u_{42}+B_1B_3u_{41}u_{32}-B_3B_4u_{11}u_{32}+B_3^2u_{11}u_{42}-B_3^2u_{12}u_{41}+B_3B_4u_{12}u_{31}\right\}$$

$$=\frac{1}{|J|}\frac{A^3\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}}{\zeta_1^2\zeta_2\zeta_3^2\zeta_4^2}\left\{-k\zeta_1\zeta_4\alpha\beta^2\gamma^2\delta\quad+k\zeta_1\zeta_4\alpha\beta^2\gamma(\gamma-1)\delta\quad-k\zeta_1\zeta_4\alpha\beta(\beta-1)\gamma(\gamma-1)\delta\right.$$

$$+k\zeta_1\zeta_4\alpha\beta(\beta-1)\gamma(\gamma-1)\delta\quad-k\zeta_1\zeta_4\alpha\beta^2\gamma^2\delta\quad+k\zeta_1\zeta_4\alpha\beta^2\gamma^2\delta\quad+l\zeta_2\zeta_4\alpha(\alpha-1)\beta\gamma^2\delta$$

$$+l\zeta_2\zeta_4\alpha^2\beta\gamma(\gamma-1)\delta\quad+n\zeta_4^2\alpha(\alpha-1)\beta(\beta-1)\gamma(\gamma-1)\quad-m\zeta_3\zeta_4\alpha(\alpha-1)\beta(\beta-1)\gamma\delta$$

$$+m\zeta_3\zeta_4\alpha(\alpha-1)\beta^2\gamma\delta\quad-n\zeta_4^2\alpha(\alpha-1)\beta^2\gamma^2\quad-l\zeta_2\zeta_4\alpha^2\beta\gamma^2\delta\quad+l\zeta_2\zeta_4\alpha^2\beta\gamma(\gamma-1)\delta$$

$$-n\zeta_4^2\alpha^2\beta^2\gamma(\gamma-1) \quad +m\zeta_3\zeta_4\alpha^2\beta^2\gamma\delta \quad -m\zeta_3\zeta_4\alpha^2\beta^2\gamma\delta \quad +n\zeta_4^2\alpha^2\beta^2\gamma^2 \quad +l\zeta_2\zeta_4\alpha^2\beta\gamma^2\delta$$

$$-l\zeta_2\zeta_4\alpha^2\beta\gamma^2\delta +n\zeta_4^2\alpha^2\beta^2\gamma^2 -m\zeta_3\zeta_4\alpha^2\beta^2\gamma\delta +m\zeta_3\zeta_4\alpha^2\beta(\beta-1)\gamma\delta -nha^2b(b-1)c^2\Big\}$$

$$+\frac{A^2\zeta_1^{2\alpha}\zeta_2^{2\beta}\zeta_3^{2\gamma}\zeta_4^{2\delta}}{\zeta_1^2\zeta_2^2\zeta_3^2\zeta_4^2}\frac{A\zeta_1^{\alpha}\zeta_2^{\beta}\zeta_3^{\gamma}\zeta_4^{\delta}\exists}{B} \qquad \Big\{-k^2\zeta_1^2\zeta_2\zeta_4\beta\gamma^2\delta \qquad +k^2\zeta_1^2\zeta_2\zeta_4\beta\gamma(\gamma-1)\delta$$

$$-kn\zeta_1\zeta_2\zeta_4^2\alpha\beta\gamma(\gamma-1) \quad +km\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta \quad +kn\zeta_1\zeta_2\zeta_4^2\alpha\beta\gamma^2 \quad -km\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta$$

$$+kl\zeta_1\zeta_2^2\zeta_4\alpha\gamma^2\delta \quad -kl\zeta_1\zeta_2^2\zeta_4\alpha\gamma(\gamma-1)\delta \quad +nl\zeta_2^2\zeta_4^2\alpha(\alpha-1)\gamma(\gamma-1) \quad -lm\zeta_2^2\zeta_3\zeta_4\alpha(\alpha-1)\gamma\delta$$

$$+lm\zeta_2^2\zeta_3\zeta_4\alpha^2\gamma\delta \quad -nl\zeta_2^2\zeta_4^2\alpha^2\gamma^2 \quad -km\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta \quad +km\zeta_1\zeta_2\zeta_3\zeta_4\alpha\beta\gamma\delta$$

$$-mn\zeta_2\zeta_3\zeta_4^2\alpha(\alpha-1)\beta\gamma +m^2\zeta_2\zeta_3^2\zeta_4\alpha(\alpha-1)\beta\delta -m^2\zeta_2\zeta_3^2\zeta_4\alpha^2\beta\delta +mn\zeta_2\zeta_3\zeta_4^2\alpha^2\beta\gamma\Big\}$$

$$=\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_2\zeta_3^2\zeta_4\exists}\ \Big\{-3\alpha\beta\gamma +2(\alpha-1)\beta\gamma +3\alpha\beta(\gamma-1) -(\alpha-1)(\beta-1)\gamma +\alpha(\beta-1)\gamma$$

$$-(2\zeta_4-1)(\alpha-1)\beta\gamma \quad +(2\zeta_4-1)(\alpha-1)(\beta-1)(\gamma-1) \quad -(2\zeta_4-1)\alpha\beta(\gamma-1) \quad +2(2\zeta_4-1)\alpha\beta\gamma$$

$$-(2\zeta_4-1)\alpha(\beta-1)\gamma\Big\} \quad +\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_2\zeta_3^2\zeta_4\exists} \quad \Big\{-(2\zeta_4-1)\alpha(\gamma-1) \quad +(2\zeta_4-1)\alpha\gamma$$

$$+(2\zeta_4-1)(\alpha-1)(\gamma-1)-(2\zeta_4-1)(\alpha-1)\gamma\Big\}$$

$$\frac{\partial\zeta_4}{\partial l}=\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta\zeta_1^{3\alpha}\zeta_2^{3\beta}\zeta_3^{3\gamma}\zeta_4^{3\delta}B}{\zeta_1^2\zeta_2\zeta_3^2\zeta_4\exists}\big\{(2\alpha\beta\gamma-\alpha-\beta-2\gamma-\beta\gamma-3\alpha\beta)+2\zeta_4(\alpha+\beta+\gamma)\big\}. \tag{31}$$

Now using $\alpha=\beta=\gamma=\delta=\frac{1}{4}$ then we get, $\exists=1$, i.e., for constant returns to scale, in (31) we get,

$$\frac{\partial\zeta_4}{\partial l}=\frac{1}{|J|}\frac{3A^3B}{2^{13}\zeta_1^{5/4}\zeta_2^{1/4}\zeta_3^{5/4}\zeta_4^{1/4}\exists}(16\zeta_4-13). \tag{32}$$

If $\zeta_4>\frac{13}{16}$ in (32) we get,

$$\frac{\partial\zeta_4}{\partial l}>0\,. \tag{33}$$

The inequality (30) indicates that if the wage rate l of the firm increases, purchasing of irregular input ζ_4 also increases. This seems that for more earnings the laborers works more working hours. Also, irregular input may be an essential material for the firm. Consequently, the firm increases purchasing of principle raw material.

If $\zeta_4<\frac{13}{16}$ in (32) we get,

$$\frac{\partial\zeta_4}{\partial l}<0\,. \tag{34}$$

The inequality (34) expresses that if the wage rate l of the firm increases, purchasing of irregular input ζ_4 decreases. In this situation, the laborers earn more money and some laborers may remain absent frequently. Perhaps they have enough money to maintain their daily expenditures with the earnings of lesser working hours. Moreover, for paying more wages, the firm cannot manage sufficient money to buy irregular input. Consequently, the firm decreases purchasing of irregular input.

11. Conclusions

In this study we have discussed the effects of various inputs of a firm if the wage rate of it is increased. To analyze this, we have considered nonlinear budget constraint. We have taken Cobb-Douglas productions function as a profit function for this study. We have also used 5×5 bordered Hessian matrix and 5×5 Jacobian to establish the economic predictions properly.

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