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Research on the Infiltration of Mathematical Culture in Class from the Perspective of Situated Cognition Theory

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Abstract

Mathematical culture contains mathematical ideas, methods, spirits, and their occurrence and evolution processes. Integrating mathematical culture into the mathematics class can develop students' thinking and promote their comprehension of knowledge. The situated cognition theory helps students connect mathematical culture with real life. This paper analyzes the possibilities of integrating situated cognition theory with mathematical culture and explores the practical paths for such integration.

Keywords: mathematical culture, situated cognition theory, high school mathematics

1. Introduction

China's Ministry of Education released the new curriculum standards for high school mathematics (2017 edition, revised in 2020) on April 21, 2022. According to its basic philosophy, we should select appropriate curriculum content for students, well deal with the relationship between mathematical core literacy and knowledge or skills, and emphasize the connection between mathematics and life as well as other subjects. We need to enhance students' ability to solve practical problems using mathematics while focusing on the penetration of mathematical culture (Ministry of Education of the People's Republic of China, 2020). Such a penetration into curriculum and teaching activities is necessary for both compulsory and optional courses since we can see more and more questions that incorporate mathematical culture are appearing in China's college entrance examination, making it urgent to integrate mathematical culture into mathematics class. In this way, students can be dominant in the learning process and change their cognitive structure regarding knowledge, while morally benefiting a lot from mathematical culture, which enables us to achieve the overall goal of "cultivating students with moral virtue" as mentioned in the new curriculum standards.

How can we infuse mathematical culture in class? Can we directly introduce the history of mathematics to students? The answer is no. If we simply list the time and main events, we can never touch them or arouse their interests since this means they just learn about a piece of history but cannot feel the humanistic value of mathematical culture. This paper therefore explores the infiltration of mathematical culture into mathematics class from the perspective of situated cognition theory and holds the belief that based on engagement in daily activities, we can introduce mathematical history into specific scenarios of teaching activities, so as to stimulate students' interest in learning mathematics and develop their emotions, attitudes, and values without distorting mathematical culture.

Traditional education now prevailing in China focuses more on the acquisition of basic knowledge and skills while emphasizing teachers' dominant roles. Considering the limited class duration, teachers have to prepare teaching resources in advance and understand the connotations of mathematical culture in depth before its infiltration, making the class more unpredictable. As a consequence, there are few practices about mathematical culture in mathematics class and also few examples of using situated cognition to infuse mathematical culture in class. In this paper, the authors discuss how we can use situated cognition theory in class and teaching activities to embody the virtue-cultivation value of mathematical culture.

2. Possibilities of Integration

Situated cognition theory derives from and further refines constructivism. It features a dialectical process of development from “outside” to “inside” and then from “inside” to “outside” (Xie Mingchu, 2009). When we place abstract and obscure mathematical problems in concrete life situations, students can naturally acquire and learn knowledge and build a knowledge structure in their minds. Meanwhile, mathematical culture allows students to know the ins and outs of knowledge development, especially the process of generating knowledge across history. Both situated cognition theory and mathematical culture are concerned with the construction and generation of knowledge, so their combination is feasible.

2.1 Relevance of Situated Cognition Theory to Mathematical Culture

Situated cognition is an important theory about study that facilitates meaningful learning and the translation from knowledge to real-life situations. The theory focuses on knowledge, cognition, and learning, with both the evaluation and perception of knowledge as its key features, and does not just see knowledge as a mental representation within the individual, but relates knowledge to socially specific contexts.

According to the situated cognition theory, any mathematical knowledge is relevant to specific situations. Theorems, formulas, and propositions divorced from situations are like soil that has lost its nutrients and lacks vitality. Similarly, the infiltration of mathematical culture into the mathematics class should be based on concrete contexts, thereby giving realistic meaning to abstract mathematical concepts, which then can be well used by students because they understand the meaning of these concepts. In this case, mathematical culture becomes a handy tool, which gives full play to the application value of mathematics.

2.2 Necessity to Infiltrate Mathematical Culture into Specific Contexts

Mathematical culture is a comprehensive discipline involving both arts and sciences. It has mathematical characteristics in addition to its inherent cultural ones, and they encompass both the mathematical ideas, spirit, language, methods, and viewpoints, as well as their formation and development; it also includes the contribution and significance of mathematics in human life, science and technology, and social development, as well as the humanistic activities related to mathematics (Yu Qingchun & Wang Xiaoqin, 2020). Therefore, mathematical culture covers extensive content that can be used as materials for textbooks, but its integrity and complexity also make general materials easy to be selected.

Students acquire knowledge mainly from textbooks, and they learn most of the mathematical knowledge to get prepared for exams. After graduation, they will forget what they have learn as time passes and cannot use mathematical knowledge to solve problems in their lives. The root cause for this is that emphasis is placed on knowledge rather than situation, skills rather than thinking, and results rather than process in math class. As a consequence, students fail to acquire mathematical thinking, research methodology, and reasoning ability. Therefore, it is both meaningful and necessary to infiltrate mathematical culture into specific situations in math class.

3. Ways of Infiltration

Under the new curriculum concept, both concrete mathematical skills and intangible mathematical culture should be the focus in the high school mathematics curriculum. Mathematical culture, as an indirect and invisible educational concept, can transform students’ thinking. To infiltrate mathematical culture into mathematics class, which cannot be done at one go and requires teachers’ continuous attempts and accumulation, one can start with a simple history and adapt historical stories before combing them with real-life situations, and then finally delve into the essence of mathematics, gradually unleashing the value of mathematical culture in educating and cultivating the young generation.

3.1 Using History to Deepen Students’ Cognition

Take “arbitrary angle” as an example (Rao Bin, 2017), making students understand the significance of arbitrary angles is the focus of this lesson and lays a solid foundation for them to realize the relationship between trigonometric functions and periodicity. The textbook uses rotary gear as an introduction; but it has been proved that if we start the lesson by introducing the fact that “trigonometric function is a model to depict the phenomenon of periodicity”, we can produce a better teaching effect. It not only reveals the need to learn the arbitrary angles, but also enables students to understand the practical significance of learning trigonometric functions.

First, we can use the Tang poem “Even a prairie fire cannot destroy the grass; it grows again when the spring breeze blows" to introduce the phenomenon that one thing moves in cycles, in mathematics which is called "periodicity", and guide students to think about similar phenomena in daily life. From this, they can feel that math originates from life and is used in life while it is also a sublimation of life. Second, we can introduce the sundial to students, the tool used by Chinese ancients to measure time as early as 3,000 years ago in the Zhou Dynasty. Specifically, we can quote the words written in the *Zhou Bi Suan Jing*, one of the oldest Chinese texts on astronomy and mathematics, "The shadow of the gnomon turns the longest in the winter solstice and shortest in the summer solstice." This can create a situation where the ancients observe the shadow to measure time, allowing students to experience the connotation of arbitrary angles. In this process, the penetration of profound Chinese culture can enhance students' cultural self-confidence, maximizing the application value of mathematics. Third, we can tell students that counterclockwise is considered to be positive since counterclockwise rotation is more common in our life by defining positive and negative angles through the onward and backward flows of time, and by using concrete life situations such as typhoon rotation and athletes' races, yet before this, they would hold the belief that clockwise rotation is positive.

Traces of mathematics can be found everywhere across the social development process. We can dig available historical materials, such as sundials, to highlight the social role of mathematical culture and find the mathematical knowledge these materials contain, thereby stimulating students' motivation to learn. Besides, by associating class with real life, we can explain the stipulation of positive and negative angles in detail while exploring the reasons behind together with students. In this way, we can correct their misunderstandings and deepen their cognition.

3.2 Context-Based Mathematical Culture Enhances Students' Understanding

In the lesson about "the concept of sequence of numbers" (Li Ling & Wang Xiaoqin, 2016), it is important and difficult for students to master the relationship between the sequence of numbers and functions. The textbook uses figured numbers proposed by Pythagoras, the ancient Greek mathematician, introducing mathematical concepts based on the history of mathematics. However, such fragmented materials can neither link old knowledge with new one nor attract students' interest. Instead, we can select other appropriate historical materials from mathematical culture. For instance, we can play a video on the changing phases of the moon, and display the Babylonian clay table that records the changing phases of the moon, which was used to guide people in farming, production, and life. This reveals the significance and the "social role" of the sequence of numbers. In addition, we can contextualize the problem of "property" in a book copied by the ancient Egyptian priest Amos, which is in fact a geometric progression, and give it a realistic context. For example:

A boss has 5 machines in the factory, each of which requires 5 workers to operate, and each worker can produce 5 products that can be sold for $5 each. How many machines, workers, and products does the boss have and how much money the boss can earn?

We can adapt mathematical culture to make mathematics more interesting. Take the classic Josephus problem as an example.

When the Romans captured their city, Josephus took refuge in a cave with 40 other Jews, one of whom was his friend. All but Joseph and his friend decided to commit suicide to avoid being tortured and humiliated by the enemy. Joseph didn't want to die, but he was smart enough not to express his thoughts directly. He saved himself through a game: All people formed a circle; then they numbered each position and randomly started counting from a certain position of the circle, anyone whose corresponding number is a multiple of "3" will be pulled out of the circle and be killed. The rest people would be renumbered and continue this process, and the last one left should commit suicide. He placed himself and his friend in positions No. 16 and 31 respectively, and eventually survived.

Discarding the violence of this historical tale while retaining the part related to the sequence of numbers, we can adapt the tale into a fun game by changing the punishment into buying chips for others:

We can randomly select ten students from the class and ask them to make a circle. Each of them will get a number ranging from 1 to 10. Then we can start to count 1, 2, and 3 repeatedly from the No. 4 student. On the counter of three, the corresponding student will be knocked out and the rest keeps repeating this process. The last student left needs to buy chips for the other nine students. Then we can ask students which position they would choose.

The above are examples of appropriately adapting mathematical culture to better facilitate teaching activities in class and allow students to feel the fun of mathematics. The law of changing phases of the moon and the game of "find the best position" in the circle embodies the essential nature of the sequence of numbers and entertaining features of mathematical culture. They can help students understand the essence of the sequence of numbers and distinguish between them and sets.

3.3 Trace the Origin of Mathematics and Replace Students in the Center of Teaching Activities

Take “the concept of function” as an example (Zhong Ping & Wang Xiaoqin, 2016), the definition of the function is the focus of the lesson, and also the cornerstone of the knowledge about algebra. It is crucial since grasping such a definition can help students understand the origin and development of functions, trace the origins of mathematics, summarize the essence of functions, and comprehend the concept of functions.

Since G.W. Leibaniz (1646-1716) put forward the usage of “function” to express the curve changes in the 17^{th} century, the concept of functions has been evolving constantly. John Bernoulli defined functions from the perspective of variables and constants, Leonhard Euler defined functions using analytic expressions and dependence, and then Dirichlet, the German mathematician used correspondence to define functions from the perspective of interval.

Looking back on the evolution of the concept of functions — from analytic expression, to dependence and correspondence of variables, and to the correspondence of sets — we can see that the concept of functions has been developing naturally. During actual teaching activities, we can first ask students to recall the functions they have learned in junior high school, so as to make them realize that “a variable varies according to the independent variable”, and to define functions using analytic expressions.

We can use real-life situations as an introduction, such as the change of temperature on a certain day, and the constantly changing stock. In these cases, we cannot use analytic expressions to depict the function, and this will trigger the cognitive conflict of students, making them eager to learn. Next, we can introduce Euler’s solution to this problem: “If the value of a certain quantity depends on another quantity, and when the latter changes, the preceding changes with it, so we can call the preceding quantity a function of the latter”, which involves the dependence. With the constant function $y = 0$ as a counterexample, it is clear that the value of x does not affect the value of y. We can guide students to think about the correspondence. From the perspective of sets, we can ask students to independently generalize the concept of function that “For every definite value of x on a certain interval, y has a uniquely definite value corresponding to it, and when x varies continuously, y varies with it, then y is called a function of x”. It is also the definition put forward by the Dirichlet.

Based on students’ understanding of the nature of functions, we can return to the application value of mathematics by providing examples associated with life situations. We can model the stopping distance under emergency braking. By analyzing the situation and the factors, we can know that the stopping distance under emergency braking is related to two functions, i.e., a function reflecting the relationship between time and the vehicle speed, and a relationship between the braking friction and the vehicle speed. Students will understand why it is important to define functions from the perspective of intervals, distinguish the difference between the definition of functions in junior high school and that in senior high school, understand that a much larger range of problems in life can be explained based on correspondences, and use correspondences from the perspective of sets to model problems and answer situational questions in life by solving abstract mathematical problems.

Gradually, with the mathematical situation in daily life as the introduction and with the sequence of mathematicians’ in-depth understanding of the function’s concept as the logic, from the perspective of mathematical culture, we can find the origin of knowledge regarding mathematical culture, penetrate its humanistic value in class, and ultimately return to the real life, which embodies the practical benefits of mathematical culture. We can encourage students to practice and apply what they have learned to real life, letting them appreciate the profound cultural charm contained in mathematics, enhancing their comprehensive literacy, as well as developing their moral emotions, who then can improve their views on the world, life, and value.

4. Conclusion

Teaching reform in mathematics class requires educationally orientated research on the mathematical history and culture, and the development of appropriate mathematical culture that can be well integrated into mathematics class, so as to cultivate students’ positive attitudes towards mathematics. Such integration can not only enliven the class atmosphere, but more importantly, let students understand the history and current situation of mathematics, as well as return to the essence of mathematical knowledge. Only by understanding the essence can they conduct in-depth research and make breakthroughs and innovations. However, the prerequisite for the rational use of mathematical culture is that teachers should understand mathematical culture, be good at applying the theory of situated cognition from the perspectives of mathematics and real life, mathematics and science and technology, as well as mathematics and the humanities and arts, so as to infiltrate mathematical culture into the mathematics class. This requires teachers to have solid professionalism, profound cultural cultivation, clever adaptability, grasp the connection between mathematical culture and situation-based teaching, and explore the “educational value” of mathematical culture in textbooks. This paper provides methods for infiltrating mathematical culture into class, ranging from the use of historical events and contextualization of mathematical culture to tracing back the origin of education, from the shallow to the deep, from the surface to the inside, in order to provide teachers with valuable ideas and practical means for creative teaching.

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