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# Study on Mathematical Concept Teaching in High School Based on 5E Instructional Model

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**Abstract**

Mathematics knowledge system centers around mathematical concepts. The ultimate goal of mathematical concept teaching is to stimulate students' innovative thinking and mathematical capabilities through their exploration and deep comprehension of the essence of concepts. 5E instructional model is a constructivism-based explorative teaching model, emphasizing the exploration of independently constructing knowledge by students, which is in line with the requirements of the curriculum standards on concept teaching. Taken the concept teaching of trigonometric functions as an example, this paper designs the 5E instructional model of mathematical concept teaching in high school. Moreover, the study provides educational researchers evidence and reference that can be developed to improve teaching strategies in the process of implementing 5E instructional model.

**Keywords:** mathematical concept teaching, 5E instructional model, high school mathematics

## 1. Introduction

*Curriculum Standards for High School Mathematics (2017 edition)* stipulates that the core of mathematics teaching is concept teaching, the key is to explore the essence of concepts, and the goal is to raise the status of mathematical concept teaching and cultivate students' core mathematical literacies via structural and situational curriculum. The new curriculum reform also proposes that concept teaching is a vital link in mathematics teaching, and students should be guided to explore and create mathematical concepts to deeply understand the nature of concepts (General Office of the State Council, 2016).

5E instructional model is a constructivism-based explorative teaching model, emphasizing independent exploration, communication, and collaboration of students. It requires students' self-construction as a way of improving their cognitive level. Studying new concepts is reconstructing the internal cognitive structure. Therefore, 5E instructional model can develop students' explorative capability and help them construct scientific mathematical concepts, which is beneficial to their life-long development.

Through designing the process of mathematical concept teaching based on 5E instructional model, this paper analyzes the strategies of applying 5E, i.e., engagement, exploration, explanation, elaboration, and evaluation to mathematical class. The analysis takes the concept of trigonometric function as an example and designs a 5E-based mathematical concept teaching case, hoping to provide a reference for teachers and contribute to studies on 5E instructional model in China.

## 2. Process Design

5E instructional model is designed based on STEM education (Science, Technology, Engineering, and Mathematics). Using 5E in mathematical concept teaching enables students to independently build knowledge system through each 5E link and encourages them to explore the nature of mathematical concepts. The core of 5E instructional model is exploration. Under this model, engagement, explanation, elaboration, and evaluation are introduced into the teaching activities, as shown in Figure 1.

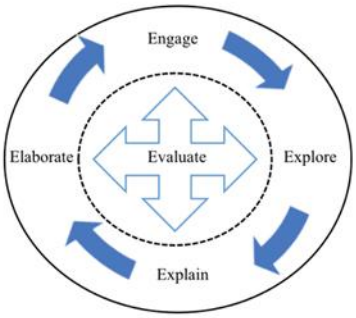


Figure 1.

*2.1 Engagement*

Engagement is the lead-in link of 5E. In the practical teaching process, with the goal of igniting students' desire to explore on their own, teachers should attract students to situations created and designed in class through mathematical history and culture, fun situations, and explorative activities.

Teachers should arouse students' interest to explore by virtue of exterior carriers. Mathematical concepts are abstract knowledge, and students cannot deeply comprehend the essence of them only through separated memory. So, teachers should use exterior carriers to lay a solid foundation for students to dig into the essence of concepts such as integrating mathematics class with real life, penetrating mathematical culture into teaching, and employing teaching aids. For example, when teaching the concept of exponential function, teachers can put forward an explorative question by virtue of historical stories to ignite students' enthusiasm, or they can present the image of exponential directly with geometric drawing board.

Teachers need to provoke the cognitive conflicts of students combined with their psychological characteristics. Teachers should grasp the students' internal psychology, assign inquiry questions according to their proximal development zone, teaching through analogy, and generate the cognitive conflicts of students, to drum up their enthusiasm for learning new concepts and enhancing their study effect. For example, as for the concept of logarithmic function, the expression and meaning of it can be summarized by analogy with the process of delving into exponential functions, and as for the concept of imaginary number, if students cannot work out the value of $x$ when based on what they have learned, it would be natural and necessary to introduce the imaginary numbers.

*2.2 Exploration*

Exploration is the core link of 5E. Based on the situations created and designed as well as the questions put forward by teachers, students can not only acquire new knowledge experience, but also master new skills and techniques in the process of solving problems through independent or cooperative exploration (Zhao Chengling, Zhao Wenjun & Jiang Zhihui, 2018).

Teachers should specify the purpose of exploration. Time for each class is limited. Students-centered exploration is not equal to blind and haphazard exploration. No specific purpose or reasonable question, no effective exploration. For example, when studying the concepts of universal and existential quantifiers, students are supposed to explore the logical similarities and differences between phrases that qualify the range of values of variables in multiple propositions, so that they can perceive the definitions through concrete examples only (Zhang Fei, Yu Mengfei & Gu Jiling, 2021).

Students need to experience the exploration process. After clarifying the exploration purpose, teachers should also regulate the class when students are allowed to independently explore: observing students' explorative activities, reminding and helping them timely, letting them experience the whole process of exploring and creating mathematical concepts and finish the explorative activities. For example, when explaining the concept of zero of a function, teachers can ask students to explore the relationship between the root of quadratic equation with one unknown and the intersection of the function graph with the $x$-axis. Then, students can continue to explore the relationship between the function, the equation and the zero of a function, going through the whole process of exploration (Meng Biao, 2020).

*2.3 Explanation*

Explanation is the improving link of 5E. In this link, students first try to summarize the essence of a concept by themselves, and the teacher then uses mathematical language to standardize the expression of concepts summarized by students. This link combines students' logical reasoning with the teacher's elaboration to help students internalize the essence of concepts.

Students explain the concepts first. Having experienced the explorative process, students can make bold guesses about the conclusions, then test their own guesses, and finally summarize the new concepts via self-explanation or group discussion. Based on students' cognitive level, the whole process develops their verbal skills, and improves their mathematical core literacies as well as capabilities such as logical reasoning, analogy, and mathematical abstraction. For example, when studying the concept of arithmetic progression, students are asked to summarize the concept based on their observation and exploration.

Teachers supplement or correct the students' explanation next. The cognitive level of students is limited, so teachers need not only to correct any obvious mistakes in the concepts summarized by students, but also to redefine the concepts in standard mathematical language and to give precise and concise concepts. Teachers can also introduce the background of the concept when supplementing it, employ a carrier to make students understand the essential properties and extensions of the concept, and help them assimilate the new concept. For example, when explaining the concept of logarithmic functions, teachers need to standardize the general expression of logarithmic functions in the language of mathematical notation.

*2.4 Elaboration*

Elaboration is a delicate link of the 5E. It further deepens the concepts and improves as well as expands students' internal cognitive structure. The vertical and horizontal elaboration allows students to construct a more complete knowledge network map and learn to see a problem from the mathematical perspectives (Li Yaoguang & He Xiaoya, 2010).

Vertical elaboration: Mathematical concepts are not single or independent. Instead, concepts are inseparable from each other. Guiding students to associate the new concept with those they already know can help them transfer and apply what they have learned, so as to build a broader and more complete system of mathematical knowledge. For example, the concept of function is the general concept of linear and quadratic functions that students have learned in junior high school, and it is also the subordinate concept of exponential functions that will be learned later. Similarly, the logarithmic functions will be learned by analogy with the exponential functions.

Horizontal elaboration: Mathematics is the fundamental subject of STEM education, but it also interpenetrates with other disciplines. The STEM education philosophy is to develop integrated and interdisciplinary thinking of students, who comprehend concepts to apply them to other disciplines and to apply mathematical ideas to other fields and real life after learning to identify mathematical problems. For example, students can look at the world from the perspective of functions: The sand pendulum experiment in physics, the repetition of musical melodies and the spinning Ferris wheel are related to periodicity in mathematical functions (Lin Jingrong, 2022).

*2.5 Evaluation*

Evaluation runs through 5E. It allows teachers to know how much students understand the concepts. Teachers should not only summarize the class at the end of the course, but also intersperse evaluation in each 5E link with timely feedback, which is formative assessment and can further improve teaching effects.

Teachers' auxiliary and instructional evaluation: By monitoring and observing the whole lesson, teachers can evaluate students in time with teaching objectives and contents as the evaluation criteria, which help them understand students' learning status, correct the teaching and learning directions according to the evaluation results, provide students with thinking strategies, and reflect on what they have taught to improve the teaching process.

Mutual evaluation among student groups: Teachers also need to encourage self-evaluation or mutual evaluation among students and groups to enhance students' awareness of participation and promote the completion of exploration. In the evaluation after exploration, test papers can be used to examine how much students have mastered, the development of students' scientific spirit, and the improvement of innovation and cooperation ability after the exploration-based teaching.

## 3. Case design

This paper takes the first lesson of Section 2, Chapter 5, *"The Concept of Trigonometric Functions"* in the first compulsory experimental mathematics textbook under the general high school curriculum standards (PEP edition) as an example to present the specific application of the 5E instructional model in mathematical concept teaching practice.

*3.1 Engagement: Creating and Design Situations to Arouse Students' Interest and Doubt*

In this case, multimedia technology is integrated at this stage to stimulate students' motivation through visual materials. And real-life related problems are also raised to let students understand the application of trigonometric function concept. The actual teaching clip is as follows:

**Teacher:** We know that functions are important mathematical models to describe the laws of change in the objective world; for example, our uniform linear motion can be represented by linear functions, projectile motion can be represented by quadratic functions, and the exponential explosion, logarithmic growth, as well as other mathematical models of various phenomena that we learned before can also be described by functions. However, in real life, there is also a phenomenon that goes round and round, such as the Ferris wheel.

The teacher shows the spinning Ferris wheel through the whiteboard, and students enjoy the animation to feel the situation intuitively.

**Teacher:** The Ferris wheel is always in uniform linear motion, so what mathematical model should we use to describe such motion? This is related to the trigonometric functions we are going to study in this lesson.

**Teacher:** As in Figure 2, now a Ferris wheel is rotating counterclockwise at a constant speed. Assume that the radius of the wheel is 1 and the height of its center $O$ above the ground is 2.

Figure 2.

**Teacher:** Now we abstract this model, as in Figure 3 and assume that initially the passenger cabin $P$ is located at $A$ on the same horizontal line as $O$. When the point $P$ rotates counterclockwise by an angle $\alpha$, the position of the point $P$ from the ground is $h$. Now please consider the following questions:

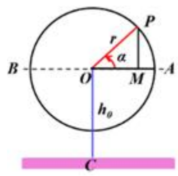


Figure 3.

**(1)** If $0 < \alpha < 90^{\circ}$, then $h = \underline{h_0 + |PM| = 2 + 1 \cdot sin\,\alpha}$.

**(2)** If $\alpha$ is an arbitrary angle, can $h$ still be expressed by an equation containing $\alpha$?

Does $h = 2 + 1 \times sin\,\alpha$ still hold?

Then, all students ponder the above questions actively and independently.

**Teacher:** Since 1 and 2 are fixed values in the above equation, it is necessary to discuss the value of $sin\,\alpha$ to make the above conclusion hold.

The Engagement stage starts from a real-life example and lets students visualize the correspondence between the height $h$ of the passenger cabin from the ground and the angle of rotation $\alpha$ through animation, and then stimulates students' curiosity through two progressively deeper problems. Students can review the trigonometric functions of acute angles in junior high school and experience the process of thinking from the particular to the general. With the teacher's further guidance, students generate new thoughts, leading to the next step of exploration.

*3.2 Exploration: Construct Concepts Through Independent Exploration*

This case allows students to experience the process of constructing a complete mathematical concept through both problem-based and experimental exploration, while strengthening students' cooperative skills and rigorous scientific attitudes. The actual teaching clip is as follows:

**Teachers:** How can the position of the point $P$ in the unit circle be inscribed by the angle $\alpha$? Because the position of the point $P$ is changing as the angle $\alpha$ rotates, we classify the discussion according to the position of the point $P$. First, as in Figure 4, when the point $P$ is above the line $AB$, $h = 2 + |PM|$. According to the exploration before class, in the unit circle, when $\alpha$ is an acute angle, $h = 2 + 1 \times \sin\alpha = 2 + \sin\alpha$, then it is easy to know $\sin\alpha$=$|PM|$.

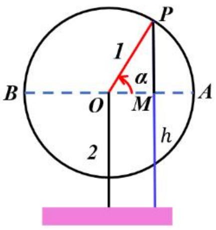


Figure 4

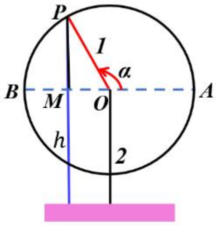


Figure 5

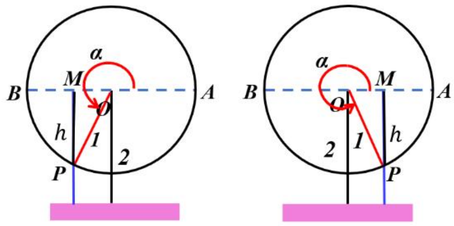


Figure6

**Teachers:** When $\alpha$ is obtuse, as in Figure 5, for unity, it is desired that $h$ is still expressed by $2 + \sin\alpha$, so $2 + |PM|$ is combined. Because of the symmetry of the circle, $\sin\alpha$ can be taken as $|PM|$ to satisfy the condition, i.e., $\sin\alpha$=$|PM|$. Similarly, when the point $P$ is below the line $AB$, as in Figure 6, $h = 2 - |PM|$ when $\pi < \alpha < 2\pi$. Similarly, to satisfy the condition of $h = 2 + \sin\alpha$, $\sin\alpha$ can be taken as $-|PM|$ to satisfy the condition, i.e., $\sin\alpha$=$-|PM|$. Therefore, $\sin\alpha$=$\pm|PM|$ can be defined to satisfy the condition.

All students think about the case when $\alpha$ is an obtuse angle by recalling the case when $\alpha$ is an acute angle explored before class, and then the teacher leads students to consider how $\sin\alpha$ can be taken to satisfy $h = 2 + \sin\alpha$. Students independently think about the case when the point $P$ is below the line $AB$ and how $\sin\alpha$ can satisfy $h = 2 + \sin\alpha$ by analogy with the case when $\alpha$ is an obtuse angle.

**Teachers:** Therefore, it can be concluded that $\sin\alpha = \pm|PM|$. The above is a categorical way of using the length of the line$PM$ to inscribe the position of the angle $\alpha$ in relation to the point $P$. What other methods can be used to inscribe their relationship? What quantity can be used to replace $\pm|PM|$ uniformly, regardless of the case, while satisfying the requirements for the sign?

In the history of mathematics, the great mathematician Descartes once said that all problems can be transformed into mathematical problems, all mathematical problems can be transformed into algebraic problems, and all algebraic problems can be transformed into equation problems. And equation and function problems can be transformed into each other, and the essence of function problems is to establish a rectangular coordinate system to present the quantitative relationship in these graphs. The teacher can lead the students to consider what else they can use to represent the value of $\sin\alpha$ other than representing it with numerical value from a geometric point of view.

**Teacher:** Do you have any ideas?

**Students:** We can build a rectangular coordinate system.

**Teacher:** How to build the system would facilitate us?

**Students:** Establish a rectangular coordinate system with $O$ as the origin, the line $AB$ as the $x$-axis and the line $OC$ as the $y$-axis. As in Figure 7.

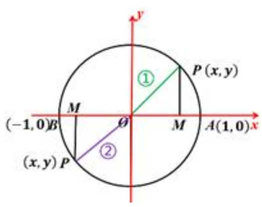


Figure 7

**Teacher:** Now please build a rectangular coordinate system on your own and think about how we can express the value of $\sin\alpha$.

**Students:** When the point $P$ is above the $x$-axis, $\sin\alpha=|PM|$. We can directly represent $|PM|$ with $y$, i.e., $y=\sin\alpha$. When the point $P$ is below the $x$-axis, $\sin\alpha=-|PM|$. In this case, it is also possible to use $y$ to express $-|PM|$directly, i.e., $y=\sin\alpha$. Therefore, the vertical coordinate $y$ of the point $P$ can be used to replace $\pm|PM|$. This leads to the conclusion that $y=\sin\alpha$.

At this stage, teachers guide students to transform the "length" relationship into "coordinate" relationship through mathematical modeling, classification discussion and other mathematical ideas, which is the most critical step in the expansion of trigonometric functions. Teachers should let students go through the whole process of exploration, transform their thinking into higher-order level, and ease the plight of transforming algebraic thinking into functional thinking.

*3.3 Explanation: Form Concept Through Deep Exploration*

In this case, after the students have constructed new concepts, the teacher consolidates the concepts in time to help them express mathematical concepts scientifically and accurately. The actual teaching clip is as follows:

**Teacher:** Is the above y=sinα a functional relationship expression?

Students are not sure if the answer is yes or not currently: some students think it is, but some think it is not.

**Teacher:** Let's review what is a function?

**Students:** If you take any number $x$ in the non-empty set of real numbers $A$, there is a unique $y$ in the non-empty set of real numbers $B$ that corresponds to it according to some correspondence, then $y$ is a function of $x$.

**Teacher:** So, for any angle $\alpha$, is there a unique point $P$ to which it corresponds?

Then the teacher uses the teaching tool Geogebra to draw a moving diagram as in Figure 8 and asks students to observe directly. Students, led by the teacher, discover the one-to-one correspondence between the angle $\alpha$ and the coordinates of the point $P$. Thus, they learn that there is a functional relationship between the angle $\alpha$ and the coordinates of the point $P$.

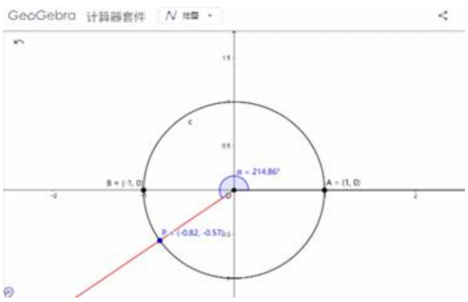


Figure 8.

**Teacher:** Should the angle $\alpha$ be in radian or angle system?

**Student:** Since $\alpha$ is an arbitrary angle and the function requires the set of real numbers, the radian system should be chosen.

**Teacher:** Please try to generalize the concept of sine function.

Then the teacher asks the students to stand up and present their concepts, but their generalization is not complete enough, so the teacher makes supplements.

Let $\alpha$ be an arbitrary angle, $\alpha\in R$, whose terminal side $OP$ intersects the unit circle at the point $P(x,y)$, and call the vertical coordinate $y$ of the point $P$ the sine function of $\alpha$, denoted as $\sin\alpha$, i.e., $y=\sin\alpha$.

By guiding students to summarize the concept of sine function and using standardized mathematical symbolic language, the teacher further strengthens students' functional thinking, which not only alleviate the teaching difficulties of this lesson, but also further develops students' core literacy of mathematical modeling and geometric intuition capability.

*3.4 Elaboration: Solve Problems and Consolidate the Concepts*

In order to understand the new concepts more deeply, students need to transfer the acquired concept and skills to new problem situations. The actual teaching clip is as follows:

**Teacher:** Now we have mastered the concept of trigonometric functions, used coordinates to represent the position of a point, and redefined a new calculation relationship for numbers. Then for a specific number, such as $2/3\pi$, what should be its value of each trigonometric function? Please try to calculate these values.

The teacher lets students think independently first, asks several students to share their thoughts, leads students to reorganize their ideas, and standardizes the steps of their answers.

**Teacher:** So, do you know how to draw the position of the passenger cabin in the Ferris wheel in the exploration before class?

**Student:** With the help of trigonometric functions.

**Teacher:** That's right. So, after learning the concept of trigonometric function, now do you know what significance it has?

**Student:** It can describe the position of the point in the circular motion.

**Teacher:** Exactly. Circular motion is a kind of periodic motion. Accurately speaking, trigonometric function is an important model to describe the periodic change law in the objective world. And we will continue to learn its periodicity next lesson.

*3.5 Evaluation: Diverse Forms of Evaluation, Summary, and Reflection*

The evaluation stage allows teachers to know how much students understand the mathematical concepts and to adjust teaching methods to students' actual conditions. The actual teaching clip is as follows:

**Teacher:** What have you learned in this lesson?

**Teacher:** What mathematical ideas have you perceived?

## 4. Conclusion

Mathematical concept teaching is difficult for teachers in practice, and it is an important carrier to train students' mathematical thinking ability. However, firstly, in practice, most teachers separate the class from actual situations, which leads to students' inability to see the world through mathematical eyes; secondly, explorative activities are rarely organized in class, which leads to students' passive acceptance of the concepts given by teachers; finally, mathematics are seldom integrated with other subjects and information technology in class, which leads to students' inability to understand the meaning of concepts intuitively (Zhan haobo, 2010). This study uses the 5E instructional model as the basic structure and "exploration" as the core to build an exploration-oriented mathematics concept teaching model, which will eventually be put into practice, and it needs to go through a tortuous process from its introduction to full-scale promotion. Based on this, educational researchers should continue to improve teaching strategies in the process of implementing this model, combine theory and practice, promote the spiral progress of 5E instructional model, and accelerate the pace of education modernization.

## Fund Project

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## References

*Implementation Plan for the Outline of Nationwide Scientific Literacy Action* (2016-2020), General Office of the State Council.

Wang Jian, Li Xiuju. (2012). The Connotation of 5E Instructional Model and Its Implication for Science Education in China. *Bulletin of Biology*, *3*, 39-42.

Zhao Chengling, Zhao Wenjun, Jiang Zhihui. (2018). *Design of a 5E Exploration-based Teaching Model Oriented toward STEM education*. *Modern Education Technology, 28*(03), 106-112.

Zhang Fei, Yu Mengfei, Gu Jiling. (2021). A Study on the Presentation and Consistency of Concepts in Junior High School Mathematics Textbooks. *Journal of Mathematics Education*, *30*(05), 21-27.

Meng Biao. (2020). *A Cases-based Discussion of the Application of 5E Instructional Model in Mathematical Concepts Teaching*. *Fujian Middle School Mathematics*, *11*, 13-16.

Li Yaoguang, He Xiaoya. (2010). A *Cognitive Review of the "Spiral" Arrangement of Mathematical Concepts in the New Curriculum*. *Journal of Mathematics Education*, *19*(4), 12-14, 91.

Lin Jingrong. (2022). *A Design Example of "The Concept and Properties of Functions" Based on the Improvement of Thinking Ability*. *Maths Teaching in Middle Schools, 4*, 49-51, 55.

Zhan haobo. (2010). *A Preliminary Study on the Structure of Explorative Mathematical Concept Teaching in High School*. *Forum on Contemporary Education*, *36*, 71-72.

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