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# Proposal for a Hypothetical Learning Path Using GeoGebra in Learning Numerical Sequences

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**Abstract**

A hypothetical learning path is proposed for the teaching-learning of numerical sequences in second grade secondary school students, it is theoretically based in a Duval's Theory of Semiotic Representation Registers and methodologically based on GeoGebra & Hypothetical Learning Path. It is intended to contribute to this field of mathematics given that there are not many the works in this regard, a fact that motivates the facilitation of better perspectives for students in learning the mathematical object numerical sequences. According to the hypothetical learning trajectory theory, each stage of learning requires a series of preconditions and appropriate instructional approaches so that the individual can progress. These stages include: 1) Acquisition of basic knowledge 2) Application of skills 3) Automation 4) Integration. It is concluded that it must be applied in the classroom to know the effectiveness of this trajectory.

**Keywords:** hypothetical learning path, GeoGebra, numerical sequences

## 1. Introduction

The teaching-learning of numerical sequences in Mexico is usually approached in a traditional way, explanation on the blackboard, presentation of the different types of sequences, and their way of solving them, with the student acting passively, and when moving on to the next stage is when you can actively interact, (Carmona, 2016; Chale, 2013) this being the objective of this proposal, to incorporate the dynamic software GeoGebra to make the teaching-learning process more active.

In the present work, a strategy is presented through the design of a hypothetical learning trajectory mediated by information and communication technology (ICT) to strengthen the development of teaching-learning of numerical sequences in second year high school students. Hypothetical learning trajectory theory provides a framework for designing effective, sequential instruction. By understanding the stages of learning, educators can adapt their approach to facilitate optimal student progress.

The importance of planning a hypothetical learning trajectory supported by GeoGebra lies in the value that this tool can contribute to the teaching and learning process of mathematics. GeoGebra is a dynamic mathematics software that allows you to create geometric constructions, graphic representations, calculations, and numerical explorations, among other functionalities. (Chavez, 2023)

Because a hypothetical learning trajectory and not a didactic sequence. The difference between a hypothetical learning trajectory and a didactic sequence lies in its focus and purpose. A hypothetical learning trajectory refers to a series of activities or tasks designed in a flexible and adaptive way, with the objective of guiding the learning process in an individualized manner. It is a suggested route based on the needs and characteristics of the student, which seeks to make the most of his or her strengths and address his or her challenges. This trajectory adjusts as the student progresses, allowing for greater personalization and accommodating the pace and learning style of everyone. On the other hand, a teaching sequence is a structured plan of activities and resources

designed to achieve specific learning objectives in an educational context. This sequence is organized in a logical and progressive manner, following a predefined order to address the contents and facilitate the acquisition of skills and knowledge. The didactic sequence is more rigid in its planning and is applied in a similar way for all students.

In summary, the hypothetical learning trajectory is more flexible and adaptive, designed to adapt to the individual needs of students, while a didactic sequence is more rigid and structured, focused on achieving specific learning objectives in a generalized way.

Research Question: "How does incorporating GeoGebra in the learning process enhance students' understanding of numerical sequences?". This research proposal aims to investigate the effectiveness of incorporating GeoGebra, a dynamic mathematics software, in the learning process of numerical sequences.

## 2. Theoretical Foundation

### *2.1 The Hypothetical Learning Trajectory Theory*

Is a perspective developed by Robert Gagné in the field of educational psychology. According to this theory, learning is considered as a series of sequential stages that must be followed to achieve competence in a certain area. (Mollo, 2017; Meza, A., & Lazarte, C, 1993)

The notion of a hypothetical learning trajectory has recently sparked interest in the mathematics education research literature. Simón and Tzur (2004) cited by Morales & Damián (2022) establish that the main characteristics of a hypothetical learning trajectory refer to the fact that it consists of the objectives for student learning, the mathematical tasks that will be used to promote learning, and hypotheses about the learning process.

Several authors have used this strategy to improve mathematics learning, Gómez, P., & Lupiáñez, J. L. (2007) reviewed the different interpretations of this notion and proposed an adaptation of it with which the future secondary school mathematics teacher can collect and organize information for the design of teaching units. Amador Saelices, M. V., & Montejo Gámez, J. (2016) work on algebraic expressions based on error analysis.

### *2.2 Duval's Theory of Semiotic Representation Registers*

The points out that mathematical activity is necessarily carried out in a context of representation, in which two kinds of transformation are distinguished. According to Jara (2020), the various registers are: Graphic Register, Figural Register, Tabular Register, Natural Language Register and Use of ICTs. The latter is what this work focuses.

### *2.3 GeoGebra & Hypothetical Learning Path*

Likewise, authors like Morales & Damian (2022) have used both strategies for assimilating the height of a triangle (Morales et.al. 2022). Assimilating the Articulated Concepts of Quadratic Function and Equation Through Variational Ideas. (Morales, A., Mojica, A. D., Balbuena, S., & Marmolejo, J. E. 2021) Isometric transformations during the calculation of the area of polygons, Henao (2014) to strengthen the concepts of notable lines and points of the triangle (Quintero, D. I., & Valderrama, N, 2017). Pythagoras theorem.

### *2.4 Hypothetical Learning Path, GeoGebra and Numerical Sequences*

Gonzalez (2012) using GeoGebra for teaching numerical sequences obtaining positive results. According to (Zapatera, 2015) a THA is a technique to project and organize learning supported by the improvement of your thinking over time.

Lukáč, S., & Sekerák, J. (2017) presented tools of spreadsheet and the system GeoGebra are used for calculation and graphical representation of different types of sequences. Investigation of sequence features is the first step to derive formulas for the calculation of sequence members. The main attention is devoted to derive analytical formulas of the n-th member for determination of the sequences. We tried to include problems which require investigation of the relationship between variables derived from real situations with which students have experiences from everyday life.

## 3. Methodological Foundation

### *3.1 Tasks or Learning Activities*

Schultz (2009) defines the term task as a learning activity that takes place in class and that directs students' attention to develop a particular mathematical idea. Smith and Stein (1998) point out that a mathematical task can be analyzed considering the type of reasoning that students put into practice to perform it and use the term cognitive demand to refer to the type and level of thinking that students require to engage. and successfully solve the task. This concept allows us to classify the type of reasoning that the task demands of students. The authors consider the following levels to classify the tasks: a) Low level of cognitive demand (Memorization); b) Low

level of cognitive demand (Procedure without connections); c) High level of cognitive demand (Procedure with connections) and d) High level of cognitive demand (Working on mathematics). García & Benítez, (2013) found evidence of the close relationship between the reasoning that students carry out when working on a task and the way the task is presented, the implicit instructions found in it, the student's prior knowledge and the concepts that are intended to be developed. Specifically, the analysis of the data allows us to conclude that tasks that can be explored through multiple representations (iconic, verbal, numerical, graphic), favor students' reasoning, regardless of their prior knowledge. In the area of mathematics education, interest has recently emerged in the design, evaluation and redesign of tasks, considering it a key aspect to achieve quality teaching and learning of mathematics (Mason & Johnston-Wilder, 2004), cited by Salazar Solórzano, L. (2014).

Likewise (Clements & Sarama, 2014), they affirm that the theoretical construct learning trajectory can be understood as a hypothetical path by which student progress in learning a mathematical topic in a way that contains conjectures about the order and character of growth of the students' mathematical understanding and on the tasks that promote the transition towards the programmed objective.

## 4. Proposal

*4.1 Steps of Hypothetical Learning Path Activities*

1). Mathematical foundations: Start by understanding the basic concepts of numerical sequences, such as the common difference (in the case of arithmetic sequences) or the recurrence relation (in the case of geometric sequences).

2). Practice with simple examples: Solve exercises and problems that involve basic numerical sequences, identifying specific patterns and characteristics.

3). Expanding knowledge: As they advance, explore different types of numerical sequences, such as quadratic sequences, Fibonacci sequences, prime number sequences, among others. They would learn to identify the unique rules and properties of each type.

4). Use of tools and resources: Make use of educational tools and resources, such as textbooks, videos, online tutorials, and machine learning programs to increase your understanding and experience of the subject.

5). Solving more complex problems: Work on more challenging problems that require identifying hidden patterns, applying factoring techniques, using formulas and making use of mathematical intuition.

6). Practical Application: Apply your knowledge of number sequences in the real world, such as in finance, statistics, computer science, or scientific research that requires sequential data analysis.

According to the hypothetical learning trajectory theory, each stage of learning requires a series of preconditions and appropriate instructional approaches so that the individual can progress. These stages include:

1) Acquisition of basic knowledge
2) Application of skills
3) Automation
4) Integration

## 5. Activities

Proposed Tasks-Activities

It is proposed to graphically resolve in GeoGebra the notable sequences as a primary task of the Hypothetical Learning Path, summarized in the following table.

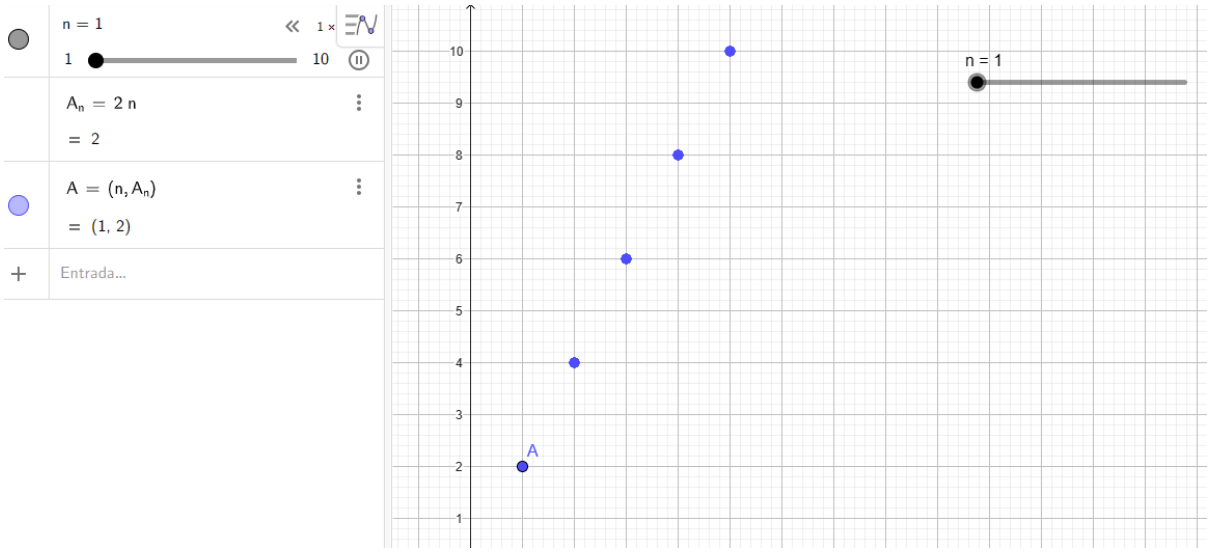
| Name Task | sequence | Rule or equation |
|---|---|---|
| 1.-Positive even numbers | 2,4,6,8.10…… | $a_n = 2n$ |
| 2.-Positive odd numbers | 1,3,5,7,9,….. | $a_n = 2n-1$ |
| 3.-Triangular numbers | 1,3,6,10,15…. | $a_n = \frac{n\,(n+1)}{2}$ |
| 4.-Tetraedic numbers | 1,4,10,20,35… | $a_n = \frac{n\,(n+1)(2n+1)}{6}$ |
| 5.-Pentagonal numbers | 1,5,12,22…. | $a_n = \frac{n\,(3n-1)}{2}$ |

| | | |
|---|---|---|
| 6.-Hexagonal numbers | 1,6,15,28…. | $a_n = n(2n-1)$ |
| 7.-Perfect squares | 1,4,9,16,25… | $a_n = n^2$ |
| 8.-Perfect cubes | 1,8,27,64,125… | $a_n = n^3$ |

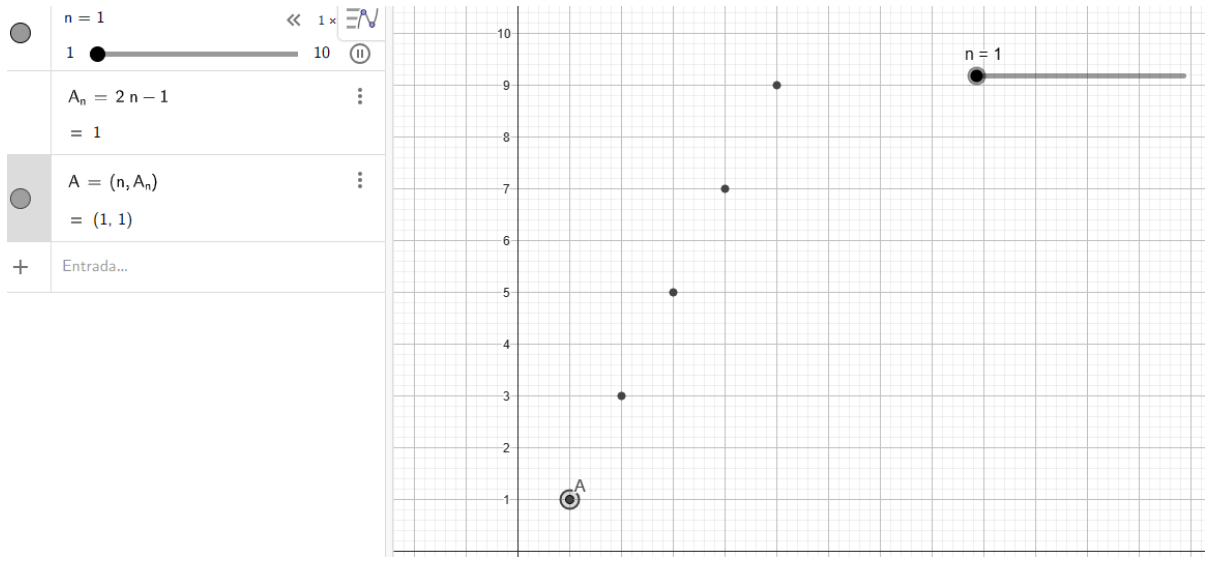
Figure 1. Adapted from https://matemathweb.com/razonamiento-matematico/sucesiones-numericas/

Some graphed numeric sequence tasks are shown in GeoGebra.

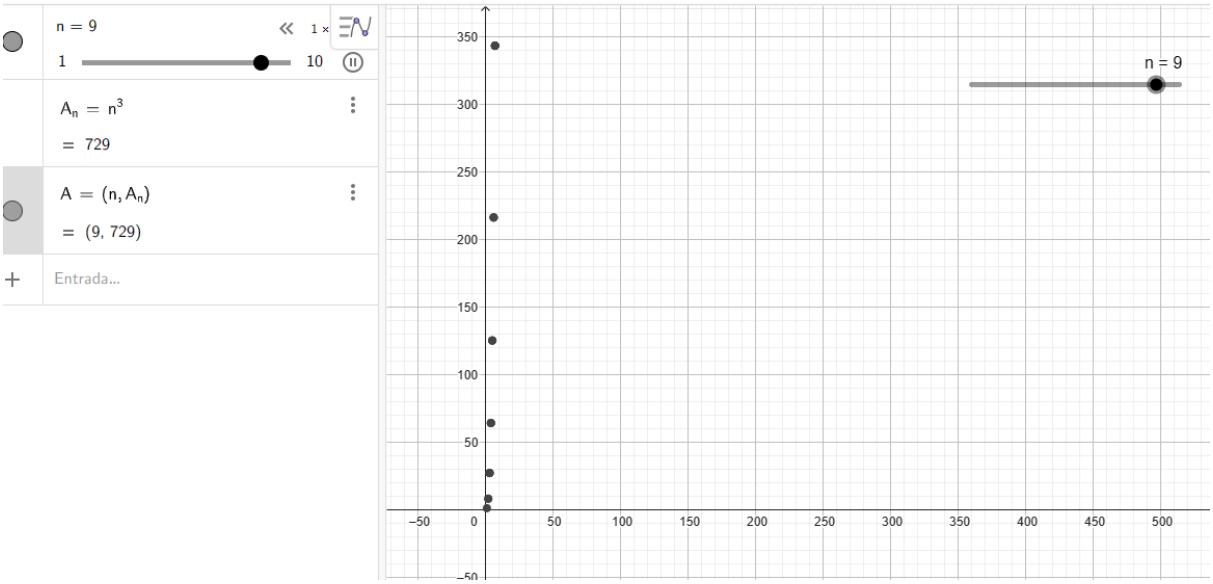
Task 1. Positive even numbers




Task 2. Positive odd numbers



Task 8. Prefect Cubes




## 6. Conclusions

Duval's theory is a pedagogical proposal that seeks to strengthen the learning of numerical sequences using tools such as GeoGebra. GeoGebra is interactive mathematics software that allows you to explore and visualize mathematical concepts.

Duval's proposal is based on the idea that learning numerical sequences benefits from combining graphical, algebraic and numerical representations. With the help of GeoGebra, students can visually manipulate numerical sequences, observe patterns and relationships, and perform numerical experiments.

GeoGebra allows the graphical representation of sequences, which helps students better visualize their properties and characteristics. In addition, the software allows algebraic calculations related to sequences, allowing students to explore how they are generated and how they evolve as they progress through the terms.

GeoGebra's interactivity and the ability to experiment with different values and settings allow students to develop a deeper understanding of numerical sequences. Furthermore, by combining the visual, algebraic and numerical aspects, a more complete and holistic understanding of sequences is promoted.

Likewise, Duval's theory focuses on the use of GeoGebra to reinforce the learning of numerical sequences, taking advantage of its ability to represent graphically, perform algebraic calculations and allow numerical experiments. This provides students with a more interactive and enriching learning experience.

This discussion answers the research question. And with this Learning Trajectory Line proposal, we contribute with a tool for teaching numerical sequences, based on Duval's Theory, and it is proposed to be used in Mexican schools.

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