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Hypothetical Learning Trajectory that Integrates Dog Feces Recycling Project for Teaching Algebraic Equations with GeoGebra Aimed at Use in High School Students in the New Mexican School Context

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Abstract

The teaching of mathematics in the context of the New Mexican School seeks to integrate innovative approaches that connect learning with the daily reality of students. In this sense, the proposal of a hypothetical learning path (HLP) that integrates a dog feces recycling project for the teaching of algebraic equations is presented as a relevant didactic alternative. Integrating a practical project such as dog feces recycling into the teaching of algebraic equations offers a unique opportunity to connect mathematical concepts with social and environmental realities. This methodology not only promotes meaningful learning, but also fosters civic values among students, aligning with the principles of the New Mexican School. At the end of the project, students will have developed skills to solve algebraic equations and apply mathematical concepts to real situations, in addition to having contributed to the improvement of their environment.

Keywords: teaching of mathematics, hypothetical learning trajectory, GeoGebra, teaching algebraic equations, Mexican School context

1. Introduction

The teaching of mathematics in the context of the New Mexican School seeks to integrate innovative approaches that connect learning with the daily reality of students. In this sense, the proposal of a hypothetical learning path (HLP) that integrates a dog feces recycling project for the teaching of algebraic equations is presented as a relevant didactic alternative. This approach not only promotes the meaningful learning of mathematical concepts, but also fosters environmental awareness and social responsibility among young people. The teaching of mathematics in the current context of the New Mexican School faces the challenge of being relevant and meaningful for students. In this framework, the proposal to develop a hypothetical learning path (HLP) that integrates a dog feces recycling project as an innovative didactic tool arises. This approach not only seeks to teach algebraic equations, but also to connect students with their social and environmental surroundings, fostering an ecological awareness that is crucial in the integral formation of the individual. (HLP) is based on the idea that learning is a dynamic process, where students build their knowledge from previous experiences and meaningful contexts. By integrating a practical project such as recycling dog feces, students are provided with a real-life scenario where they can apply mathematical concepts, specifically algebraic equations, to solve concrete problems. This method allows students to not only learn how to handle equations, but also understand their application in everyday situations, which enhances their interest and motivation for the subject. In addition, this approach promotes collaborative learning, where students work in teams to collect data, analyze it, and present their findings. In this way, not only the development of mathematical skills is encouraged, but also social and civic competencies, aligning with the educational objectives of the New Mexican School. The combination of mathematics and environmental education in this proposal offers a unique opportunity to form responsible

citizens who are committed to their community and the environment. The use of technology such as GeoGebra, already used in teaching mathematics, is used to integrate it into the learning process.

1.1 Objectives

1) Understand the concept of algebraic equations and their application in real situations.

2) Develop skills to solve algebraic equations in a logical and systematic manner.

3) Promote environmental awareness and responsibility in waste management.

4) Promote interdisciplinarity and collaboration between students and the community.

In summary, this hypothetical learning path represents a creative and effective response to current educational needs, by integrating mathematical content with projects that directly impact the social and environmental reality of students.

2. Theoretical Framework

2.1 Hypothetical Learning Trajectories

Hypothetical learning trajectories are a construct that allows planning the educational process based on students' prior conceptions. Simon (1995) defines a HLT as a vehicle for organizing learning, composed of hypothetical goals, tasks, and processes that guide students in their mathematical understanding. This approach has been successfully used in various investigations to structure activities that facilitate the understanding of complex concepts (Daro et al., 2011; Edgington et al., 2016). Recent research on hypothetical learning trajectories (HLT) for algebraic equations highlights their potential to improve mathematics education. Studies have explored HLTs for several algebraic concepts, including algebraic expressions (Amador Saelices & Gámez, 2016), quadratic equations (Pereira & Proença, 2023; Morales Carballo et al., 2022), and systems of linear equations (Cárcamo & Fuentealba, 2023). These studies emphasize the importance of error analysis and problem-solving approaches in designing effective HLTs (Amador Saelices & Gámez, 2016; Pereira & Proença, 2023). The use of dynamic software, such as GeoGebra, has been proposed to improve the understanding of quadratic functions and equations (Morales Carballo et al., 2022). Likewise, Negrete (2024) carried out an HLT where he proposes the use of the aforementioned software for teaching numerical sequences. Researchers have also developed models for constructing preliminary HLTs, particularly for college-level courses (Cárcamo & Fuentealba, 2023). Overall, these studies suggest that well-designed HLTs can help prevent common errors, provide context for mathematical concepts, and improve students' understanding of algebraic equations at various educational levels.

2.2 NEM, Environmental Education and Mathematics

The New Mexican School (NEM) aims to improve the quality of education by integrating knowledge in the classroom (Alvarado Monroy et al., 2024). This approach aligns with ongoing efforts to improve mathematics teaching in secondary schools, where challenges persist despite various proposed strategies (Sánchez Moreno et al., 2024). NEM emphasizes interdisciplinary learning, community-based knowledge, and project-based work (Alvarado Monroy et al., 2024). It builds on previous initiatives, such as the incorporation of information and communication technologies in mathematics and science education (Rojano, 2003). NEM seeks to transform schools by focusing on the comprehensive development of students and revaluing the teaching profession (Secretaría de Educación Pública, 2019). This new educational model aims to create a more harmonious, inclusive, and productive society, reflecting the broader goals of Mexico' s "Fourth Transformation" (Secretaría de Educación Pública, 2019). Implementation of the NEM involves the collaborative design of learning sequences and teacher training to ensure effective integration of knowledge across disciplines.

Environmental education has become an essential component in the comprehensive training of students. Integrating practical projects, such as recycling dog feces, not only addresses ecological problems, but also allows students to apply mathematical concepts in real-life situations. The connection between mathematics and the environment can increase motivation and interest in learning (Ivars, Fernández & Llinares, 2020).

2.3 GeoGebra

GeoGebra is a software widely used in teaching mathematics, in Mexico. Carballo et al. (2022) and Negrete (2024) have used it in teaching mathematics of various mathematical objects. Algebraic Equations in Context. On the other hand, Duval' s Theory of Semiotic Representation is endorsed, since the use of this technology allows the student to use the graphic representation record and algebraic representation of the object that is intended to be taught.

The study of algebraic equations is fundamental in the secondary mathematics curriculum. Through contextualized activities, students can develop critical problem-solving skills and apply their knowledge in everyday situations. The proposed THA will focus on how equations can be used to analyze data related to the recycling project, thus promoting active and collaborative learning (Fernández & Choy, 2020).

2.4 Basic Concepts of Algebraic Equations

Definition: Explain what an algebraic equation is. An algebraic equation is a mathematical expression that establishes equality between two algebraic expressions. These expressions can contain numbers, variables (letters that represent unknown values), and mathematical operations such as addition, subtraction, multiplication, and division. Components of an Algebraic Equation.

2.5 Elements of an Equation

1) Variables: These are symbols (usually letters such as x, y, z) that represent unknown values.

2) Constants: These are fixed numbers that do not change.

3) Operations: These include addition (+), subtraction (−), multiplication (×), division (÷), and other more complex operations such as exponents.

4) Equality: The equal sign (=) indicates that what is on the left is equal to what is on the right.

2.6 Methodological Framework

The methodological proposal is based on the implementation of a THA that articulates practical activities related to recycling and the use of algebraic equations. The following steps will be followed:

1) Definition of the objective: Understand and apply algebraic equations in the context of recycling.

2) Selection of tasks: Design activities that involve the collection and analysis of data on the amount of dog waste recycled.

3) Evaluation of the process: Implement formative evaluations that allow activities to be adjusted according to the needs and progress of the group.

2.7 Proposal

The proposal consists of carrying out a school project where students collect data on dog feces in their community. Through statistical analysis and the formulation of algebraic equations, they will be able to calculate:

1) The total amount of waste collected.

2) Proportions and percentages related to its recycling.

3) Projections on the environmental impact of recycling.

2.8 Activities

1) Research and diagnosis: Students will research the environmental impact of dog feces in their community and diagnose the current waste management situation in their school or neighborhood.

2) Designing a recycling system: In groups, students will design a dog feces recycling system that includes collecting, processing, and transforming waste into useful products (such as compost or biogas).

3) Algebraic modeling: Students will create algebraic models to describe the recycling process, including variables such as the amount of dog feces collected, processing time, number of products generated, etc.

4) Solving equations: Students will solve algebraic equations related to the recycling process, such as:

• How many tons of dog feces are needed to produce 100 kg of compost?

• How long does it take to process 500 kg of dog feces?

• What is the cost of producing 1 kg of biogas from dog feces?

1) Simulation and Analysis: Students will simulate different recycling scenarios and analyze the results to assess the efficiency and environmental impact of their recycling system.

2) Presentation and Reflection: Students will present their projects and reflect on what they learned, the challenges they faced, and possible improvements for their recycling system.

2.9 Assessment

1) Portfolio of Work: Students will maintain a portfolio of their work, which will include their research, designs, algebraic models, equation solving, and simulations.

2) Final Presentation: The final presentation of the projects will be evaluated based on the clarity, creativity, and coherence of the proposal.

3) Participation and Collaboration: Active participation and collaboration between students will be evaluated throughout the project.

3. Discussion and Results

3.1 Exercise 1 Solving Equations

Practical Problem: If feces are collected and composted to produce fertilizer, how many kilograms would be needed to fertilize a garden that requires 10 kg?

Enter the data from the table in GeoGebra and obtain the equation

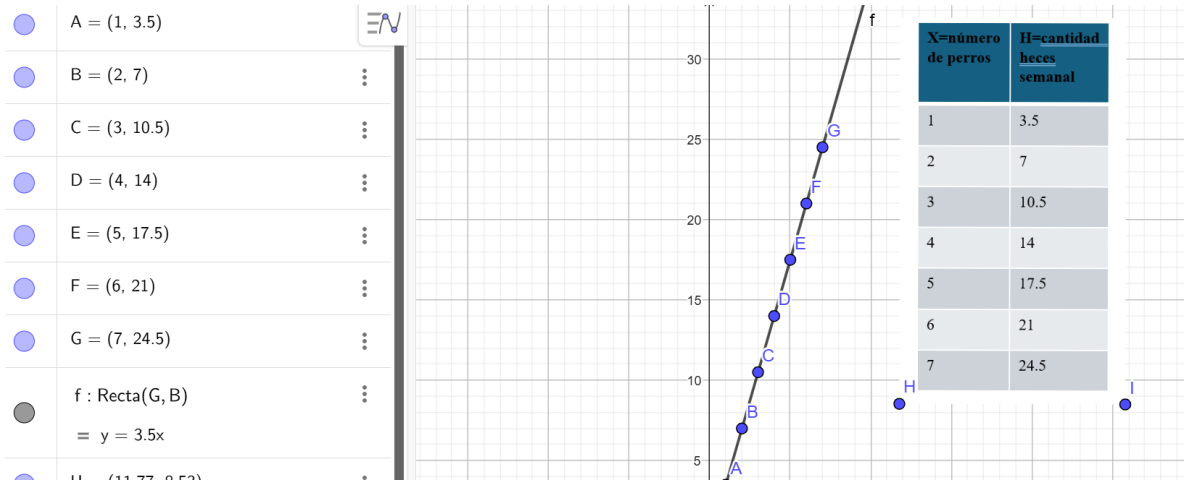

Figure 1. Table and drawing in GeoGebra to obtain the algebraic formula of exercise 1

3.2 Mathematical Modeling of the Composting Process

Students can model the composting process using equations that represent the relationship between the materials used (leaf litter, kitchen waste, and dog feces). Sample Problem: If 5 liters of leaf litter, 5 liters of kitchen waste, and 1 liters of dog feces are used to make compost, how much total material is needed to produce z liters of compost?

Equation:

x=5+5+y

Activity: Students can investigate how many liters are required for different amounts of compost and solve the equation to find y.

Input the data from the table into GeoGebra and obtain the equation

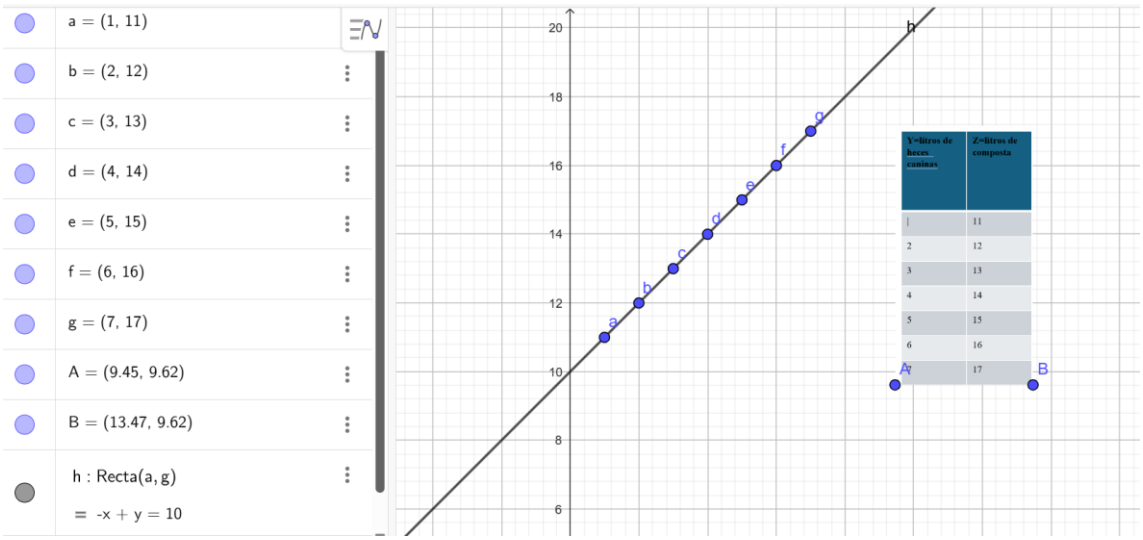

Figure 2. Table and drawing in GeoGebra to obtain the algebraic formula of exercise 2

4. Conclusions

Integrating a practical project such as dog feces recycling into the teaching of algebraic equations offers a unique opportunity to connect mathematical concepts with social and environmental realities. This methodology not only promotes meaningful learning, but also fosters civic values among students, aligning with the principles of the New Mexican School. This hypothetical learning trajectory seeks to integrate the teaching of algebraic equations with dog feces recycling project, fostering interdisciplinarity, environmental awareness, integrating GeoGebra software and collaboration among students. At the end of the project, students will have developed skills to solve algebraic equations and apply mathematical concepts to real situations, in addition to having contributed to the improvement of their environment.

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