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Study on the Logical Reasoning Ability Development of Junior High School Students Based on SOLO Taxonomy

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Abstract

Logical reasoning ability, as a vital part of China's mathematics curriculum objectives, is one of the key abilities that junior high school students should possess. SOLO (Structure of the Observed Learning Outcome) taxonomy is an analysis method pertaining to students' response to a certain question. Based on SOLO taxonomy, this paper sets up an assessment scale that evaluates the logical reasoning ability level of junior high school students. After analyzing four specific cases, the paper obtains some enlightenment on mathematics teaching, which is supposed to help cultivate the logical reasoning ability of students in actual teaching.

Keywords: logical reasoning, SOLO taxonomy, core literacy

1. Background of the Study

With the continuous curriculum reform of basic education, countries around the world are attaching greater importance to students' literacy and competency. In 2014, China's Ministry of Education issued an official document of its suggestions on deepening the curriculum reform, and fostering virtue through education, which is the fundamental task of the reform. The document proposed to formulate and develop a core literacy system for students and cultivate their core literacy in the teaching activities of each subject (*Opinions on Comprehensively Deepening Curriculum Reform and Implementing the Fundamental Task of Fostering Virtue through Education*, 2014). The General *High School Mathematics Curriculum Standards (2017)* identified logical reasoning as one of the core literacies of mathematics. And the 2011 edition of the standards also listed logical reasoning as one of the core words. It is an important method of obtaining conclusions and building a mathematical system, an essential guarantee for mathematics rigor, as well as the basic thinking quality people should possess to communicate in mathematical activities (*Mathematics Curriculum Standards for Compulsory Education, 2011*).

Mathematics teaching needs and aims to cultivate the strong logical reasoning ability of students. Based on SOLO taxonomy, this paper classifies the logical reasoning into various levels, which are explained via specific cases, and ultimately puts forward suggestions that would help cultivate students' logical reasoning ability.

2. SOLO Taxonomy Theory-Based Classification of Logical Reasoning Levels

The SOLO taxonomy was pioneered by Australian educational psychologist John B. Biggs in 1982 as a way of classifying students' academic levels. Based on the analysis of students' responses to a specific question, SOLO taxonomy classifies the students' understanding levels into 5 basic grades, which from low to high are: pre-structural level, uni-structural level, multi-structural level, relational level, and extended abstract level.

Liu Jingli introduced the approach of using SOLO taxonomy to develop test tools from the perspective of mathematics, analyzed the test results, and obtained inspiration of SOLO taxonomy for teaching and the limitations of its application, which laid the foundation for later research on evaluating construction of test items with SOLO taxonomy in China (Liu Jingli, 2005). In *Preliminary Study on SOLO Scoring Methods for Open-ended Questions in Mathematics*, Li Xiangzhao firstly introduced the basic content of SOLO taxonomy and used two open-ended questions to illustrate how to apply SOLO taxonomy to the scoring of such questions. He came up with several suggestions on how teachers can use SOLO taxonomy to score students' answers (Li Xiangzhao, 2006). Chen Ang and Ren Zizhao used the SOLO taxonomy to develop comprehensive evaluation rules for the questions of the mathematics college entrance examinations. Specifically, these questions test students' arithmetic solving ability, logical thinking ability, data processing ability, and innovative application awareness. Chen and Ren also emphasized that the evaluation rules should be considered comprehensively in practice(Chen Ang, Ren Zizhao, 2014).

Although the starting points and research perspectives of various scholars are different, they generally agree that SOLO taxonomy is theory-based, practical and operational, which can be adopted to assess student's problem-solving and focus on examining their knowledge. SOLO model determines the students' understanding level according to their ability of combining knowledge in problem solving, which is the main feature of this model.

Experts of the high school curriculum standards revision group divide the 6 core literacies into three levels: academic test, college entrance examination, and independent recruitment examination level, each placing several requirements on the 6 core literacies. This division is in line with the characteristics of mathematics studying in high school, but it will not adapt to and cannot be adopted in junior high school. Therefore, with reference to Professor Wang Guangming's *Operational Definition of High School Students' Mathematical Literacy (*Wang Guangming, Zhang Nan & Zhou Jiushi, 2016*)*, a general humanities and social science project of the Ministry of Education in 2013, this paper interprets the division of logical reasoning literacy level from the perspective of SOLO taxonomy and analyzes the description of each level as follows:

Table 1. Classification of logical reasoning ability levels with SOLO taxonomy

SOLO thinking level	Description
Pre-structural	Students are unwilling to learn mathematics, and unable to repeat relevant concepts and theorems or think of relevant knowledge, let alone relate them to each other. They respond to questions with irrelevant comments.
Uni-structural	Students have a preliminary understanding of the learned concepts and theorems. They can quickly associate a theorem or a concept they have learned, roughly identifying simple or a single piece of information according to the question. However, if the answer involves two more situations, their conclusion may be incorrect. They are eager to work out the question but fail to present the complete solving process, finally answering the questions via trial and error.
Multi-structural	Stimulated by multiple basic graphical structures and textual condition information in the questions, students can immediately display various concepts and theorems involved as representational and abstract semantics in their minds. They may obtain correct or mostly correct answers by finding clues from different given conditions.
Relational	Students at this stage do not rush to write down answers. Instead, they will consider what concepts and theorems correspond to the conditions of the question, how to relate these theorems to each other and use them to work out the conclusion. They can solve problems in a holistic way and express their answers in rigorous mathematical language.
Extended abstract	Facing an unfamiliar question, students identify and analyze the conditions in the question, make connection and comparison with the questions they have encountered (similar ones), and make variations based on them. They can see the essence behind the question and transform an unfamiliar knowledge situation into a familiar one to solve new problems.

3. Case study

Case 1: Xiaoming throws a stone towards a board as follows, and the probability that the stone hits the blank part is _____.

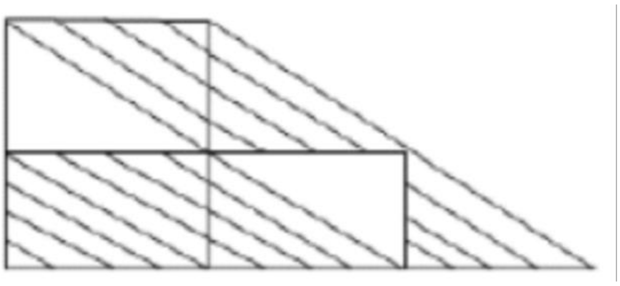

Figure 1.

Pre-structural level: Students cannot figure out the answer since they are unable to understand the question or are not interested in mathematics.

Uni-structural level: Students know that this is a geometric probability question and can derive the right answer 1/4 from calculating the ratio of the blank part to the shadowed part.

Case 2: Observe the following equations：

$7^1=$	**7**，
$7^2=$	4**9**，
$7^3=$	34**3**，
$7^4=$	240**1**，
$7^5=$	1680**7**，
$7^6=$	11764**9**，
$7^7=$	82354**3**，
$7^8=$	576480**1**，
$7^9=$	4035360**7**，

……

According to the law of the above equations, please guess the last digit of 7^{190}: ________.

Pre-structural level: Students cannot answer the question or write down an irrelevant number casually.

Uni-structural level: After observing the above equations, students can find that the last digit of 7^1 and 7^5 is the same, and the last digit of 7^2 and 7^6 is the same..., and the cycle period is 4.

Multi-structural level: Students understand that the last number of 7^{190} depends on the result of dividing 190 by 4. After calculation, 190÷4=47…2. Therefore, the last digit of 7^{190} is the same as that of 7^2, and the answer is 9.

Case 3: In Figure 2, triangles ΔABC and ΔCDE are equilateral triangles. Connecting A and E, B and D, then AE and BD intersects at the point O. Please prove $\angle AOB = 60°$.

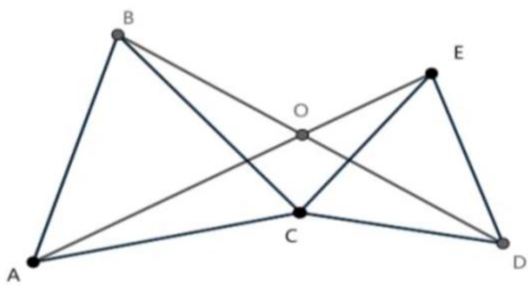

Figure 2.

Pre-structural level: Students cannot prove it or just restate the conditions of the question without any idea of how to prove it.

Uni-structural level: Students know the three sides of an equilateral triangle are equal and that all three angles are 60° according to the conditions in the question. However, they cannot think of the next step of reasoning.

Multi-structural level: Students arrive at $\angle ACE = \angle BCD$ by equivalent substitution, associate it with the knowledge of congruent triangles, and use SAS to prove $\Delta ACE \cong \Delta BCD$. After obtaining congruence, they conclude that $\angle PAC = \angle PBO$ (corresponding angles are equal).

Relational level: Students observe whether there is a connection between the $\angle AOB$ and the known angles, and realize that the sum of the interior angles of ΔACP and ΔBPO are both 180° (let BC and AE intersect at point P). Moreover, opposite angles are equal. With another equivalent substitution, it can be concluded that $\angle AOB$ and $\angle ACB$ are equivalent since $\angle PAC = \angle PBO$, which is obtained via congruence. And the following is their proving process:

Proving: $\because \Delta ABC$、ΔCDE are equilateral triangles.

$\therefore \angle ACB = \angle DCE = 60°$.

And $\because \angle ACE = \angle ACB + \angle BCE$,

$\angle BCD = \angle DCE + \angle BCE$. $\therefore \angle ACE = \angle BCD$

In ΔACE and ΔBCD,

$$\begin{cases} AC = BC \\ \angle ACE = \angle BCD \\ CE = CD \end{cases}$$

$\therefore ACE \cong \Delta BCD(SAS)$.

$\therefore \angle CBD = \angle CAE$. (Opposite angles are equal.)

Supposing AE and BC intersect at the point P,

In ΔACP and ΔBOP,

$\because \angle APC = \angle BPO$. (Opposite angles are equal.)

$\angle OBP = \angle$CAP. (Opposite angles are equal.)

Therefore, $\angle AOB = \angle BCA = 60°$. (The sum of the interior angles of a triangle is 180°.)

Case 4: In Figure 3, in ΔABC, $\angle ABC$=45°, $CD \perp AB$, $BE \perp AC$, the foots of the perpendiculars are D and E respectively. F is the midpoint of BC. BE, DF and DC respectively intersect at point G and H. ∠ABE=∠CBE.

Please prove: $BG^2 - GE^2 = EA^2$

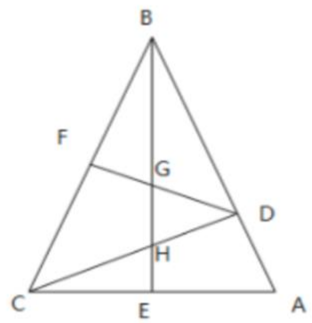

Figure 3.

Pre-structural level: Students cannot prove it or just restate the conditions of the question without any idea of how to prove it.

Uni-structural level: Given $\angle ABE = \angle CBE$ and $BE \perp AC$, students know ΔABC is an isosceles triangle, and BE bisects AC vertically. But they cannot think of the next step of reasoning.

Multi-structural level: Students reread the question and reason that ΔBCD is an isosceles right triangle and DF bisects BC vertically since $\angle ABC$=45°, $CD \perp AB$, and F is the midpoint of BC. They can realize auxiliary lines are needed to link the known conditions.

Relational level: By deforming the equation, students obtain $BG^2 = GE^2 + EA^2$. By observing the equation, they find that the equation to be proved is similar in structure to the Pythagorean Theorem, so they connect C or A with G.

Extended abstract level: By drawing an auxiliary line CG, students can obtain $CG^2 = GE^2 + CE^2$ in the right triangle ΔCEG. The points on the perpendicular bisector are equidistant from the two ends of the line segment. And through equivalent substitution, they get $CG = BG$ and $CE = AE$, thereby proving that $BG^2 = GE^2 + EA^2$. Their specific answer is as follows:

Proving $\because \angle ABE = \angle CBE$, $BE \perp AC$,

$\therefore AB = BC$, BE bisects AC vertically.

$\because F$ is the midpoint of BC, $ABC = 45°$, and ΔBCD is an isosceles right triangle.

$\therefore DF$ bisects BC vertically.

Connecting C with G, in $\varDelta CEG$,

$CE^2 + GE^2 = CG^2$.

Via equivalent substitution, $CG = BG$, and $CE = AE$.

$\therefore BG^2 - GE^2 = EA^2$.

From the above cases, we can know that when combining SOLO taxonomy with specific mathematics knowledge to analyze students' level of logical reasoning ability, the analysis of such ability is refined, which can help students identify their own logical reasoning level, find what should be focused on, scientifically allocate time for each part in mathematics, and improve their studying efficiency. Also, this refinement can help teachers discover students' weaknesses, and adjust their teaching strategies.

4. Enlightenment on Teaching

Mathematics curricula enable students to master necessary basic knowledge and skills and cultivate their abstract thinking as well as reasoning ability. And reasoning ability is developed in the whole process of studying mathematics, during which it is formed and improved step by step in the long term.

4.1 Build a Bridge Between Prior and New Knowledge and Cultivate Analogical Reasoning Ability

Learning mathematics is a process of actively constructing a new knowledge structure based on prior knowledge and experience, during which new and prior knowledge are linked. Teachers should build this linking bridge according to the proximal development zone of students' cognition. For example, they can illustrate the properties of rhombuses by analogy with rectangles and parallelograms, equilateral triangles by analogy with isosceles triangles, quadratic equation with one unknown by analogy with linear equation with one unknown, etc. Students can find the shared features of these knowledge under the guidance of teachers. And their confidence will be enhanced by recalling what they have learned. Teaching them new knowledge in time will make it easier for them to understand and accept.

4.2 Explore More Deeply and Cultivate Plausible Reasoning Ability

Teachers are supposed to chew over the requirements in the curriculum standards, exploring more effective and diverse activity methods in their class to cultivate students' reasoning ability. They should also be good at processing teaching materials, integrating more practical activities into mathematics class, and letting students guess the conclusion through specific questions. Students will enjoy the fun of logical mathematical reasoning by conjecturing conclusions from specific problems and then verifying their findings through general proofs. For example, when teaching the perfect square formula, teachers can ask students to derive the formula from a geometric perspective by cutting out paper to assemble large squares and small squares, thus enhancing students' understanding of the formula. Students can arrive at conclusions in exploration through plausible reasoning. In this process, they can appreciate basic mathematical ideas and gather experience of mathematical activities, which can greatly improve their plausible reasoning ability.

4.3 Standardize Answering Steps and Cultivate Deductive Reasoning Ability

Mathematics is rigorous and so is the process of deductive reasoning. Therefore, it is important that students are disciplined in answering questions. When writing down proofs, they should pay attention to the logical relationship between each step. To master more proving skills, they should make more efforts in logical proof, mainly through doing more exercises. Teachers are the role model for students, and every word and action of them will have an invisible influence on students. Therefore, teachers always need to standardize their own steps of answers to the example questions and exercises, which can better develop students' deductive reasoning ability. For example, there are strict rules for proving a triangle is congruent, which requires teachers to emphasize the answering format in their class so that students can realize the importance of standardized steps.

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