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The Responses of an Organization for the Increase in Wage Rates: Profit Maximization Cases

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Abstract

Every organization expects to achieve maximum profit for its survival economic environment. To develop profit maximization strategy an organization must be sincere in economic sensitivity analysis. In this study sensitivity analysis is inspected to predict about the economic situation for the efficient production of the organization. Here scientific based production policy is indicated subject to a budget constraint, using Lagrange multiplier technique.

Keywords: profit maximization, Lagrange multiplier, wage, sensitivity analysis

1. Introduction

At present mathematical modeling in economics becomes popular in economics and social sciences (Samuelson, 1947). It plays a governing role in modern economics to build local and global financial structures (Ferdous & Mohajan, 2022). The economists not only see their own benefits but also see welfare of the society (Eaton & Lipsey, 1975). No doubt, an organization wants to achieve maximum profit, but it must be sincere about environment pollution in every step of its operation (Mohajan et al., 2013; Mohajan & Mohajan, 2022b, 2023h).

Cobb-Douglas production function is a popular mathematical model in economics that can be applied for the test of profit maximization (Cobb & Douglas, 1928). In this study, we have tried to discuss sensitivity analysis for inputs of the organization with respect to the increased wage rate. We have used determinant of bordered 6×6 Hessian matrix and 6×10 Jacobian matrix to analyze optimization problems.

2. Literature Review

In any research procedure, the literature review is an introductory section and a researcher presents published works of famous authors (Polit & Hungler, 2013). It deals with a secondary research source and does not thinks about coming research works (Gibbs, 2008). In 1928, two American professors: mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976) have obtained the functional distribution of income between capital and labor (Cobb & Douglas, 1928). Later in 1984, another two American professors: mathematician John V. Baxley and economist John C. Moorhouse have provided a mathematical formulation for nontrivial constrained optimization problem (Baxley & Moorhouse, 1984).

Bangladeshi famous mathematician Jamal Nazrul Islam and his coworkers have worked on optimization problems with a reasonable interpretation of the Lagrange multipliers (Islam et al., 2009a, b, 2010, 2011). Starting with four variable inputs; Devajit Mohajan and Haradhan Kumar Mohajan have discussed profit maximization procedures (Mohajan & Mohajan, 2022a, 2023g). They have also worked on the sensitivity analysis in a series of papers (Mohajan & Mohajan, 2022c, d).

Pahlaj Moolio and his coauthors have operated on optimization structure (Moolio et al., 2009). Lia Roy and her coworkers have shown that cost minimization is essential for the sustainable development of an industry (Roy et al., 2021). Jannatul Ferdous and Haradhan Kumar Mohajan have established a profit maximization problem (Ferdous & Mohajan, 2022).

3. Research Methodology of the Study

Research is a fundamental and significant tool to the academicians to lead in academic world (Pandey & Pandey, 2015). Methodology is a guideline to prepare a good research that follows scientific methods efficiently (Kothari, 2008). Research methodology is the collection of a set of principles for planning, designing, organizing, and conducting a good research (Legesse, 2014).

We have started our research work with a Cobb-Douglas production function. Then we have used Lagrange multiplier, 6×6 bordered Hessian matrix, and 6×6 Jacobian to discuss sensitivity analysis. Reliability and validity are soul of a seminal research and we have stressed on these as far as possible (Mohajan, 2017b, 2018a, 2020). To make this article attractive to the readers we have depended on the secondary data sources of optimization, such as published and unpublished articles, published books, conference papers, internet, websites, etc. (Mohajan, 2017a, 2018b).

4. Objective of the Study

The chief objective of this study is to discuss sensitivity analysis of various inputs when wage rate is increased. Other trivial but related objectives are as follows:

- to explain Cobb-Douglas production function,
- to predict the economic results, and
- to show the results properly.

5. An Economic Model

Let us consider that an organization deals with h_1 quantity of capital, h_2 quantity of labor, h_3 quantity of principal raw materials, and h_4 quantity of other inputs. The profit function of the organization can be represented by the Cobb-Douglas production function as (Cobb & Douglass, 1928; Mohajan & Mohajan, 2022c,d),

$$P = f\left(h_1, h_2, h_3, h_4\right) = A h_1^a h_2^b h_3^c h_4^d \qquad (1)$$

where *A* is the technical process of economic system that indicates total factor productivity. Here *a*, *b*, *c*, and *d* are parameters, where *a* is the output of elasticity of capital measures the percentage change in *P* for 1% change in h_1; while h_2, h_3, and h_4 are held constants. Similar properties carry the other parameters *b*, *c*, and *d*. The values of *a*, *b*, *c*, and *d* are determined by the available technologies, and must satisfy the following four inequalities (Roy et al., 2021; Mohajan & Mohajan, 2023a,b):

$$0 < a < 1,\ 0 < b < 1,\ 0 < c < 1, \text{ and } 0 < d < 1. \qquad (2)$$

A strict Cobb-Douglas production function, in which $\Sigma = a + b + c + d = 1$ indicates constant returns to scale, $\Sigma > 1$ indicates increasing returns to scale, and $\Sigma < 1$ indicates decreasing returns to scale (Moolio et al., 2009; Mohajan et al., 2013).

The budget constraint of the organization can be expressed as,

$$B\left(h_1, h_2, h_3, h_4\right) = k h_1 + l h_2 + m h_3 + n h_4, \qquad (3)$$

where *k* is rate of interest or services of capital per unit of capital h_1; *l* is the wage rate per unit of labor h_2; *m* is the cost per unit of principal raw material h_3; and *n* is the cost per unit of other inputs h_4.

Now we introduce a single Lagrange multiplier μ, as a device; and by using equations (1) and (3) we can represent the Lagrangian function $v\left(h_1, h_2, h_3, h_4, \mu\right)$, in a 5-dimensional unconstrained problem as (Baxley & Moorhouse, 1984; Mohajan & Mohajan, 2022b, 2023c,d),

$$v\left(h_1, h_2, h_3, h_4, \mu\right) = Ah_1^a h_2^b h_3^c h_4^d + \mu\left(B - kh_1 - lh_2 - mh_3 - nh_4\right). \tag{4}$$

where $\frac{\partial B}{\partial h_1} = B_1$, $\frac{\partial B}{\partial h_2} = B_2$, $\frac{\partial v}{\partial h_1} = v_1$, $\frac{\partial^2 v}{\partial h_1 \partial h_3} = v_{31}$, $\frac{\partial^2 v}{\partial h_2^2} = v_{22}$, etc. are partial derivatives. Let us consider the determinant of the 5×5 bordered Hessian matrix as (Mohajan, 2021b),

$$H = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & v_{11} & v_{12} & v_{13} & v_{14} \\ -B_2 & v_{21} & v_{22} & v_{23} & v_{24} \\ -B_3 & v_{31} & v_{32} & v_{33} & v_{34} \\ -B_4 & v_{41} & v_{42} & v_{43} & v_{44} \end{vmatrix}. \tag{5}$$

Taking first-order partial differentiations of (3) we get,

$$B_1 = k,\ B_2 = l,\ B_3 = m, \text{ and } B_4 = n. \tag{6}$$

Taking second-order and cross partial derivatives of (4) we get (Islam et al., 2010a; Mohajan, 2021a),

$$\begin{aligned} v_{11} &= a(a-1)Ah_1^{a-2}h_2^b h_3^c h_4^d, \\ v_{22} &= b(b-1)Ah_1^a h_2^{b-2} h_3^c h_4^d, \\ v_{33} &= c(c-1)Ah_1^a h_2^b h_3^{c-2} h_4^d, \\ v_{44} &= d(d-1)Ah_1^a h_2^b h_3^c h_4^{d-2}, \\ v_{12} &= v_{21} = abAh_1^{a-1}h_2^{b-1}h_3^c h_4^d, \\ v_{13} &= v_{31} = acAh_1^{a-1}h_2^b h_3^{c-1} h_4^d, \\ v_{14} &= v_{41} = adAh_1^{a-1}h_2^b h_3^c h_4^{d-1}, \\ v_{23} &= v_{32} = bcAh_1^a h_2^{b-1} h_3^{c-1} h_4^d, \\ v_{24} &= v_{42} = bdAh_1^a h_2^{b-1} h_3^c h_4^{d-1}, \\ v_{34} &= v_{43} = cdAh_1^a h_2^b h_3^{c-1} h_4^{d-1}. \end{aligned} \tag{7}$$

Now we expand the bordered Hessian (5) as,

$$|H| = \frac{A^3 B^2 abcd h_1^{3a} h_2^{3b} h_3^{3c} h_4^{3d}}{h_1^2 h_2^2 h_3^2 h_4^2 \Sigma} > 0 \tag{8}$$

where $A > 0$, $a, b, c, d > 0$, and budget, $B > 0$, therefore, $|H| > 0$. Hence, the profit is maximized (Islam et al., 2011; Mohajan & Mohajan, 2023e).

6. Sensitivity Analysis

We have observed that the second order condition is satisfied, so that the determinant of (5) survive at the optimum, i.e., $|J| = |H|$; hence, we can apply the implicit function theorem. Let $\mathbf{G}$ be the vector-valued function of ten variables $\mu^*, h_1^*, h_2^*, h_3^*, h_4^*, k, l, m, n$, and *B*, and we define the function $\mathbf{G}$ for the point $(\mu^*, h_1^*, h_2^*, h_3^*, h_4^*, k, l, m, n, B) \in R^{10}$, and take the values in R^5. By the implicit function theorem of multivariable calculus, the equation,

$$F(\mu^*, h_1^*, h_2^*, h_3^*, h_4^*, k, l, m, n, B) = 0, \tag{9}$$

may be solved in the form of

$$\begin{bmatrix} \mu \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \mathbf{G}(k, l, m, n, B). \tag{10}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}$; regarded as $J_G = \dfrac{\partial(\mu, h_1, h_2, h_3, h_4)}{\partial(k, l, m, n, B)}$, and can be represented by (Baxley & Moorhouse, 1984; Mohajan & Mohajan, 2023f);

$$J_G = \begin{bmatrix} \frac{\partial\mu}{\partial k} & \frac{\partial\mu}{\partial l} & \frac{\partial\mu}{\partial m} & \frac{\partial\mu}{\partial n} & \frac{\partial\mu}{\partial B} \\ \frac{\partial h_1}{\partial k} & \frac{\partial h_1}{\partial l} & \frac{\partial h_1}{\partial m} & \frac{\partial h_1}{\partial n} & \frac{\partial h_1}{\partial B} \\ \frac{\partial h_2}{\partial k} & \frac{\partial h_2}{\partial l} & \frac{\partial h_2}{\partial m} & \frac{\partial h_2}{\partial n} & \frac{\partial h_2}{\partial B} \\ \frac{\partial h_3}{\partial k} & \frac{\partial h_3}{\partial l} & \frac{\partial h_3}{\partial m} & \frac{\partial h_3}{\partial n} & \frac{\partial h_3}{\partial B} \\ \frac{\partial h_4}{\partial k} & \frac{\partial h_4}{\partial l} & \frac{\partial h_4}{\partial m} & \frac{\partial h_4}{\partial n} & \frac{\partial h_4}{\partial B} \end{bmatrix}. \tag{11}$$

$$= -J^{-1} \begin{bmatrix} -h_1 & -h_2 & -h_3 & -h_4 & 1 \\ -\mu & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & -\mu & 0 \end{bmatrix}$$

The inverse of Jacobian matrix is, $J^{-1} = \dfrac{1}{|J|} C^T$, where $C = (C_{ij})$, the matrix of cofactors of J, where T stands for transpose, then (11) becomes (Islam et al., 2010; Mohajan & Mohajan, 2023g),

$$= -\frac{1}{|J|}\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} -h_1 & -h_2 & -h_3 & -h_4 & 1 \\ -\mu & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & -\mu & 0 \end{bmatrix}$$

$$J_G = -\frac{1}{|J|}\begin{bmatrix} -h_1C_{11}-\mu C_{21} & -h_2C_{11}-\mu C_{31} & -h_3C_{11}-\mu C_{41} & -h_4C_{11}-\mu C_{51} & C_{11} \\ -h_1C_{12}-\mu C_{22} & -h_2C_{12}-\mu C_{32} & -h_3C_{12}-\mu C_{42} & -h_4C_{12}-\mu C_{52} & C_{12} \\ -h_1C_{13}-\mu C_{23} & -h_2C_{13}-\mu C_{33} & -h_3C_{13}-\mu C_{43} & -h_4C_{13}-\mu C_{53} & C_{13} \\ -h_1C_{14}-\mu C_{24} & -h_2C_{14}-\mu C_{34} & -h_3C_{14}-\mu C_{44} & -h_4C_{14}-\mu C_{54} & C_{14} \\ -h_1C_{15}-\mu C_{25} & -h_2C_{15}-\mu C_{35} & -h_3C_{15}-\mu C_{45} & -h_4C_{15}-\mu C_{55} & C_{15} \end{bmatrix}. \quad (12)$$

In (11) total 25 comparative statics are available, and for sensitivity analysis we will try only on $\frac{\partial h_1}{\partial l}$, $\frac{\partial h_2}{\partial l}$, $\frac{\partial h_3}{\partial l}$, and $\frac{\partial h_4}{\partial l}$ for the prediction of the economic analysis during the profit maximization investigation (Mohajan & Mohajan, 2023h).

Now we observe the effect on capital h_1 when the wage rate per unit of labor, *l* increases. Taking T_{22} (i.e., term of 2nd row and 2nd column) from both sides of (12) we get (Mohajan, 2021a; Wiese, 2021),

$$\frac{\partial h_1}{\partial l} = \frac{h_2}{|J|}[C_{12}] + \frac{\mu}{|J|}[C_{32}]$$

$$= \frac{h_2}{|J|}\text{Cofactor of } C_{12} + \frac{\mu}{|J|}\text{Cofactor of } \mathrm{C}_{32}$$

$$= -\frac{h_2}{|J|}\begin{vmatrix} -B_1 & v_{12} & v_{13} & v_{14} \\ -B_2 & v_{22} & v_{23} & v_{24} \\ -B_3 & v_{32} & v_{33} & v_{34} \\ -B_4 & v_{42} & v_{43} & v_{44} \end{vmatrix} - \frac{\mu}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & v_{12} & v_{13} & v_{14} \\ -B_3 & v_{32} & v_{33} & v_{34} \\ -B_4 & v_{42} & v_{43} & v_{44} \end{vmatrix}$$

$$= -\frac{h_2}{|J|}\left\{ -B_1\begin{vmatrix} v_{22} & v_{23} & v_{24} \\ v_{32} & v_{33} & v_{34} \\ v_{42} & v_{43} & v_{44} \end{vmatrix} - v_{12}\begin{vmatrix} -B_2 & v_{23} & v_{24} \\ -B_3 & v_{33} & v_{34} \\ -B_4 & v_{43} & v_{44} \end{vmatrix} + v_{13}\begin{vmatrix} -B_2 & v_{22} & v_{24} \\ -B_3 & v_{32} & v_{34} \\ -B_4 & v_{42} & v_{44} \end{vmatrix} - v_{14}\begin{vmatrix} -B_2 & v_{22} & v_{23} \\ -B_3 & v_{32} & v_{33} \\ -B_4 & v_{42} & v_{43} \end{vmatrix} \right\}$$

$$-\frac{\mu}{|J|}\left\{ B_2\begin{vmatrix} -B_1 & v_{13} & v_{14} \\ -B_3 & v_{33} & v_{34} \\ -B_4 & v_{43} & v_{44} \end{vmatrix} - B_3\begin{vmatrix} -B_1 & v_{12} & v_{14} \\ -B_3 & v_{32} & v_{34} \\ -B_4 & v_{42} & v_{44} \end{vmatrix} - B_4\begin{vmatrix} -B_1 & v_{12} & v_{13} \\ -B_3 & v_{32} & v_{33} \\ -B_4 & v_{42} & v_{43} \end{vmatrix} \right\}$$

$$= -\frac{h_2}{|J|}\left[-B_1\{v_{22}(v_{33}v_{44} - v_{43}v_{34}) + v_{23}(v_{42}v_{34} - v_{32}v_{44}) + v_{24}(v_{32}v_{43} - v_{42}v_{33})\}\right.$$

$$-v_{12}\{-B_2(v_{33}v_{44} - v_{43}v_{34}) + v_{23}(-B_4v_{34} + B_3v_{44}) + v_{24}(-B_3v_{43} + B_4v_{33})\}$$

$$+v_{13}\{-B_2(v_{32}v_{44} - v_{42}v_{34}) + v_{22}(-B_4v_{34} + B_3v_{44}) + v_{24}(-B_3v_{42} + B_4v_{32})\}$$

$$-v_{14}\left\{-B_2\left(v_{32}v_{43}-v_{42}v_{33}\right)+v_{22}\left(-B_4v_{33}+B_3v_{43}\right)+v_{23}\left(-B_3v_{42}+B_dv_{32}\right)\right\}\Big]$$

$$-\frac{\mu}{|J|}\Big[B_2\left\{-B_1\left(v_{33}v_{44}-v_{43}v_{34}\right)-v_{13}\left(-B_3v_{44}+B_4v_{34}\right)+v_{14}\left(-B_3v_{43}+B_4v_{33}\right)\right\}$$

$$-B_3\left\{-B_1\left(v_{32}v_{44}-v_{42}v_{34}\right)-v_{12}\left(-B_3v_{44}+B_4v_{34}\right)+v_{14}\left(-B_3v_{42}+B_4v_{32}\right)\right\}$$

$$-B_4\left\{-B_1\left(v_{32}v_{43}-v_{42}v_{33}\right)-v_{12}\left(-B_3v_{43}+B_4v_{33}\right)+v_{13}\left(-B_3v_{42}+B_4v_{32}\right)\right\}\Big]$$

$$=-\frac{h_2}{|J|}\left\{-B_1v_{22}v_{33}v_{44}\quad+B_1v_{22}v_{43}v_{34}\quad-B_1v_{23}v_{42}v_{34}\quad+B_1v_{23}v_{32}v_{44}\quad-B_1v_{24}v_{32}v_{43}\quad+B_1v_{24}v_{42}v_{33}\right.$$

$$+B_2v_{12}v_{33}v_{44}\ -B_2v_{12}v_{43}v_{34}\ +B_4v_{12}v_{23}v_{34}\ -B_3v_{12}v_{23}v_{44}\ +B_3v_{12}v_{24}v_{43}\ -B_4v_{12}v_{24}v_{33}\ -B_2v_{13}v_{32}v_{44}$$

$$+B_2v_{13}v_{42}v_{34}\ -B_4v_{13}v_{22}v_{34}\ +B_3v_{13}v_{22}v_{44}\ -B_3v_{13}v_{24}v_{42}\ +B_4v_{13}v_{24}v_{32}\ +B_2v_{14}v_{32}v_{43}\ -B_2v_{14}v_{42}v_{33}$$

$$\left.+B_4v_{14}v_{22}v_{33}\quad-B_3v_{14}v_{22}v_{43}\quad+B_3v_{14}v_{23}v_{42}\quad-B_4v_{14}v_{23}v_{32}\right\}\quad-\frac{\mu}{|J|}\left\{-B_1B_2v_{33}v_{44}\quad+B_1B_2v_{43}v_{34}\right.$$

$$-B_2B_3v_{13}v_{44}\quad-B_2B_4v_{13}v_{34}\quad-B_2B_3v_{14}v_{43}\quad+B_2B_4v_{14}v_{33}\quad+B_1B_3v_{32}v_{44}\quad-B_1B_3v_{42}v_{34}\quad-B_3^2v_{12}v_{44}$$

$$+B_3B_4v_{12}v_{34}\quad+B_3^2v_{14}v_{42}\quad-B_3B_4v_{14}v_{32}\quad+B_1B_4v_{32}v_{43}\quad-B_1B_4v_{42}v_{33}\quad-B_3B_4v_{12}v_{43}\quad+B_4^2v_{12}v_{33}$$

$$\left.+B_3B_4v_{13}v_{42}-B_4^2v_{13}v_{32}\right\}$$

$$=-\frac{h_2}{|J|}\left\{-B_1v_{22}v_{33}v_{44}\quad+B_1v_{22}v_{34}^2\quad-B_1v_{23}v_{42}v_{34}\quad+B_1v_{23}^2v_{44}\quad-B_1v_{24}v_{32}v_{43}\quad+B_1v_{24}^2v_{33}\quad+B_2v_{12}v_{33}v_{44}\right.$$

$$-B_2v_{12}v_{34}^2\quad+B_4v_{12}v_{23}v_{34}\quad-B_3v_{12}v_{23}v_{44}\quad+B_3v_{12}v_{24}v_{43}\quad-B_4v_{12}v_{24}v_{33}\quad-B_2v_{13}v_{32}v_{44}\quad+B_2v_{13}v_{42}v_{34}$$

$$-B_4v_{13}v_{22}v_{34}\quad+B_3v_{13}v_{22}v_{44}\quad-B_3v_{13}v_{24}^2\quad+B_4v_{13}v_{24}v_{32}\quad+B_2v_{14}v_{32}v_{43}\quad-B_2v_{14}v_{42}v_{33}\quad+B_4v_{14}v_{22}v_{33}$$

$$\left.-B_3v_{14}v_{22}v_{43}\quad+B_3v_{14}v_{23}v_{42}\quad-B_4v_{14}v_{23}^2\right\}\quad-\frac{\mu}{|J|}\left\{-B_1B_2v_{33}v_{44}\quad+B_1B_2v_{34}^2\quad-B_2B_3v_{13}v_{44}\quad-B_2B_4v_{13}v_{34}\right.$$

$$+B_2B_3v_{14}v_{43}\quad+B_2B_4v_{14}v_{33}\quad+B_1B_3v_{32}v_{44}\quad-B_1B_3v_{42}v_{34}\quad-B_3^2v_{12}v_{44}\quad+B_3B_4v_{12}v_{34}\quad+B_3^2v_{14}v_{42}$$

$$\left.-B_3B_4v_{14}v_{32}+B_1B_4v_{32}v_{43}-B_1B_4v_{42}v_{33}-B_3B_4v_{12}v_{43}+B_4^2v_{12}v_{33}+B_3B_4v_{13}v_{42}-B_4^2v_{13}v_{32}\right\}$$

$$=-\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1^2h_2h_3^2h_4^2}\qquad\left\{-kh_1^2b(b-1)c(c-1)d(d-1)\qquad+kh_1^2b(b-1)c^2d^2\qquad-kh_1^2b^2c^2d^2\right.$$

$$+kh_1^2b^2c^2d(d-1)\quad-kh_1^2b^2c^2d^2\quad+kh_1^2b^2c(c-1)d^2\quad+lh_1h_2abc(c-1)d(d-1)\quad-lh_1h_2abc^2d^2$$

$$+nh_1h_4ab^2c^2d\quad-mh_1h_3ab^2cd(d-1)\quad+mh_1h_3ab^2cd^2\quad-nh_1h_4ab^2c(c-1)d\quad-lh_1h_2abc^2d(d-1)$$

$$+lh_1h_2abc^2d^2 \quad -nh_1h_4ab(b-1)c^2d \quad +mh_1h_3ab(b-1)cd(d-1) \quad -mh_1h_3ab^2cd^2 \quad +nh_1h_4ab^2c^2d$$

$$+lh_1h_2abc^2d^2 \quad -lh_1h_2abc(c-1)d^2 \quad +nh_1h_4ab(b-1)c(c-1)d \quad -mh_1h_3ab(b-1)cd^2 \quad +mh_1h_3ab^2cd^2$$

$$-nh_1h_4ab^2c^2d\Big\} - \frac{\mu}{|J|}\frac{A^2h_1^{2a}h_2^{2b}h_3^{2c}h_4^{2d}}{h_1^2h_2^2h_3^2h_4^2}\Big\{-klh_1^2h_2^2c(c-1)d(d-1)+klh_1^2h_2^2c^2d^2-lmh_1h_2^2h_3acd(d-1)$$

$$-nlh_1h_2^2h_4ac^2d \quad +lmh_1h_2^2h_3acd^2+nlh_1h_2^2ac(c-1)d^2+kmh_1^2h_2h_3bcd(d-1)-kmh_1^2h_2h_3bcd(d-1)$$

$$-m^2h_1h_2h_3^2abd(d-1) \qquad +mnh_1h_2h_3h_4abcd \qquad +m^2h_1h_2h_3^2abd^2 \qquad -mnh_1h_2h_3h_4abcd$$

$$-knh_1^2h_2h_4bc(c-1)d \qquad +knh_1^2h_2h_4bc^2d \qquad -mnh_1h_2h_3h_4abcd \qquad +n^2h_1h_2h_4^2abc(c-1)$$

$$+mnh_1h_2h_3h_4abcd \quad -n^2h_1h_2h_4^2abc^2\Big\}$$

$$=-\frac{1}{|J|}\frac{A^3abcdh_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1^2h_2h_3^2h_4^2}\qquad\Big\{-kh_1^2a^{-1}(b-1)(c-1)(d-1)\qquad+kh_1^2a^{-1}(b-1)cd\qquad-kh_1^2a^{-1}bcd$$

$$+kh_1^2a^{-1}bc(d-1)\quad -kh_1^2a^{-1}bcd\quad +kh_1^2a^{-1}b(c-1)d\quad +lh_1h_2(c-1)(d-1)\quad -lh_1h_2cd\quad +nh_1h_4bc$$

$$-mh_1h_3b(d-1)\quad +mh_1h_3bd\quad -nh_1h_4b(c-1)\quad -lh_1h_2c(d-1)\quad +lh_1h_2cd\quad -nh_1h_4(b-1)c$$

$$+mh_1h_3(b-1)(d-1)\quad -mh_1h_3bd\quad +nh_1h_4bc\quad +lh_1h_2cd\quad -lh_1h_2(c-1)d\quad +nh_1h_4(b-1)(c-1)$$

$$-mh_1h_3(b-1)d\quad +mh_1h_3cd\quad -nh_1hbc\Big\} - \frac{\mu}{|J|}\frac{A^2cdh_1^{2a}h_2^{2b}h_3^{2c}h_4^{2d}}{h_1^2h_2^2h_3^2h_4^2}\Big\{-klh_1^2h_2^2(c-1)(d-1)\quad +klh_1^2h_2^2cd$$

$$-lmh_1h_2^2h_3a(d-1)\quad -nlh_1h_2^2h_4ac\quad +lmh_1h_2^2h_3ad\quad +nlh_1h_2^2a(c-1)\quad -m^2h_1h_2h_3^2abc^{-1}(d-1)$$

$$+m^2h_1h_2h_3^2abc^{-1}d\quad -knh_1^2h_2h_4b(c-1)-knh_1^2h_2h_4bc\quad +n^2h_1h_2h_4^2ab(c-1)d^{-1}-n^2h_1h_2h_4^2abcd^{-1}\Big\}$$

$$=-\frac{1}{|J|}\frac{A^3abcdh_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1h_2h_3^2h_4^2}\qquad\left\{\frac{B}{\Sigma}(1-\Sigma)+\frac{B}{\Sigma}(a+b+c+d)\right\}\qquad-\frac{1}{|J|}\frac{A^2abcdh_1^{2a}h_2^{2b}h_3^{2c}h_4^{2d}}{h_1^2h_2^2h_3^2h_4^2}$$

$$\frac{Ah_1^xh_2^yh_3^zh_4^wh_1h_2\Sigma}{B}\left\{\frac{B^2}{\Sigma^2}(c+d)+c\frac{B^2}{\Sigma^2}\right\}$$

$$=-\frac{1}{|H|}\frac{A^3abcdh_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1h_2h_3^2h_4^2}\;\frac{B}{\Sigma}(1+2c+d)<0. \tag{13}$$

From (13) we observe that when the wage rate increases, the capital h_1 of the organization decreases. It seems that due to increase of labor cost, production cost of the organization increases. On the other hand, the laborers work for fewer hours due to income effect. Consequently, for the sustainable profit maximization the organization should decrease its capital.

Now we analyze the effect on labor h_2 when the wage rate per unit of labor, *l* increases. Taking T_{32} (i.e., term of

3rd row and 2nd column) from both sides of (12) we get (Mohajan, 2021c; Mohajan & Mohajan, 2022k,l),

$$\frac{\partial h_2}{\partial l}=\frac{h_2}{|J|}\left[C_{13}\right]+\frac{\mu}{|J|}\left[C_{33}\right]$$

$$=\frac{h_2}{|J|}\text{Cofactor of } C_{13}+\frac{\mu}{|J|}\text{Cofactor of C}_{33}$$

$$=\frac{h_2}{|J|}\begin{vmatrix}-B_1 & v_{11} & v_{13} & v_{14}\\ -B_2 & v_{21} & v_{23} & v_{24}\\ -B_3 & v_{31} & v_{33} & v_{34}\\ -B_4 & v_{41} & v_{43} & v_{44}\end{vmatrix}+\frac{\mu}{|J|}\begin{vmatrix}0 & -B_1 & -B_3 & -B_4\\ -B_1 & v_{11} & v_{13} & v_{14}\\ -B_3 & v_{31} & v_{33} & v_{34}\\ -B_4 & v_{41} & v_{43} & v_{44}\end{vmatrix}$$

$$=\frac{h_2}{|J|}\left\{-B_1\begin{vmatrix}v_{21} & v_{23} & v_{24}\\ v_{31} & v_{33} & v_{34}\\ v_{41} & v_{43} & v_{44}\end{vmatrix}-v_{11}\begin{vmatrix}-B_2 & v_{23} & v_{24}\\ -B_3 & v_{33} & v_{34}\\ -B_4 & v_{43} & v_{44}\end{vmatrix}+v_{13}\begin{vmatrix}-B_2 & v_{21} & v_{24}\\ -B_3 & v_{31} & v_{34}\\ -B_4 & v_{41} & v_{44}\end{vmatrix}-v_{14}\begin{vmatrix}-B_2 & v_{21} & v_{23}\\ -B_3 & v_{31} & v_{33}\\ -B_4 & v_{41} & v_{43}\end{vmatrix}\right\}$$

$$+\frac{\mu}{|J|}\left\{-B_1\begin{vmatrix}-B_1 & v_{13} & v_{14}\\ -B_3 & v_{33} & v_{34}\\ -B_4 & v_{43} & v_{44}\end{vmatrix}+B_3\begin{vmatrix}-B_1 & v_{11} & v_{14}\\ -B_3 & v_{31} & v_{34}\\ -B_4 & v_{41} & v_{44}\end{vmatrix}-B_4\begin{vmatrix}-B_1 & v_{11} & v_{13}\\ -B_3 & v_{31} & v_{33}\\ -B_4 & v_{41} & v_{43}\end{vmatrix}\right\}$$

$$=\frac{h_2}{|J|}\left[-B_1\left\{v_{21}\left(v_{33}v_{44}-v_{43}v_{34}\right)+v_{23}\left(v_{41}v_{34}-v_{31}v_{44}\right)+v_{24}\left(v_{31}v_{43}-v_{41}v_{33}\right)\right\}\right.$$

$$-v_{11}\left\{-B_2\left(v_{33}v_{44}-v_{43}v_{34}\right)+v_{23}\left(-B_4v_{34}+B_3v_{44}\right)+v_{24}\left(-B_3v_{43}+B_4v_{33}\right)\right\}$$

$$+v_{13}\left\{-B_2\left(v_{31}v_{44}-v_{41}v_{34}\right)+v_{21}\left(-B_4v_{34}+B_3v_{44}\right)+v_{24}\left(-B_3v_{41}+B_4v_{31}\right)\right\}$$

$$\left.-v_{14}\left\{-B_2\left(v_{31}v_{43}-v_{41}v_{33}\right)+v_{21}\left(-B_4v_{33}+B_3v_{43}\right)+v_{23}\left(-B_3v_{41}+B_4v_{31}\right)\right\}\right]$$

$$+\frac{\mu}{|J|}\left[\left\{-B_1\left\{-B_1\left(v_{33}v_{44}-v_{43}v_{34}\right)+v_{13}\left(-B_4v_{34}+B_3v_{44}\right)+v_{14}\left(-B_3v_{43}+B_4v_{33}\right)\right\}\right.\right.$$

$$+B_3\left\{-B_1\left(v_{31}v_{44}-v_{41}v_{34}\right)+v_{11}\left(-B_4v_{34}+B_3v_{44}\right)+v_{14}\left(-B_3v_{41}+B_4v_{31}\right)\right\}$$

$$\left.-B_4\left\{-B_1\left(v_{31}v_{43}-v_{41}v_{33}\right)+v_{11}\left(-B_4v_{33}+B_3v_{43}\right)+v_{13}\left(-B_3v_{41}+B_4v_{31}\right)\right\}\right\}\Big]$$

$$=-\frac{h_2}{|J|}\{B_1v_{21}v_{33}v_{44}\quad -B_1v_{21}v_{43}v_{34}\quad +B_1v_{23}v_{41}v_{24}\quad -B_1v_{23}v_{31}v_{44}\quad +B_1v_{24}v_{31}v_{43}\quad -B_1v_{24}v_{41}v_{33}$$

$$-B_2v_{11}v_{33}v_{44}\ +B_2v_{11}v_{43}v_{34}\ -B_4v_{11}v_{23}v_{34}\ +B_3v_{11}v_{23}v_{44}\ -B_3v_{11}v_{24}v_{43}\ +B_4v_{11}v_{24}v_{33}\ +B_2v_{13}v_{31}v_{44}$$

$$-B_2v_{13}v_{41}v_{34}\ +B_4v_{13}v_{21}v_{34}\ -B_3v_{13}v_{21}v_{44}\ +B_3v_{13}v_{24}v_{41}\ -B_4v_{13}v_{24}v_{31}\ -B_2v_{14}v_{31}v_{43}\ +B_2v_{14}v_{41}v_{33}$$

$$-B_4v_{14}v_{21}v_{33} + B_3v_{14}v_{21}v_{43} - B_3v_{14}v_{23}v_{41} + B_4v_{14}v_{23}v_{31}\} + \frac{\mu}{|J|}\{B_1^2v_{33}v_{44} - B_1^2v_{43}v_{34} + B_1B_4v_{13}v_{34}$$

$$-B_1B_3v_{13}v_{44} + B_1B_3v_{14}v_{43} - B_1B_4v_{14}v_{33} - B_1B_3v_{31}v_{44} + B_1B_3v_{41}v_{34} - B_3B_4v_{11}v_{34} + B_3^2v_{11}v_{44} - B_3^2v_{14}v_{41}$$

$$+B_3B_4v_{14}v_{31} + B_1B_4v_{31}v_{43} - B_1B_4v_{41}v_{33} + B_4^2v_{11}v_{33} - B_3B_4v_{11}v_{43} + B_3B_4v_{13}v_{41} - B_4^2v_{13}v_{31}\}$$

$$= -\frac{h_2}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1^2h_2^2h_3^2h_4^2}\{kh_1h_2abc(c-1)d(d-1) - kh_1h_2abc^2d^2 + kh_1h_2abc^2d^2$$

$$-kh_1h_2abc^2d(d-1) + kh_1h_2abc^2d^2 - kh_1h_2abc(c-1)d^2 - lh_2^2a(a-1)c(c-1)d(d-1)$$

$$+lh_2^2a(a-1)c^2d^2 - nh_2h_4a(a-1)bc^2d + mh_2h_3a(a-1)bcd(d-1) - mh_2h_3a(a-1)bcd^2$$

$$+nh_2h_4a(a-1)bc(c-1)d + lh_2^2a^2c^2d(d-1) - lh_2^2a^2c^2d^2 + nh_2h_4a^2bc^2d - mh_2h_3a^2bcd(d-1)$$

$$+mh_2h_3a^2bcd^2 - nh_2h_4a^2bc^2d - lh_2^2a^2c^2d^2 + lh_2^2a^2c(c-1)d^2 - nh_2h_4a^2bc(c-1)d$$

$$+mh_2h_3a^2bcd^2 - mh_2h_3a^2bcd^2 + nh_2h_4a^2bc^2d\}$$

$$+\frac{\mu}{|J|}\frac{A^2h_1^{2a}h_2^{2b}h_3^{2c}h_4^{2d}}{h_1^2h_2^2h_3^2h_4^2}\{k^2h_1^2h_2^2c(c-1)d(d-1) - k^2h_1^2h_2^2c^2d^2 + knh_1h_2^2h_4ac^2d$$

$$-kmh_1h_2^2h_3acd(d-1) + kmh_1h_2^2h_3acd^2 - knh_1h_2^2h_4ac(c-1)d - kmh_1h_2^2h_3acd(d-1)$$

$$+kmh_1h_2^2h_3acd^2 - nlh_2^2h_3h_4a(a-1)cd + m^2h_2^2h_3^2a(a-1)d(d-1) - m^2h_2^2h_3^2a^2d^2 + nlh_2^2h_3h_4a^2cd$$

$$+knh_1h_2^2h_4ac^2d - knh_1h_2^2h_4ac(c-1)d + n^2h_2^2h_4^2a(a-1)c(c-1) - mnh_2^2h_3h_4a(a-1)cd$$

$$+mnh_2^2h_3h_4a^2cd - n^2h_2^2h_4^2a^2c^2\}$$

$$= -\frac{1}{|J|}\frac{abcdA^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1^2h_3^2h_4^2}\{kh_1(c-1)(d-1) - kh_1c(d-1) + kh_1cd - kh_1(c-1)d + lh_2ab^{-1}c(d-1)$$

$$-2lh_2ab^{-1}cd + lh_2ab^{-1}(c-1)d - lh_2(a-1)b^{-1}(c-1)(d-1) + lh_2(a-1)b^{-1}cd + mh_3(a-1)(d-1)$$

$$-2mh_3a(d-1) + mh_3ad + nh_4(a-1)(c-1) - nh_4(a-1)c - nh_4a(c-1) + nh_4ac\}$$

$$+\frac{1}{|J|}\frac{A^2cdh_1^{2a}h_2^{2b}h_3^{2c}h_4^{2d}}{h_1^2h_3^2h_4^2}\frac{Ah_1^xh_2^yh_3^zh_4^w\Sigma}{B}\{k^2h_1^2(c-1)(d-1) - k^2h_1^2cd + knh_1h_4ac - kmh_1h_3a(d-1)$$

$$+kmh_1h_3ad - knh_1h_4a(c-1) - kmh_1h_3a(d-1) + kmh_1h_3ad - nlh_3h_4a(a-1)$$

$$+m^2h_3^2a(a-1)c^{-1}(d-1) - m^2h_3^2a^2c^{-1}d + nlh_3h_4a^2 + knh_1h_4ac - knh_1h_4a(c-1)$$

$$+n^2h_4^2a(a-1)(c-1)d^{-1}-mnh_3h_4a(a-1)+mnh_3h_4a^2-n^2h_4^2a^2cd^{-1}\}$$

$$=-\frac{1}{|J|}\frac{abcdA^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_3^2h_4^2\Sigma}\{2a(c-1)(d-1)-(a-1)(c-1)(d-1)-2ca(d-1)+(a-1)(c-1)d$$

$$-a(c-1)d+acd\}+\frac{1}{|J|}\frac{acdA^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_3^2h_4^2\Sigma}\{a(c-1)(d-1)-2ac(d-1)+3acd-2a(c-1)d$$

$$+(a-1)c(d-1)+(a-1)(c-1)d-2(a-1)cd\}$$

$$=-\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_3^2h_4^2\Sigma}acd(abc-bcd+b)+\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_3^2h_4^2\Sigma}acd(a+c+d)$$

$$=\frac{1}{|J|}\frac{acdA^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_3^2h_4^2\Sigma}\{(\Sigma+bcd)-b(ac+2)\} \tag{14}$$

From (14) we observe that if $(\Sigma+bcd)>b(ac+2)$ we get,

$$\frac{\partial h_2}{\partial l}>0. \tag{15}$$

The inequality (15) indicates that if the wage rate *l* of the organization increases, the total labor h_2 of the organization also increases, which is reasonable. Due to substitution effect for more income the total labor hours among the laborers increase.

From (14) we observe that if $(\Sigma+bcd)<b(ac+2)$ we get,

$$\frac{\partial h_2}{\partial l}<0. \tag{16}$$

The inequality (16) indicates that if the wage rate *l* of the organization increases, the total labor h_2 of the organization decreases, which is not reasonable. It seems that due to income effect laborers need less total labor hours to manage necessary money of daily expenditure. Consequently some laborers remain absent, because they have enough money to carry on their absent periods.

Now we analyze the effect on principal raw materials h_3 when the wage rate per unit of labor, *l* increases. Taking T_{42} (i.e., term of 4th row and 2nd column) from both sides of (12) we get (Mohajan, 2022; Mohajan & Mohajan, 2023i),

$$\frac{\partial h_3}{\partial l}=\frac{h_2}{|J|}[C_{14}]+\frac{\mu}{|J|}[C_{34}]$$

$$=\frac{h_2}{|J|}\text{Cofactor of } C_{14}+\frac{\mu}{|J|}\text{Cofactor of } C_{34}$$

$$=-\frac{h_2}{|J|}\begin{vmatrix}-B_1 & v_{11} & v_{12} & v_{14}\\ -B_2 & v_{21} & v_{22} & v_{24}\\ -B_3 & v_{31} & v_{32} & v_{34}\\ -B_4 & v_{41} & v_{42} & v_{44}\end{vmatrix}-\frac{\mu}{|J|}\begin{vmatrix}0 & -B_1 & -B_2 & -B_4\\ -B_1 & v_{11} & v_{12} & v_{14}\\ -B_3 & v_{31} & v_{32} & v_{34}\\ -B_4 & v_{41} & v_{42} & v_{44}\end{vmatrix}$$

$$=-\frac{h_2}{|J|}\left\{-B_1\begin{vmatrix} v_{21} & v_{22} & v_{24} \\ v_{31} & v_{32} & v_{34} \\ v_{41} & v_{42} & v_{44} \end{vmatrix}-v_{11}\begin{vmatrix} -B_2 & v_{22} & v_{24} \\ -B_3 & v_{32} & v_{34} \\ -B_4 & v_{42} & v_{44} \end{vmatrix}+v_{12}\begin{vmatrix} -B_2 & v_{21} & v_{24} \\ -B_3 & v_{31} & v_{34} \\ -B_4 & v_{41} & v_{44} \end{vmatrix}-v_{14}\begin{vmatrix} -B_2 & v_{21} & v_{22} \\ -B_3 & v_{31} & v_{32} \\ -B_4 & v_{41} & v_{42} \end{vmatrix}\right\}$$

$$-\frac{\mu}{|J|}\left\{B_1\begin{vmatrix} -B_1 & v_{12} & v_{14} \\ -B_3 & v_{32} & v_{34} \\ -B_4 & v_{42} & v_{44} \end{vmatrix}-B_2\begin{vmatrix} -B_1 & v_{11} & v_{14} \\ -B_3 & v_{31} & v_{34} \\ -B_4 & v_{41} & v_{44} \end{vmatrix}+B_4\begin{vmatrix} -B_1 & v_{11} & v_{12} \\ -B_3 & v_{31} & v_{32} \\ -B_4 & v_{41} & v_{42} \end{vmatrix}\right\}$$

$$=-\frac{h_2}{|J|}\left[-B_1\left\{v_{21}\left(v_{32}v_{44}-v_{42}v_{34}\right)+v_{22}\left(v_{41}v_{34}-v_{31}v_{44}\right)+v_{24}\left(v_{31}v_{42}-v_{41}v_{32}\right)\right\}\right.$$

$$-v_{11}\left\{-B_2\left(v_{32}v_{44}-v_{42}v_{34}\right)+v_{22}\left(-B_4v_{34}+B_3v_{44}\right)+v_{24}\left(-B_3v_{42}+B_4v_{32}\right)\right\}$$

$$+v_{12}\left\{-B_2\left(v_{31}v_{44}-v_{41}v_{34}\right)+v_{21}\left(-B_4v_{34}+B_3v_{44}\right)+v_{24}\left(-B_3v_{41}+B_4v_{31}\right)\right\}$$

$$\left.-v_{14}\left\{-B_2\left(v_{31}v_{42}-v_{41}v_{32}\right)+v_{21}\left(-B_4v_{32}+B_3v_{42}\right)+v_{22}\left(-B_3v_{41}+B_4v_{31}\right)\right\}\right]$$

$$-\frac{\mu}{|J|}\left[-B_1\left\{-B_1\left(v_{32}v_{44}-v_{42}v_{34}\right)+v_{12}\left(-B_4v_{34}+B_3v_{44}\right)+v_{14}\left(-B_3v_{42}+B_4v_{32}\right)\right\}\right.$$

$$-B_2\left\{-B_1\left(v_{31}v_{44}-v_{41}v_{34}\right)+v_{11}\left(-B_4v_{34}+B_3v_{44}\right)+v_{14}\left(-B_3v_{41}+B_4v_{31}\right)\right\}$$

$$\left.+B_4\left\{-B_1\left(v_{31}v_{42}-v_{41}v_{32}\right)+v_{11}\left(-B_4v_{32}+B_3v_{42}\right)+v_{12}\left(-B_3v_{41}+B_4v_{31}\right)\right\}\right]$$

$$=-\frac{h_2}{|J|}\left\{-B_1v_{21}v_{32}v_{44}+B_1v_{21}v_{42}v_{34}-B_1v_{22}v_{41}v_{34}+B_1v_{22}v_{31}v_{44}-B_1v_{24}v_{31}v_{42}+B_1v_{24}v_{41}v_{32}\right.$$

$$+B_2v_{11}v_{32}v_{44}-B_2v_{11}v_{42}v_{34}+B_4v_{11}v_{22}v_{34}-B_3v_{11}v_{22}v_{44}+B_3v_{11}v_{24}v_{42}-B_4v_{11}v_{24}v_{32}$$

$$-B_2v_{12}v_{31}v_{44}+B_2v_{12}v_{41}v_{34}-B_4v_{12}v_{21}v_{34}+B_3v_{12}v_{21}v_{44}-B_3v_{12}v_{24}v_{41}+B_4v_{12}v_{24}v_{31}$$

$$\left.+B_2v_{14}v_{31}v_{42}-B_2v_{14}v_{41}v_{32}+B_4v_{14}v_{21}v_{32}-B_3v_{14}v_{21}v_{42}+B_3v_{14}v_{22}v_{41}-B_4v_{14}v_{22}v_{31}\right\}$$

$$-\frac{\mu}{|J|}\left\{B_1^2v_{32}v_{44}-B_1^2v_{42}v_{34}+B_1B_4v_{12}v_{34}-B_1B_3v_{12}v_{44}+B_1B_3v_{14}v_{42}-B_1B_4v_{14}v_{32}\right.$$

$$+B_1B_2v_{31}v_{44}-B_1B_2v_{41}v_{34}+B_2B_4v_{11}v_{34}-B_2B_3v_{11}v_{44}+B_2B_3v_{14}v_{41}-B_2B_4v_{14}v_{31}$$

$$\left.-B_1B_4v_{31}v_{42}+B_1B_4v_{41}v_{32}-B_4^2v_{11}v_{32}+B_3B_4v_{11}v_{42}-B_3B_4v_{12}v_{41}+B_4^2v_{12}v_{31}\right\}$$

$$=-\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}}{h_1^2h_2h_3^2h_4^2}\left\{-kh_1h_3ab^2cd(d-1)+kh_1h_3ab^2cd^2-kh_1h_3ab(b-1)cd^2-kh_1h_3ab^2cd^2\right.$$

$$+kh_1h_3ab(b-1)cd(d-1)+kh_1h_3ab^2cd^2+lh_2h_3a(a-1)bcd(d-1)-lh_2h_3a(a-1)bcd^2$$

$$+nh_3h_4a(a-1)b(b-1)cd \quad -mh_3^2a(a-1)b(b-1)d(d-1) \quad +mh_3^2a(a-1)b^2d^2 \quad -nh_3h_4a(a-1)b^2cd$$

$$-lh_2h_3a^2bcd(d-1) \quad +lh_2h_3a^2bcd^2 \quad -nh_3h_4a^2b^2cd \quad +mh_3^2a^2b^2d(d-1) \quad -mh_3^2a^2b^2d^2$$

$$+nh_3h_4a^2b^2cd \quad +lh_2h_3a^2bcd^2 \quad -lh_2h_3a^2bcd^2 \quad +nh_3h_4a^2b^2cd \quad -mh_3^2a^2b^2d^2 \quad +mh_3^2a^2b(b-1)d^2$$

$$-nh_3h_4a^2b(b-1)cd\}$$

$$-\frac{\mu}{|J|}\{k^2h_1^2h_2h_3bcd(d-1) \quad -k^2h_1^2h_2h_3bcd^2 \quad +knh_1h_2h_3h_4abcd \quad -kmh_1h_2h_3^2bcd(d-1)$$

$$+kmh_1h_2h_3^2bcd^2 \quad -knh_1h_2h_3h_4abcd \quad +klh_1h_2^2h_3acd(d-1) \quad -klh_1h_2^2h_3acd^2 \quad +nlh_2^2h_3h_4a(a-1)cd$$

$$+lmh_2^2h_3^2a^2d^2 \quad -lmh_2^2h_3^2a(a-1)d(d-1) \quad -nlh_2^2h_3h_4a^2cd \quad -knh_1h_2h_3h_4abcd \quad +knh_1h_2h_3h_4abcd$$

$$-n^2h_2h_3h_4^2a(a-1)bc \quad +mnh_2h_3^2h_4a(a-1)bd \quad -mnh_2h_3^2h_4a^2bd \quad +n^2h_2h_3h_4^2a^2bc\}$$

$$=-\frac{1}{|J|}\frac{A^3abcdh_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_2h_3h_4^2\Sigma}+\frac{1}{|J|}\frac{A^3abcdh_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_2h_3h_4^2\Sigma}=0\,. \tag{17}$$

From (17) we have realized that there is no relation between principle raw material and wage rate. It seems that although if the wage rate increase, the organization may increase or decrease principle raw material depending on its demand.

Now we observe the effect on other inputs h_4 when the wage rate per unit of labor, *l* increases. Taking T_{52} (i.e., term of 5th row and 2nd column) from both sides of (12) we get (Islam et al., 2011; Mohajan, 2017a; Ferdous & Mohajan, 2023j),

$$\frac{\partial h_4}{\partial l}=\frac{h_2}{|J|}[C_{15}]+\frac{\mu}{|J|}[C_{35}]$$

$$=\frac{h_2}{|J|}\text{Cofactor of } C_{15}+\frac{\mu}{|J|}\text{Cofactor of C}_{35}$$

$$=\frac{h_2}{|J|}\begin{vmatrix}-B_1 & v_{11} & v_{12} & v_{13}\\ -B_2 & v_{21} & v_{22} & v_{23}\\ -B_3 & v_{31} & v_{32} & v_{33}\\ -B_4 & v_{41} & v_{42} & v_{43}\end{vmatrix}+\frac{\mu}{|J|}\begin{vmatrix}0 & -B_1 & -B_2 & -B_3\\ -B_1 & v_{11} & v_{12} & v_{13}\\ -B_3 & v_{31} & v_{32} & v_{33}\\ -B_4 & v_{41} & v_{42} & v_{43}\end{vmatrix}$$

$$=\frac{h_2}{|J|}\left\{-B_1\begin{vmatrix}v_{21} & v_{22} & v_{23}\\ v_{31} & v_{32} & v_{33}\\ v_{41} & v_{42} & v_{43}\end{vmatrix}-v_{11}\begin{vmatrix}-B_2 & v_{22} & v_{23}\\ -B_3 & v_{32} & v_{33}\\ -B_4 & v_{42} & v_{43}\end{vmatrix}+v_{12}\begin{vmatrix}-B_2 & v_{21} & v_{23}\\ -B_3 & v_{31} & v_{33}\\ -B_4 & v_{41} & v_{43}\end{vmatrix}-v_{13}\begin{vmatrix}-B_2 & v_{21} & v_{22}\\ -B_3 & v_{31} & v_{32}\\ -B_4 & v_{41} & v_{42}\end{vmatrix}\right\}$$

$$+\frac{\mu}{|J|}\left\{B_1\begin{vmatrix}-B_1 & v_{12} & v_{13}\\ -B_3 & v_{32} & v_{33}\\ -B_4 & v_{42} & v_{43}\end{vmatrix}-B_2\begin{vmatrix}-B_1 & v_{11} & v_{13}\\ -B_3 & v_{31} & v_{33}\\ -B_4 & v_{41} & v_{43}\end{vmatrix}+B_3\begin{vmatrix}-B_1 & v_{11} & v_{12}\\ -B_3 & v_{31} & v_{32}\\ -B_4 & v_{41} & v_{42}\end{vmatrix}\right\}$$

$$=\frac{h_2}{|J|}\left[-B_1\left\{v_{21}\left(v_{32}v_{43}-v_{42}v_{33}\right)+v_{22}\left(v_{41}v_{33}-v_{31}v_{43}\right)+v_{23}\left(v_{31}v_{42}-v_{41}v_{32}\right)\right\}\right.$$

$$-v_{11}\left\{-B_2\left(v_{32}v_{43}-v_{42}v_{33}\right)+v_{22}\left(-B_4v_{33}+B_3v_{43}\right)+v_{23}\left(-B_3v_{42}+B_4v_{32}\right)\right\}$$

$$+v_{12}\left\{-B_2\left(v_{31}v_{43}-v_{41}v_{33}\right)+v_{21}\left(-B_4v_{33}+B_3v_{43}\right)+v_{23}\left(-B_3v_{41}+B_4v_{31}\right)\right\}$$

$$\left.-v_{13}\left\{-B_2\left(v_{31}v_{42}-v_{41}v_{32}\right)+v_{21}\left(-B_4v_{32}+B_3v_{42}\right)+v_{22}\left(-B_3v_{41}+B_4v_{31}\right)\right\}\right]$$

$$+\frac{\mu}{|J|}\left[B_1\left\{-B_1\left(v_{32}v_{43}-v_{42}v_{33}\right)+v_{12}\left(-B_4v_{33}+B_3v_{43}\right)+v_{13}\left(-B_3v_{42}+B_4v_{32}\right)\right\}\right.$$

$$-B_2\left\{-B_1\left(v_{31}v_{43}-v_{41}v_{33}\right)+v_{11}\left(-B_4v_{33}+B_3v_{43}\right)+v_{13}\left(-B_3v_{41}+B_4v_{31}\right)\right\}$$

$$\left.+B_3\left\{-B_1\left(v_{31}v_{42}-v_{41}v_{32}\right)+v_{11}\left(-B_4v_{32}+B_3v_{42}\right)+v_{12}\left(-B_3v_{41}+B_4v_{31}\right)\right\}\right]$$

$$=\frac{h_{21}}{|J|}\left\{-B_1v_{21}v_{32}v_{43}+B_1v_{21}v_{42}v_{33}-B_1v_{22}v_{41}v_{33}+B_1v_{22}v_{31}v_{43}-B_1v_{23}v_{31}v_{42}+B_1v_{23}v_{41}v_{32}\right.$$

$$+B_2v_{11}v_{32}v_{43}-B_2v_{11}v_{42}v_{33}+B_4v_{11}v_{22}v_{33}-B_3v_{11}v_{22}v_{43}+B_3v_{11}v_{23}v_{42}-B_4v_{11}v_{23}v_{32}$$

$$-B_2v_{12}v_{31}v_{43}+B_2v_{12}v_{41}v_{33}-B_4v_{12}v_{21}v_{33}+B_3v_{12}v_{21}v_{43}-B_3v_{12}v_{23}v_{41}+B_4v_{12}v_{23}v_{31}$$

$$\left.+B_2v_{13}v_{31}v_{42}-B_2v_{13}v_{41}v_{32}+B_4v_{13}v_{21}v_{32}-B_3v_{13}v_{21}v_{42}+B_3v_{13}v_{22}v_{41}-B_4v_{13}v_{22}v_{31}\right\}$$

$$+\frac{\mu}{|J|}\left\{-B_1^2v_{32}v_{43}+B_1^2v_{42}v_{33}-B_1B_4v_{12}v_{33}+B_1B_3v_{12}v_{43}-B_1B_3v_{13}v_{42}+B_1B_4v_{13}v_{32}\right.$$

$$+B_1B_2v_{31}v_{43}-B_1B_2v_{41}v_{33}+B_2B_4v_{11}v_{33}-B_2B_3v_{11}v_{43}+B_2B_3v_{13}v_{41}-B_2B_4v_{13}v_{31}$$

$$\left.-B_1B_3v_{31}v_{42}+B_1B_3v_{41}v_{32}-B_3B_4v_{11}v_{32}+B_3^2v_{11}v_{42}-B_3^2v_{12}v_{41}+B_3B_4v_{12}v_{31}\right\}$$

$$=\frac{1}{|J|}\frac{A^3h_1^{3x}h_2^{3y}h_3^{3z}h_4^{3w}}{h_1^2h_2h_3^2h_4^2}\left\{-kh_1h_4ab^2c^2d+kh_1h_4ab^2c(c-1)d-kh_1h_4ab(b-1)c(c-1)d\right.$$

$$+kh_1h_4ab(b-1)c(c-1)d-kh_1h_4ab^2c^2d+kh_1h_4ab^2c^2d+lh_2a(a-1)bc^2d$$

$$-lh_2h_4a(a-1)bc(c-1)d+nh_4^2a(a-1)b(b-1)c(c-1)-mh_3h_4a(a-1)b(b-1)cd$$

$$+mh_3h_4a(a-1)b^2cd-nh_4^2a(a-1)b^2c^2-lh_2h_4a^2bc^2d+lh_2h_4a^2bc(c-1)d-nh_4^2a^2b^2c(c-1)$$

$$+mh_3h_4a^2b^2cd-mh_3h_4a^2b^2cd+nh_4^2a^2b^2c^2+lh_2h_4a^2bc^2d-lh_2h_4a^2bc^2d+nh_4^2a^2b^2c^2$$

$$\left.-mh_3h_4a^2b^2cd+mh_3h_4a^2b(b-1)cd-nha^2b(b-1)c^2\right\}$$

$$+\frac{\mu}{|J|}\{-k^2h_1^2h_2h_4bc^2d \quad +k^2h_1^2h_2h_4bc(c-1)d \quad -knh_1h_2h_4^2abc(c-1) \quad +kmh_1h_2h_3h_4abcd$$

$$+knh_1h_2h_4^2abc^2 \ -kmh_1h_2h_3h_4abcd \ +klh_1h_2^2h_4ac^2d \ -klh_1h_2^2h_4ac(c-1)d \ +nlh_2^2h_4^2a(a-1)c(c-1)$$

$$-lmh_2^2h_3h_4a(a-1)cd \quad +lmh_2^2h_3h_4a^2cd \quad -nlh_2^2h_4^2a^2c^2 \quad -kmh_1h_2h_3h_4abcd \quad +kmh_1h_2h_3h_4abcd$$

$$-mnh_2h_3h_4^2a(a-1)bc \ +m^2h_2h_3^2h_4a(a-1)bd \ -m^2h_2h_3^2h_4a^2bd \ +mnh_2h_3h_4^2a^2bc\}$$

$$=\frac{1}{|J|}\frac{A^3abcdh_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_2h_3^2h_4\Sigma} \quad \{-abc \quad +ab(c-1) \quad +(a-1)bc \quad -(a-1)b(c-1) \quad -(a-1)(b-1)c$$

$$+(a-1)(b-1)(c-1)\} \ +\frac{1}{|J|}\frac{A^2abcdh_1^{2a}h_2^{2b}h_3^{2c}h_4^{2d}B^2}{h_1^2h_2h_3^2h_4\Sigma^2} \ \frac{Ah_1^xh_2^yh_3^zh_4^w\Sigma}{B} \ \{ac \ -a(c-1) \ +(a-1)(c-1)$$

$$-2(a-1)c \ +(a-1)c\}$$

$$=\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_2h_3^2h_4\Sigma}abcd(-ab+a-1)+\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_2h_3^2h_4\Sigma}abcd(-ac-a+1)$$

$$=-\frac{1}{|J|}\frac{A^3h_1^{3a}h_2^{3b}h_3^{3c}h_4^{3d}B}{h_1^2h_2h_3^2h_4\Sigma}a^2bcd(b+c)<0\,. \tag{18}$$

From (18) we have realized that when the wage rate per unit of labor, *l* increases; the amount of other inputs h_4 decreases. It seems that other inputs are not essential materials for the organization, and labor level of it may decrease from this sector.

7. Conclusions

In this study we have studied sensitivity analysis of various inputs, such as capital, labors, principal raw materials and other inputs with the increase of wage rate. We have started the discussion with the Cobb-Douglas production function by considering the budget as constraint. This study will be beneficial for the organizations where wage rate is increased remarkably. In this paper we have tried to present mathematical calculations in some details.

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